Transport Processes in Multi-Component Bubbly Flows

Final report

Research goals

In the framework of this OTKA project we aimed to achieve advances in the following topics:

1. The transport of dilute material in (possibly boiling/cavitating) bubbly water. This includes the transfer of chemical species through an individual bubble's interface and the larger scale convective transport in the bubbly flow — often driven by the buoyancy of the bubbles itself.
2. The investigation of cavitating/boiling, dense, typically non-polar mixtures.

In the course of the project it became evident that our results in the field of bubble research as offshoots of our original goals also gather under two additional topics, namely: flow structure detection — especially Lagrangian aspects — and bubble dynamics.

Methods

In order to pursue the above goals we planned to combine experimental, theoretical and numerical methods. This complex approach is actually well reflected in our results as, indeed, our 14 journal papers describe physical experiments, numerical methods and include theoretical results as well. In several papers the combination of these methods appears.

Results

Dynamics of aqueous bubbles

In a multicomponent environment the thermodynamic principles, the prevailing conditions, the material properties of the liquid and the gas fully determine the state of the bubble in mechanical and thermal equilibrium. The equilibrium state includes the radius and the chemical composition of the mixture in the gaseous and the liquid phases. When a single bubble is deviated from this equilibrium, it starts to oscillate. The mechanical oscillation is coupled to changes in thermodynamic state and chemical composition. Often the oscillation is so strong that it becomes highly nonlinear. It is known that the strongly nonlinear interplay between the bubble oscillation and the diffusion (of both momentum and heat) in the vicinity of the bubble controls a nontrivial way the dynamic behaviour of bubbles even in a single phase environment.

Our first task was to investigate how the size changes of the bubble are coupled to changes in its thermodynamic state and chemical composition, i.e. species transport through the bubble interface in a multicomponent environment. The motivation of this research was a demonstration experiment in which we used indicator to visualize the dissolution of CO_2 from gas bubbles in water by taking advantage of its effect on the local pH value of the water. The dissolution of CO_2 has also substantial relevance from industrial and environmental points of view. We investigated the dissolution process of a CO_2 gas bubble during both free [1] and forced [2] oscillation. In both cases the theoretical problem contains a mutually coupled system of four nonlinear, but different kind of equations (an algebraic, an ODE, a PDE and a boundary condition), the treatment of which requires adequate numerical tools.

In the case of free oscillations [1], by formulating the governing equations using stretched Lagrangian spatial coordinates, the model system can be effectively solved by the method of lines. We determined the minimum equilibrium radius necessary to the adiabatic free oscillation and the related damped frequency. It was found that the gas dissolution significantly enhances the damping of the free oscillations. In addition, a periodic detachment of the concentration boundary layer takes place at the bubble wall, which induces short periods of gas desorption within a cycle of the oscillation. This causes a retarding effect on the dissolution.

In the case of acoustically forced oscillations [2] the desorption process of the CO_2 gas can be accelerated, an effect termed as *rectified diffusion.* If the gas concentration level in the ambient liquid is undersaturated, a competing dissolution process also takes place. In this case a pseudo-spectral numerical approach was used to discretise the diffusion problem and to resolve the concentration boundary layer around the bubble. We investigated both the fully coupled problem and a partially decoupled variant of it, in order to reveal any significance of rectified diffusion on the bubble oscillation. We determined a critical acoustical frequency above which rectified bubble growth can occur.

The above mentioned full equilibrium at the bubble interface does not exist upon arbitrary conditions. By decreasing the ambient pressure below the so called Blake threshold, the equilibrium disappears. In the last decade many attempts were made to stabilize the bubble motion by applying ultrasonic radiation. We managed to prove for incompressible liquid phase (for which the bubble dynamics is described by the Rayleigh–Plesset equation), without any restriction of nonlinearities, that the bubble motion can be kept in a 1:1 stable resonance by ultrasonic excitation even in this genuinely unstable regime. The proof is based on a special numerical continuation technique of modern nonlinear dynamics and bifurcation theory, the so called pseudo-arclength method. [3]

By combining the pseudo-arclength method with the so called shooting method, we managed to improve on the analysis of the problem. [4] By this technique even the higher order resonances became tractable. We managed to explore the complete structure of the periodic attractors in the pressure amplitude—excitation frequency parameter space. The topological structure of these attractors are equivalent to the self-similar hierarchy of a Farey-ordered graph. Besides proving that bubbles can also be stabilized in higher order resonant states, this result has an implication beyond bubble dynamics itself, as it seems to have a fairly general validity for a wide class of nonlinear oscillators.

In case of extremely large oscillations the maximum velocity of a gas bubble wall during collapse can reach the order of the speed of sound of the ambient water. Under such conditions liquid compressibility becomes important and the above mentioned Rayleigh–Plesset equation is not satisfactory any longer. We studied the problem when the dynamic equation is replaced by the Keller–Miksis equation, which reflects the acoustic attenuation too in the system. We systematically explored the bifurcation structure of the system on the control parameter plane of acoustic excitation (i.e. amplitude and frequency) with the other environmental and material parameters kept fixed at characteristic values. We identified a *giant response region* in which the bubble wall velocity becomes supersonic while there is a strong collapse. We found numerically a very good correlation between the asymptotic bubble radius and the Mach number, and thus concluded that the latter can be used to quantify the strength of collapse. By changing the equilibrium radius of the gas bubble over more than two magnitudes, we found no significant change in the bifurcation structure; indicating the structurally stability of the system. [5]. We have also explored the bifurcation structure on the pressure amplitude vs. equilibrium radius control parameter plane. It was found that it contrast to the expectation of other authors, it is sensitive to the viscosity and the surface tension, the former playing determining role in case of water microbubbles. The regions of periodic and chaotic oscillations were detected via the maximum Lyapunov exponent. In this research we have also used the Mach number for the quantification of the collapse strength as explained briefly in [6]. Further details are expected to appear soon in [7]. We note that practical limits for safe medical therapeutic applications have been derived from this model [8].

Fluid mixtures

Concerning the 2nd topic, our starting proposition was that in dense mixtures the phenomena of cavitation and boiling must be different and more complex than in single component fluids. The reason is that in such systems there is no simple saturation curve, rather, the bubble and dew points deviate and become of a function of the chemical composition as well. The thermodynamics of the bubbles therefore do not depend exclusively on the variation of pressure and temperature, but are also sensitive to the small variations in the concentrations. So mass transport of the constituent plays an essential and distinguishing role in such systems. We made several attempts to substantiate this hypothesis in experiments.

In a simple experimental setup we examined bubble growth and collapse processes in gasoline under different circumstances. [9] Bubbles were indeed found to collapse at considerably higher pressure than the one they were produced, causing hysteresis in the bubble volume—absolute pressure diagram. We showed that a time-dependent deterministic model can be developed for the contraction phase. Investigating the response of the system to small disturbances the experienced characteristics were found very similar to those of heat conduction and diffusion equations. This lead to the conclusion that size changes of bubbles seem to be controlled by the heat and mass transport processes in their vicinity and the multicomponent diffusion effects can be responsible for the observed hysteresis. These experimental results established a good starting point for future numerical model development e.g. for simulating flows in automotive fuel pumps.

With the assistance of an industrial partner we further achieved the study of fuel cavitation. We have constructed a large scale physical model of an actual (production design) side channel automotive fuel pump with a complete test rig. The facility was aimed at providing experimental data for the validation of Computational Fluid Dynamics (CFD) tools for the redesign and further development of the existing side channel fuel pump design. The installation of the test facility and the measurement of its characteristic curves are briefly described in the paper [10]. The top part of the model pump was produced from a thick Plexiglas plate, this arrangement allowed to optically access the flow in between the impeller blades and to record the motion of bubbles whenever cavitation sets up. The apparatus was tested with single phase pure water and a special multiphase (tetralin-decalin) mixture to investigate the possible effects of test fuel. The hydraulic rig facility provided us the characteristic curves (total pressure rise vs. volume flow rate) and the overall efficiency (hydraulic performance over electric power input to the electric drive) at various conditions. Again, the differences between the single and multiple phase runs in the dimensionless characteristic curves became evident.

In order to discover the conditions during cavitation, Laser Doppler Velocimetry (LDV) and Particle Image Velocimetry (PIV) measurements were also carried out inside the suction side channel through the transparent Plexiglas cover of the pump; however the latter results have not been published due to reasons attributed to our industrial partner.

Yet another experimental apparatus was built in the context of automobile industry and multiphase dispersed hydrocarbon flows: a simplified model in which gearbox lubrication was investigated [11]. However , in our apparatus water was used instead of oil. A partially immersed gear was rotated, while the emerging bubbles and droplets were recorded with a camera. Flash and laser sheet visualizations of the global flow field and the boundary layer were used. In the experiments we observed both the bubbles in the liquid phase and the droplets in the gas phase. Resolved mass flow distributions in the fluid sheet above the water surface were obtained. The relation between the circumferential velocity and integrated mass flux of the fluid sheet was found to be linear above a critical speed, for which we gave an analytical formula. Using dimensional analysis, we showed that the relation between the mass flux and gear speed signifies a Reynolds-number independent behaviour of the boundary layer around the immersed part of the gear.

Flow structures

A common experimental method in the previous two measurement systems is the use of video recording and PIV to document and later assess the visually observable flow field, including the motion of bubbles and droplets. As a matter of fact both processes provide only Eulerian information on the flow field. In most (and not only engineering) cases Eulerian information on the momentary flow field — including turbulence characteristics and Eulerian flow structures — is enough. However, whenever mixing processes and material transport is important – like in case of multicomponent cavitation and boiling – the need arises for the ability to track the path (and dispersion) of individual fluid elements, with potentially different chemical composition. In other words we would like to collect Lagrangian information on the flow: distinguish individual fluid particles, identify them and find the Lagrangian flow structures, which eventually determine the mixing (and dissolving) processes in the flow.

We tried this agenda first in surface flow experiments discussed in the paper [12]. They included hydraulic field and laboratory experiments as well. In order to detect signatures of chaotic advection in river surface motion, surface buoys equipped with GPS were deployed in a field experiment in River Danube, Hungary. The buoys were released in the vicinity of groynes (obstacle structures perpendicular to the flow) where complex mixing processes occur. The buoys detected their own trajectories, which were later analysed, focusing on the time evolution of the distance between buoy pairs. The analysis aimed the determination of Lagarangian flow characteristics, such as the local Lyapunov exponents and descriptors of finite-time hyperbolic behaviour, which is related to strong mixing. Despite of the small number of applied buoys, we found evidence on Lagrangian chaos in the wake of a groyne field. Since field data is scarce due to the very limited number of buoys, an analogous series of hydraulic experiments were also carried out, in a small-scale laboratory model of an open channel. In these experiments the motion of a multitude of surface floaters was detected by a camera, and the particle paths were later reconstructed from the recorded images. This method is called Particle Tracking Velocimetry (PTV), which provides Lagrangian flow information directly. Lagrangian flow structures and characteristics were obtained from these data in the subsequent analysis. From the point of view of this project, the methodology in this paper is more important than the hydraulic results themselves.

In bubbly flows the bubbles themselves can be used as tracers, so one does not need to intervene the natural behaviour of the flow field in order to gain Lagrangian information on bubble motion. However, unlike surface floaters, bubbles move in three dimensions in the liquid, so one needs stereographic, or three dimensional PTV (3D PTV) instead of 2D PTV. 3D PTV has a number of challenges with respect to 2D PTV. In the latter there is a one-to-one relationship between an image point and a surface position, while in 3D, a single camera provides only a projection of the position, losing information on a spatial dimension. In addition, the tracer particles (bubbles) can obscure each other from time to time, so their identity cannot be maintained. Therefore, in a 3D PTV system several cameras have to be used in order to provides enough information on the spatial position of the tracers. In the framework of this project we have designed and developed a 3D PTV system that includes three relatively high speed cameras. The three different perspectives provides redundant information on the spatial position of the tracers. The redundancy in our system is twofold; this makes the identification process more robust than the 2D PTV. With three cameras, hundreds of particles can be tracked at the same time. This way not only the tracer tracks can be observed, but the (Eulerian and Lagrangian) flow pattern structures prevailing in the vicinity of the tracer swarms as well. The resolution and frame rate of the cameras allow to observe the turbulent characteristics of the flow as well.

A disadvantage of the 3D process is the huge information bandwidth the system requires. The three cameras can produce as much as 1.5 Gbyte image information per second. In order to continuously record long image sequences, each component of the system — including cables, frame grabber cards and the DMA channels — has to be able to manage this transfer rate. This demand usually exceeds the capabilities of most present day computers. In order to overcome this problem we applied the achievements of machine vision systems. Notably we use cameras and frame grabber cards on which the hardware itself performs certain image processing steps in real time. This can be achieved by the Field Programmable Gate Array (FPGA) technology. By this, one can reconfigure the logical circuits of the frame grabber boards so that it executes the whole chain of image processing up to and including the binarization and segmentation steps. As a result we has to treat only the positional and basic shape information of the particle (bubble) candidates , rather than the whole images. In practice this reduces the bandwidth requirement to the order of 100 kbyte/s rate, a ten thousand times reduction. This allows not only that the data can be recorded, but means it can even be post processed and assessed real time. The recording of intermediate data is unnecessary unless for a posteriori performance and quality analysis.

3D PTV experiments require a thorough and individual hydraulic and optical planning concerning flow speed, bubble size, field of view, managing the problems of multiple refracting and reflecting surfaces, depth of field limitations and illumination. Therefore our system has been designed for certain pre determined test experiments. These include

- A single vertical vortex flow in a small container, designed for optimal 3D PTV redundancy. This setup reflects some aspects of detaching cavitation vortex cores, and it is also possible to measure some turbulence characteristics inside the vortex core.

- Bubble entrainment in a bore, or in a hydraulic jump in an open channel. This can be relevant e.g. from the point of view of aeration of natural waters using hydraulic structures.
- Investigation of the flow of a bubble column representative of chemical process or sewage treatment facilities.
- Repetition of the surface flow experiment of [12], mentioned above, for performance and quality control analysis.

The test experiments are still underway. The publication of the equipment design, the measurement process, including the applied 3D vision logic [22] is targeted to the Measurement Science and Technology (IF= 1.433), however the manuscript cannot be finalized before the test experiments are completed. The results of individual experiments are planned to be published in the Experiments in Fluids (IF=1.670).

The counterpart of PTV in computational fluid modelling is Smoothed Particle Hydrodynamics (SPH). This genuinely Lagrangian numerical approach is completely mesh free in which the flow is represented by numerical particles that follow the flow under the action of external forces and each other. We developed an in-house software code, using the immense parallel computational capacities of Graphical Processor Unit (GPU) programming, in order to support and supplement our Lagrangian experiments. We studied the flow structure detection capabilities of SPH in [13], implementing both the above mentioned Lagrangian and some traditional Eulerian structure detection methods. In a related study [14], a thorough analysis was given upon the reasons of uncertainty in traditional first order flow structure detection techniques. We pointed out that this uncertainty can be related to and quantified by the next order structure quantities, which by the way are related to the difference between Lagrangian and Eulerian approach. It might be worth mentioning the interesting analogy between the geometry of the flows studied in [12] and [14].

Bubbles in high viscosity mixtures

Besides hydrocarbon mixtures, in an exciting series of research work, we have obtained significant results in studying water–glycerol mixtures. The special property of this medium is that its viscosity can be controlled, and raised to extremely high values. This provides a unique opportunity to single out the effects of viscosity in bubble dynamical phenomena.

First, the free oscillations of a single spherical gas bubble in glycerol was examined experimentally and numerically. [15] The bubble was generated by a laser impulse and its unsteady radius was measured by a novel technique, the Laser Shadow Method (LSM). The experimental results were compared to computations obtained from two models, first taking into consideration the liquid compressibility (Keller–Miksis dynamics) and then assuming an incompressible liquid (Rayleigh–Plesset dynamics), respectively. For high amplitude oscillations the incompressible model provided poor agreement with the measurements and the inclusion of the liquid compressibility has been proved to be necessary. In contrast to the standard method, a practical region of applicability for the incompressible approach was determined as a function of the instantaneous Mach and Reynolds numbers, rather than specifying a simple threshold.

In another series of experiments the shock wave emission of laser-induced bubbles in the vicinity of the free surface was investigated both in glycerine and in distilled water during the first bubble collapse. [16] Experimental investigations were carried out, using the LSM method, as a function of temperature and viscosity. At high viscosities (i.e. at low temperatures), the bubble tends toward a rebound without emitting a shock wave if the liquid temperature is less than a critical value. For higher temperatures the viscosity decreases rapidly and collapse shocks are present. For water, at higher temperature, where the vapour pressure of the liquid becomes important, the bubble rebound takes place without a collapse shock. These are the first experimental verifications of relating theories.

We analysed the theoretical aspects of the above mentioned critical temperature value. [17] By applying the sophisticated numerical techniques of modern bifurcation theory, we revealed the regions in the excitation pressure amplitude—ambient temperature parameter plane where collapse-like motion of an acoustically driven gas bubble in highly viscous glycerine exists. We reported evidence that below a threshold temperature the bubble, modelled by the Keller–Miksis equation, becomes an overdamped oscillator and suppresses collapse-like behaviour. In addition, we have found periodic windows interspersed with chaotic regions indicating the presence of transient chaos, which is important from the point of view applications if predictability is required. We managed to obtain the critical temperature below which overdamping occurs at arbitrary intensity of excitation.

In a recent development, using similar methods in this highly viscous system, we found giant response regions at low enough relative frequencies and high enough pressure amplitudes, where the bubble wall velocity may even become supersonic. The region of the opposite behaviour(at high frequencies and small pressure amplitudes) has been identified as well, in such cases the bubble wall velocity never reaches extreme values. Both domains can be important for certain applications. These results are briefly summarised in [18] and [19], using the Rayleigh–Plesset and Keller–Miksis Equations, respectively.

We have thoroughly analysed the bifurcation diagram on the parameter plane of acoustic excitations in this system using the numerical techniques described above. We found that the topological structure of the resonances follow the same Farey ordering that we have already identified in the case of the much less viscous compressible system, (c.f. [4]). Likewise in its counterpart, low order resonances exist in this system as well, however high dissipation suppresses their hysteresis or period doubling. [20]. Further publications from this ongoing research topic is expected to appear this year [21]

Progress of research and changes in the participants

There were two significant setbacks in the course of the project.

First, the decision of the acceptance on the proposal, likewise the first year funding, appeared very late — almost at the end of the first research year of the project. Due to this uncertainty and delay, the planning and preparation for the more serious experiments could not started in the first year.

The second setback happened at the end of 2012, when a key participant of the project, **Károly Czáder**, not long after he had completed his absolutorium, abruptly discontinued his PhD studies in the beginning of its active research phase and accepted a position in the industry instead. On the one hand, this caused a disruption in the project because as a BsC in chemical engineering he was supposed to provide the necessary background for the chemical aspects of the research. In addition, it was his task to perform the scheduled the experiments. On the other hand, as he was scheduled to be employed from this grant throughout the research phase of his PhD, his unexpected leave caused a budgetary problem as well.

In order to substitute him in the experimental work, two new participants were invited to join the project. The in house experimental work was taken over by **Zsolt Várhegyi**, while **Ferenc Hegedűs**, who had already been involved in an international cooperation experimental work on bubbles in dense solutions, received support to continue his experiments in Emden, Germany. In both cases the experiments were successful and have eventually been published in journals [11,16]. The remaining financial resources were used for the development of the 3D PTV apparatus.

Zsolt Várhegyi had been employed for a short period until he finished his experiments. Then another new participant, **Roxána Varga** became employed, who had successfully joined the team with her theoretical and numerical work on bubble dynamics. Her ongoing publications [5–8] confirm this decision.

Even though these changes eventually turned out to be very successful, a considerable time had to pass before they became effective. Because of that the PI requested twice the prolongation of the project, which was granted both times.

Zsolt Várhegyi left the project after completing his experiments, while **János Vad** decided not to take part in the joint work during the second prolonged year. **Ferenc Hegedűs** has remained in the team and contributed by a substantial amount of results [3–4,6–8,15–17,19–21].

Budget changes

The salaries of Károly Czáder for one year period that remained unused after his leave, was partly used to employ Zsolt Várhegyi for two months while he was finishing the experiments with the lubrication gearbox; and Roxána Varga for ten months.

The changes in the participants forced us to redistribute the project’s experimental tasks according to the personal expertise and the scientific interest of the new and remaining participants. Rather than building our own acoustic device, it seemed more purposeful to support Ferenc Hegedűs in his acoustic experiments in Emden by partially covering his travel costs. It was faster, safer and more economic.

In the meantime our experience with Lagrangian structures indicated that instead of the originally planned (Eulerian) PIV flow measurement technique, we should try to apply PTV. Having finished with the planning and design of the PTV experiments in 2014, we requested a significant restructuring of the budget plan in favour of the investment costs early in 2015. This was necessary to be able to put together an operational PTV equipment. Later a small change was also requested in order to purchase a magnetic stirrer for the vortex PTV test measurement. Both of our requests have been approved by the OTKA Office.

Publications

Despite the fluctuations in the ranks of the participants we managed to achieve a continuous flow of results. Our publication rate was almost constant – some 2 journal papers annually. 12 papers have already appeared or accepted in different international and Hungarian journals (out of which 8 have impact factor) and further 2 are submitted to (impact factor) journals.

In addition to journal papers, we have prepared 11 conference material — posters, lectures, abstracts and proceedings — in the course of the project. Out of these 7 are listed here which, at least partially, contain scientific information not yet published in journals. It worth emphasising that 6 of our papers are strictly experimental ones.

1. Czáder K; Szabó KG: ***Numerical investigation of the dissolution mechanism of a freely oscillating CO_2 gas bubble by the method of lines***, Periodica Polytechnica — Mechanical Engineering 57: 63-73, 2013
2. Czáder K; Szabó KG: ***Numerical investigation of rectified diffusion for an acoustically induced CO_2 gas bubble***, In: Diffusion in Solids and Liquids VIII (editors: Öchsner A; Murch GE; Shokuhfar A; Delgado JMPQ - ISBN: 978-3-03785-662-8), pp. 207-212. (paper 367), 2013
3. Hegedűs F: ***Stable bubble oscillations beyond Blake's critical threshold.***, Ultrasonics 54(4), 1113-1121, 2014
4. Hegedűs F: ***Topological analysis of the periodic structures in a harmonically driven bubble oscillator near Blake's critical threshold: Infinite sequence of two-sided Farey ordering trees***, Physics Letters A 380, 1012-1022, 2016
5. Varga R; Paál Gy: ***Numerical investigation of the strength of collapse of a harmonically excited bubble***, Chaos, Solitons & Fractals 76, 56–71, 2015
6. Varga R; Hegedűs F: ***On the investigation of two-dimensional bifurcation structure of an acousticaly excited gas bubble***, Conference on Modelling Fluid Flow (CMFF'15), The 16th International Conference on Fluid Flow Technologies, paper119, pp 7, 2015
7. Varga R; Hegedűs F: ***Classification of the bifurcation structure of a periodically driven gas bubble***, Nonlinear Dynamics (submitted), 2016
8. Varga R; Klapcsik K; Hegedűs F: ***Towards physical modeling of the utilization of ultrasound in modern medical therapeutic applications***, First European Biomedical Engineering Conference for Young Investigators (ISBN: 978-981-287-573-0), Volume 50 of the series IFMBE Proceedings pp 114-117, 2015
9. Jesch D; Kristóf G: ***Experimental investigation of characteristics of cavitation in gasoline***, Periodica Polytechnica - Mechanical Engineering, 55: 43-47, 2011
10. Jesch D; Kristóf G; Vad J: ***Oldalcsatornás üzemanyagszivattyúban kialakuló áramlások tanulmányozására szolgáló tesztberendezés kialakítása és beüzemelése***, GÉP 61 (11): 9-14, 2010
11. Várhegyi Zs; Kristóf G: ***Mass flux distribution measurements and visualizations of a fluid sheet generated by a partially immersed dip-lubricated gear***, Periodica Polytechnica - Mechanical Engineering, paper 7764 (to appear), 2016
12. Zsugyel M; Szabó KG; Ciraolo G; Nasello C; Napoli E; Kiss ZsM; Tél T; Józsa J: ***Detecting the chaotic nature of advection in complex river flows***, Periodica Polytechnica Civil Engineering 56(1), 97-106, 2012
13. Tóth B; Szabó K G: ***Flow Structure Detection with Smoothed Particle Hydrodynamics***, Proceedings of the 9th SPHERIC International Workshop (ISBN: 88-7617-020-0, ISBN 13: 978-88-7617-020-1, Paris, 2014), 2014
14. Farkas B; Paál G; Szabó KG: ***Descriptive analysis of a mode transition of the flow over an open cavity***, Physics of Fluids 24(2), 2012

15. Hegedűs F; Koch S; Garen W; Pandula Z; Paál Gy; Kullmann L; Teubner U: ***The effect of high viscosity on compressible and incompressible Rayleigh–Plesset-type bubble models***, International Journal of Heat and Fluid Flow 42, 200–208, 2013
16. Garen W; Hegedűs F; Kai Y; Koch S; Meyerer B; Neu W; Teubner U: ***Shock wave emission during the collapse of cavitation bubbles***, Shock Waves (published online pp 1-10), 2016
17. Hegedűs F; Klapcsik K: ***The effect of high viscosity on the collapse-like chaotic and regular periodic oscillations of a harmonically excited gas bubble***, Ultrasonics Sonochemistry 27, 153–164, 2015
18. Klapcsik K; Hegedűs: ***Harmonikusan gerjesztett gázbuborék nemlineáris dinamikai vizsgálata nagy viszkozitású folyadékban***, XXII. Nemzetközi Gépészeti Találkozó – EMT – OGÉT-2014, 2014
19. Klapcsik K; Hegedűs F: ***Two-parameter bifurcation analysis for the seeking of high amplitude oscillation of a periodically driven gas bubble in glycerine***, Conference on Modelling Fluid Flow (CMFF'15), The 16th International Conference on Fluid Flow Technologies, paper 116, pp 8, 2015
20. Klapcsik K; Hegedűs F: ***The effect of high viscosity on the bifurcation set of a periodically excited gas bubble: Patterns in the bifurcation structure***, Nonlinear Dynamics (submitted), 2016
21. Hegedűs F: ***Harmonikusan gerjesztett gázbuborék nemlineáris dinamikai vizsgálata nagy viszkozitású folyadékban***, XXIV. Nemzetközi Gépészeti Találkozó – EMT – OGÉT-2016, 201, 2016
22. K G Szabó: ***Experimental Realization of Real-Time Stereoscopic Particle Tracking***, (manuscript), 2016