In the present project, following our research plan, we have done research and established a number of significant results in the following areas:

- Set-theoretic topology
- Descriptive set theory
- Real analysis and measure theory
- Combinatorics
- Model theory and philosophy of mathematics

We presented our results in 40 publications, almost all of which appeared or will appear in the leading international journals of these fields (6 of these papers have been submitted but not accepted as yet). Our research group consisted of the PI and 8 participants, moreover 2 PhD students. One of the students has received his degree in 2013 and is now a postdoc with us and the other is finishing momentarily. Both of them have contributed significantly to our results.

Members of our group participated at a large number of international conferences, four of us (Elekes, Juhász, Sági, Soukup) as plenary and/or invited speakers at many of these. We now give an overview of our results.

I. Set-theoretic topology

A topological space X is called κ-resolvable if it contains κ disjoint dense subsets, and maximally resolvable if it is $\Delta(X)$-resolvable where $\Delta(X)$ is the smallest size of a non-empty open set in X. We have continued in [38.] the study of resolvability properties of topological spaces. Our main result here is the following significant improvement of a theorem of O. Pavlov: If the extent $e(X)$ of a regular space X, i.e. the supremum of sizes of its closed discrete subspaces, is less than $\Delta(X)$, then the space is ω-resolvable. Earlier it wasn't even known if a regular Lindelöf space is 3-resolvable if all non-empty open sets in it are uncountable. (Note that the extent of a Lindelöf space is clearly countable.) The proof of this easily stated result is extremely complex, it takes up almost 20 pages.

We also established in [38.] a stepping up result which, in particular, implies that any regular Lindelöf space in which all non-empty open sets are of size ω_1 is even ω_1-resolvable. (This is remarkable because earlier it was not even known if such a space is 3-resolvable!) This also shows that our efforts to prove maximal resolvability of these spaces are not hopeless.

On the other hand, compact Hausdorff spaces are well-known to be maximally resolvable. This classical result was extended in [16.] where it was shown that every compact Hausdorff space is maximally G_δ-resolvable as well, i.e. it has as many disjoint G_δ-dense subsets as the smallest size of a non-empty G_δ subset.

In [32.] a completely new type of resolvability question (due to Barnabás Farkas) was studied: When can we partition any given base for a topological space into two bases? Spaces having this property are called *base resolvable.* Clearly, any space with an isolated point is not base resolvable; hence, in this part, by *space* we always mean a *dense-in-itself topological space.* In the first part of the article [32.] we showed that all members of certain natural classes of spaces are base resolvable. In the second part we construct non base resolvable spaces.

We started with general observations about bases and we prove that metric spaces and weakly separated spaces are base resolvable. Then we proved one of our main results: every regular (locally) Lindelöf space is base resolvable.

We also investigated base resolvability from a purely combinatorial viewpoint which leads to further results: every hereditarily Lindelöf space (without any separation axioms) is base resolvable and any base for a T_1 topology which is closed under finite unions can be partitioned into two bases, The second part of the paper started with the isolation of a partition property, denoted by $\mathbb{P} \to (I_\omega)^1_2$, of the partial order $\mathbb{P} = (\mathbb{B}, \supseteq)$ associated to a base $\mathbb{B}$ which is closely related to base resolvability. We constructed a partial order $\mathbb{P}$ with this property and deduced the existence of a T_0 non base resolvable topology (in ZFC).

Next we presented a ccc forcing (of size $\omega_{1 1}$) which introduces a first-countable, 0-dimensional, Hausdorff space X of size 2^{ω_1} and weight ω_1 such that X is not base resolvable.

Let $\mathcal{P}$ be any fixed property of subspaces of topological spaces. A space X is called $\mathcal{P}$-bounded if every subsspace of X has compact closure in X. In particular, a space is ω-bounded if every countable subset has compact closure in it.

In connection with some classical problems of P. Nyikos, we executed in [11.] a thorough study of numerous variations of this concept obtained by replacing countability with various other countability properties, like the (hereditary, weak) Lindelöf, the CCC, and σ-compact properties. All these follow from countability, hence the resulting "boundedness" properties are all stronger than ω-boundedness but weaker than compactness. In most cases, with basically one exception, we could produce (at least consistent) examples that distinguish these classes of spaces.

On the other hand, in [39.], we have investigated boundedness properties for certain strengthenings of countability, like "countable and discrete", "countable and nowhere-dense", or "second countable" that yield properties weaker than ω-bounded but still stronger than countably compact. Again, we could produce numerous new examples of spaces that distinguish the various classes of spaces having these boundedness properties and their combinations.

Moreover, we proved that the class of "countable discrete"-bounded ($= \omega D$-bounded) spaces is particularly interesting. For instance, if $\mathfrak{b} > \omega$ then countably tight regular ωD-bounded spaces are automatically ω-bounded, while the continuum hypothesis implies the existence of an even first countable, locally compact Hausdorff ωD-bounded space that is not ω-bounded. As an interesting by-product of the proof we showed that countably tight regular countably compact spaces are discretely determined, i.e. if in such a space a point x is in the closure of a set A then x is in the closure of a (countable) discrete subset of A. Earlier this was only known for countably tight compact spaces.

We also showed that ωD-bounded spaces behave much more nicely under taking products than countably compact spaces do.

It has been known that any non-isolated point of a compact T_2 space is the accumulation point of a discrete subset. In [37.] we produced ZFC examples of 0-dimensional (hence Tychonov) κ-bounded spaces (and also Lindelöf) spaces in which this fails, i.e. which have "discretely untouchable" non-isolated points. This shows that the above property of compact T_2 spaces breaks down if compactness is weakened in any way. We point out that the ω-bounded case of this result solves a more than 10 year old problem of Dow, Tkachenko, Tkachuk, and Wilson.

Topologists and analysts have often considered properties stating that a space has a *small* dense set. The most popular of them is *separability*, that is, the property of having a countable dense set.

Smallness conditions for dense sets other than separability have also been considered. A space is called *d-separable* if it has a dense set which is the countable union of discrete subsets. This property was introduced by Kurepa in his PhD dissertation under the name of *property* K_0 as part of his study of the Suslin Problem. d-separability has a much better behavior than separability: arbitrary products of d-separable spaces are d-separable, and for every space X there is a cardinal κ such that X^κ is d-separable. We introduced a natural property called *nwd-separability* which is obtained by replacing *discrete* with *nowhere dense* in the definition.

A new class of smallness conditions for dense sets has been introduced as part of the program known as *selection principles in mathematics* and attracted a lot of attention recently. The general idea is that a small dense set can be obtained by diagonalizing over a countable sequence of dense sets. In this way, one can define a selective strengthening of any of the properties we mentioned above: a space is *D-separable* iff for every sequence $\{D_n : n < \omega\}$ of dense sets there is a discrete set $E_n \subset D_n$ for every $n < \omega$ such that $\bigcup_{n<\omega} E_n$ is dense. We defined NWD-separability as a selective version of *nwd*-separability in a similar way and compare it with D-separability.

[19.] was devoted mostly to the analysis of these properties and via constructing a great wealth of examples.

We proved that $\omega^* \times 2^\omega$ is a compact space which is *nwd*-separable but not d-separable. We also answered a question of Tkachuk by showing that there is a Corson-compact space with non-d-separable square.

Next we dealt with selective versions of separability. We presented a new construction of a countable M-separable, non-R-separable space which also serves as an answer to a question of Hutchinson We present a general framework to deal with selective separability properties and conclude that the class of D-and NWD-separable spaces are close under finite unions.

Our next aim was to construct ZFC examples separating the newly introduced properties. We presented an NWD-separable space which is not d-separable and countable, dense subsets of $2^{\mathfrak{c}}$ which are not NWD-separable. We finish by investigating some related cardinal invariants and answering several questions of A. Bella, M. Matveev and S. Spadaro

Next we showed , by forcing, that even in the class of first-countable spaces, d-and D-separability (*nwd*-and NWD-separability) can be different; compare this with the result that every separable Fréchet space is M-separable.

Finally, we finished with some positive results: we prove that every monotonically normal, *nwd*-separable space is D-separable and show that the additional assumption of compactness even yields a σ-disjoint π-base. The last part of the section deals with the question whether $\sigma(2^{\omega_1})$ is D-separable.

In [7.] we have studied the question under what condition is there a free ultrafilter on a topological space that has a base consisting of connected sets. It is easy to see that there is a T_1 space on which every ultrafilter is like that. On the other hand, we proved the surprising and non-trivial fact that there can be no such ultra- filter on any Tychonov space. This question for the classes of T_2 and T_3 spaces remains open and looks very hard.

Continuing our earlier research, in [13.] we have studied further the Fodor-type Reflection Principle (FRP) and its topological consequences. We showed that FRP is equivalent to the following combinatorial statement: There is no almost essentially disjoint ladder system on a stationary subset of a regular uncountable cardinal consisting of ordinals of countable cofinality. Using this characterization, we could show that FRP is equivalent to many known purely mathematical reflection statements. For example, FRP is equivalent to the following topological statement: If all subspaces of cardinality at most ω_1 of a locally countably compact space are metrizable then so is the space itself.

II. Descriptive set theory

In [27.] we proved that consistently there exists a nowhere dense compact subspace C of a compact metrizable group G so that every non-meager set in G is relatively non-meager in a translate of C. (This fails under the Continuum Hypothesis.) This theorem generalizes results of Bartoszyński and Burke-Miller, and also answers the category analogue of an old open problem of Fremlin concerning Haar null sets in non-locally compact groups, hence the solution gives new insights to this problem. In the same paper, we also solved another problem of Fremlin concerning a possible alternative definition of Haar null sets.

We studied in [6.] cardinal invariants connected with certain pre-orderings of the family of ideals on the set of natural numbers. We gave topological and analytic characterizations of these invariants using the idealized version of the Fréchet-Urysohn property and, in a special case, using sequential properties of the space of finitely-supported probability measures with the weak* topology. We also investigated the consistency of some inequalities between these invariants and other related combinatorial questions. We also discussed certain maximality properties of almost-disjoint families related to a pre-ordering on the ideals.

The classical Bruckner-Garg theorem describes the topological structure of the level sets of the generic continuous functions. Generic means here that the set of exceptional functions is of first Baire category. In [10.] we managed to prove a prevalent version of this result, that is, when the exceptional functions form a Haar null set (in the sense of Christensen).

In [28.] we solved a more than 20 year old question, first asked by Mycielski, concerning Haar null sets (in the sense of Christensen, also called shy sets) in non-locally compact groups, e.g. in infinite dimensional Banach spaces. Namely, we showed that in every non-locally compact Polish group with an invariant metric there is a (Borel) Haar null sets that is not contained in any G_δ Haar null set.

In [33.] we considered decompositions of the real line into pairwise disjoint Borel pieces such that each piece is closed under addition, and investigated the possible number of pieces. We found a model of set theory in which this number is strictly between ω and continuum.

Following earlier work of Kuratowski, Laczkovich, Komjáth, Elekes-Steprans and Elekes-Kunen, in [26.] we investigated the possible order types of linear orderings consisting of Baire class 1 functions (i.e. pointwise limits of continuous functions) ordered by the pointwise ordering. We eventually managed to find the long-sought complete characterization of these order types.

In a seminal paper A. Kechris and A. Louveau introduced three rank functions measuring the complexity of a Baire class 1 function. In [25.], we managed to find very well-behaved generalizations of this classical result in the case of the Baire class ξ functions. As an application we answered a problem of Elekes and Laczkovich concerning the so called solvability cardinals, arising from paradoxical geometric decompositions. We also found that certain other very natural generalizations, surprisingly, turn out to be degenerate.

We proved in [1.] that if A_n is a sequence of measurable sets in a σ-finite measure space $(X, \mathcal{A}, \mu)$ that covers μ-a.e. point of X infinitely many times, then there exists a sequence of integers n_i of density zero so that A_{n_i} still covers μ-a.e. $x \in X$ infinitely many times. The proof is a probabilistic construction. As an application we gave a simple direct proof of the known theorem that the ideal of density zero subsets of the natural numbers is random-indestructible, that is, random forcing does not add a co-infinite set of naturals that almost contains every ground model density zero set. This answered a question of B. Farkas.

It was asked in [1.] whether other variants of this covering theorem hold where the density ideal is replaced by some other ideal J on the set of the natural numbers. In [3.] some negative results concerning this problem were presented. We introduced the general notion of the J-covering property for a pair (A, I) where A is a sigma-algebra and I is an ideal on the same underlying set. We investigated connections between these properties and the forcing-indestructibility of ideals. Also, we studied the J-covering property on product spaces.

III. Real analysis and measure theory

In [4.] we discovered a genuinely new notion of fractal dimension and already found numerous applications of it. Perhaps most importantly this notion completely describes the classical Hausdorff dimension of the level sets of the generic continuous functions defined on compact spaces. But it also provides new insight to Mandelbrot's fractal percolation process and Brownian motion, moreover it yields a far reaching generalization of a result of B. Kirchheim.

More than 80 years ago Kolmogorov asked the following question: Let E be a bounded measurable set in the plane and $\varepsilon > 0$. Does there exist a contraction f such that $f(E)$ is a polygon whose Lebesgue measure is ε-close to the Lebesgue measure of E? In [22.] we answered this question in the negative; moreover, the counterexample is simply connected and open, hence homeomorphic to the unit disc. Our construction can easily be modified to yield analogous examples in higher dimensions.

In [11.] we investigated the following tomography-related geometric reconstruction problem. Let us fix a class of compact subsets of a euclidean space. Let us say that certain sets (called the test sets) reconstruct a member of this class if the Lebesgue measure of the intersection of this member with the test sets determine the member of the class. We proved numerous negative and positive results, often exactly determining the minimal (typically finite) number of test sets required. Besides measure theory, surprisingly, algebraic topology, probability theory and Fourier analysis also played key roles here.

The notions of shyness and prevalence generalize the property of being zero and full Haar measure to arbitrary (not necessarily locally compact) Polish groups. The main goal of the paper [23.] is to answer the following question: What can we say about the Hausdorff and packing dimension of the fibers of prevalent continuous maps? Let K be an uncountable compact metric space. We prove that the prevalent $f \in C(K, \mathbb{R}^d)$ has many fibers with almost maximal Hausdorff dimension. This generalizes a theorem of Dougherty and yields that the graph of the prevalent member of $C(K, \mathbb{R}^d)$ has maximal Hausdorff dimension, generalizing a result of Bayart and Heurteaux. Similar results hold for the packing dimension. Also, for the prevalent $f \in C([0,1]^m, \mathbb{R}^d)$ the set $\{y \in f([0,1]^m) : \dim_H f^{-1}(y) = m\}$ contains a dense open set of full measure with respect to the occupation measure $\lambda^m \circ f^{-1}$, where $\dim_H$ and λ^m denote the Hausdorff dimension and the m-dimensional Lebesgue measure, respectively. We also proved an analogous result where $[0,1]^m$ is replaced by any self-similar set satisfying the open set condition. The occupation measure cannot be replaced by Lebesgue measure in the above statement: We show that the functions for which positively many level sets are singletons form a non-shy set in $C[0,1]$. To see this, we generalized a theorem of Antunović, Burdzy, Peres and Ruscher. We also prove sharper results in which large Hausdorff dimension is replaced by positive measure with respect to generalized Hausdorff measures, thus answering a problem of Fraser and Hyde.

IV. Combinatorics

A set system $\mathcal{A}$ is *μ-almost disjoint* iff $|A \cap A'| < \mu$ for distinct $A, A' \in \mathcal{A}$. We say that $\mathcal{A}$ is *essentially disjoint* iff to each each $A \in \mathcal{A}$ we may assign a subset $F(A) \subset A$ with $|F(A)| < |A|$ such that the family $\{A \setminus F(A) : A \in \mathcal{A}\}$ is disjoint.

Using Shelah's celebrated revised GCH theorem, we proved in [20.] that if $\mu < \beth_\omega \le \lambda$, then every μ-almost disjoint family $\mathcal{A} \subset \big[\lambda\big]^{\beth_\omega}$ is *essentially disjoint.*

We also showed that if $\mu \le \kappa \le \lambda$ with $\kappa \ge \omega$, moreover every μ-almost disjoint family $\mathcal{A} \subset \big[\lambda\big]^{\kappa}$ is essentially disjoint then every μ-almost disjoint family $\mathcal{B} \subset \big[\lambda\big]^{\ge\kappa}$ has *a conflict-free coloring with κ colors.* In other words, this means that there is a coloring $f : \lambda \to \kappa$ such that for all $B \in \mathcal{B}$ there is a color $\xi < \kappa$ such that $|\{\beta \in B : f(\beta) = \xi\}| = 1$. Putting together these two results we obtain that every μ-almost disjoint family $\mathcal{B} \subset \big[\lambda\big]^{\ge\beth_\omega}$ has a conflict-free coloring with $\beth_\omega$ colors whenever $\mu < \beth_\omega \le \lambda$.

In the course of proving these results we established a certain singular compactness theorem that is of independent interest.

An *r-edge colouring* of a graph (or hypergraph) $G = (V, E)$ is a map $c : E \to r$ where r is some cardinal. R. Rado proved in the 80s that every r-edge colored ($r \in \omega$) complete graph on ω can be partitioned into r monochromatic paths of distinct colors.

We extended in [34.] Rado's result for hypergraphs, and answered questions of Rado, moreover of Gyárfás and Sárközy by proving that every r-edge colored complete n-uniform hypergraph on ω can be partitioned into r monochromatic tight paths of distinct colors. (A tight path is a sequence of distinct vertices such that every set of k consecutive vertices forms an edge.)

We also proved that for any natural numbers r and m there is a natural number M such that every r-edge colored complete graph on ω can be partitioned into M monochromatic m^{th} powers of paths, apart from a finite set. Using a recent result of Pokrovskiy on finite graphs we could show that every 2-edge colored complete graph on ω can be partitioned into 5 squares of paths.

Finally, we could extend Rado's result for some uncountable graphs: every 2-edge colored infinite complete graph on ω_1 can be partitioned into two monochromatic transfinite paths with distinct colors.

There are many generalizations of the celebrated Erdős-Ko-Rado theorem. Our new results (and some problems) in [35.] concern generalizations for families of t-intersecting k-element multisets in an n-set. We have verified the conjecture that for $n \ge t(k - t) + 2$ such a family can have at most $\binom{n+k-t-1}{k-t}$ members. We also pointed out several interesting connections with coding theory and geometry.

Given a graph G and a positive integer R we addressed in [40.] the following combinatorial search theoretic problem: What is the minimum number of queries of the form "does an unknown vertex $v \in V(G)$ belong to the ball of radius r around u?", with $u \in V(G)$ and $r \le R$, that is needed to determine v. We consider both the adaptive case when the jth query might depend on the answers to the previous queries and the non-adaptive case when all queries must be made at once. We obtained some bounds on the minimum number of queries for hypercubes, the Erdős-Rényi random graphs and graphs of bounded maximum degree.

V. Model theory and philosophy of mathematics

In [2.], continuing some investigations initiated by T. Sayed-Ahmed, we constructed a large family of non-representable ω-dimensional G-Polyadic Equality Algebras ($G-PEA_\omega$) for short) with a representable cylindric reduct. More concretely, we proved that if $G \subseteq {}^\omega\omega$ is a semigroup that contains at least one constant function, then for an arbitrary ω-dimensional cylindric set algebra $\mathcal{A}$ with an infinite base set there exists a nonrepresentable $G-PEA_\omega$ with a representable cylindric reduct that contains an isomorphic copy of $\mathcal{A}$.

For a first order theory T and a cardinal κ the cardinality of pairvise non-isomorphic κ-sized models of T is denoted by $I(T,\kappa)$. The celebrated Vaught's conjecture is the following statement: If T is a complete theory in a countable language then

$$I(T,\aleph_0) > \aleph_0 \text{ implies } I(T,\aleph_0) = 2^{\aleph_0}.$$

Endow ω with the discrete topology and ${}^\omega\omega$ with the product topology. Suppose for convenience that all countable models of the theory T have universe ω. Then any elementary embedding between models is an injective selfmap of ω. Let S be a fixed σ-compact submonoid of injective selfmaps of ω. The main result of the paper [9.] reads as follows: If there are at least $\aleph_1$ many countable models of T that are mutually not elementarily embeddable into each other via members of S, then there are $2^{\aleph_0}$ many models of T that are not elementarily embeddable into each other via members of S. Actually, it is not needed to assume here that T is a complete theory. The proof uses techniques from the theory of cylindric algebras (for T formulated in a language with equality) and from polyadic algebras (for T formulated in a language without equality).

A set C of multivariable functions over a given domain X is defined to be a clone iff C contains the projection functions and it is closed under composition. Investigating clones over finite domains has a long tradition. Recently, definite progress has been made in the study of clones over infinite domains - this is much harder from the technical point of view.

In the main result of the work [14.] we proved that over a countably infinite domain one can find $2^{2^{\aleph_0}}$ many pairwise different clones, all with the same unary fragment, namely the set of all unary operations. We also gave, for each n, a family of $2^{2^{\aleph_0}}$ clones all with the same n-ary fragment, and all containing the set of all unary operations.

By a celebrated theorem of Morley, if a theory T is $\aleph_1$-categorical then it is κ-categorical for all uncountable cardinals κ. In the paper [15.] we were taking the first steps towards extending Morley's categoricity theorem "to the finite". In more detail, we were presenting conditions which imply that certain finite subsets of certain $\aleph_1$-categorical theories T have at most one n-element model for each natural number n (up to isomorphism, of course).

Let I be a fixed finite set and let L be a fragment of usual first order logic. The epistemic formulas of L over I are the members of the smallest set E containing the formulas of L and $\Box_i\,\varphi$ whenever $i \in I$ and $\varphi \in E$. The intended meaning of $\Box_i\,\varphi$ is that "the ith agent (or participant) knows φ".

In [18.] we proposed a reasonable semantics of these epistemic extensions. We proved that if the consequence relation of L is decidable, then - according to our semantics - the consequence relations of its respective epistemic extensions remain decidable as well.

Formula interpolation and related problems have been intensively studied in the literature of algebraic logic. It turned out that interpolation properties of different logics are strongly related to various amalgamation properties of certain classes of algebras.

In the paper [29.] we introduced a local variant of Craig's interpolation property and it was shown that, in the context of certain universal homogeneous structures, this interpolation property is equivalent to a new kind of amalgamation property of the age of such a structure.

The paper [30.] takes the Abstract Principal Principle to be a norm demanding that subjective degrees of belief of a Bayesian agent be equal to the objective probabilities once the agent has conditionalized his subjective degrees of beliefs on the values of the objective probabilities, where the objective probabilities can be not only chances but any other quantities determined objectively. Weak and strong consistency of the Abstract Principal Principle are defined in terms of classical probability measure spaces. It is proved that the Abstract Principal Principle is both weakly and strongly consistent. It is argued that it is desirable to strengthen the Abstract Principal Principle by adding a stability requirement to it. Weak and strong consistency of the resulting Stable Abstract Principal Principle are defined, and the strong consistency of the Abstract Principal Principle is interpreted as necessary for a non-omniscient Bayesian agent to be able to have rational degrees of belief in all epistemic situations. It is shown that the Stable Abstract Principal Principle is weakly consistent, but its strong consistency remains open.

It is shown in [36.] that the classical interpretation of probability and the related Principle of Indifference can be consistently formulated for probability spaces with an infinite number of random events having their probability given by a Haar measure. Bertrand's Paradox is interpreted as the mathematically provable statement of violation of invariance of the Haar measure with respect to re-labeling of random events. It is argued that the mistaken conflation of labeling invariance of the Haar measure with the correct intuition that labeling of random events is a matter of convention is the reason why violation of labeling invariance of the Haar measure is felt paradoxical but that there is in fact nothing paradoxical in Bertrand's Paradox.