Final report on OTKA-72856

Investigations in Game Theory

October 29, 2012

We have investigated various topics in game theory, some of them are related (in various degrees) to both the cooperative and the strategic approaches, some are concerned with foundational issues. Major part of the research has been a natural continuation of the work we had performed in a previous OTKA project (T46194: Game Theory). During the four and a half years of the current project, however, we have also addressed new questions originally not foreseen and planned. In 2010, the original team of the three senior researchers (Forgó, Pintér, Solymosi) was extended by Dezső Bednay, a PhD student working on closely related issues.

In the proposal we stated that "our goal is to achieve results publishable in high-level journals, and to have a total of 10-12 presentations at significant conferences, and eventually a total of 6-8 articles in respected periodicals of the field". We have succeeded to surpass all these figures: already 9 articles appeared in well-respected international journals (and 2-4 more articles are expected to come out in the near future from the available pool of submissions and manuscripts); 5 papers in domestic journals; 2 papers in an edited volume; more than 20 presentations at international conferences and workshops; and more than 10 talks at domestic conferences.

On the other hand, we came short on our "secondary" goal to produce an improved and extended "second edition" of our "electronic" book "Játékelmélet" ([9]).

The Corvinus Game Theory Seminar has become a regular forum (with at least 10 talks each semester) for colleagues and students interested in Game Theory and its applications to Economics and other social sciences. To disseminate information we maintain the webpage http://gametheory.uni-corvinus.hu/index.html .

Following the structure of the project proposal, we summarize the achieved results topic by topic. We list our papers related to this project separately from the general literature and refer to them in a different style.

1 Implementing the L-Nash bargaining solution

The "Nash program" initiated in [24] is a research agenda aiming at representing every axiomatically determined cooperative solution to a game as a Nash outcome of a reasonable non-cooperative bargaining game. The L-Nash solution first defined by Forgó ([8]) is obtained as the limiting point of the Nash bargaining solution when the disagreement point goes to negative infinity in a fixed direction.

Forgó and Fülöp (2008) establish finite bounds for the penalty of disagreement in certain special two-person games, making it possible to apply all the implementation models designed for Nash bargaining problems with a finite disagreement point to obtain the L-Nash solution as well. For another set of problems where this method does not work, a version of Rubinstein's alternative offer game ([31]) is shown to asymptotically implement the L-Nash solution. If penalty is internalized as a decision variable of one of the players, then a modification of Howard's game ([17]) also implements the L-Nash solution.

2 New protocols for correlated equilibria

It is a long-standing problem in multiperson conflict/cooperation situations (games) how to achieve the best social outcome in equilibrium with indirect methods, that is, by devising pre-game scenarios (protocols) leading as close as possible to the maximum social welfare (measured as the sum of utilities) while agents (players) are driven by self-interest. Correlated equilibrium and its generalizations can do this in many classes of games. The different pre-game scenarios may improve on the achievable social welfare while maintaining the self-enforcing character of equilibrium.

Correlated equilibrium (CE), introduced by Aumann ([2]), is a particular generalization of Nash equilibrium (NE). It has proved to be very useful in several ways. One of them is that it allows to realize, in equilibrium, payoffs strictly better for all players than those of any NE's. Yet, for some classes of games, where intuition would call for solutions that are more in line with conventional wisdom, correlated equilibrium does not help. Moulin and Vial ([23]) provide the first departure from Aumann's protocol of a CE. Their concept, called weak correlated equilibrium (WCE) could provide strictly better pay-offs that can be realized in any CE. There are games, however, where even the WCE protocol does not help in achieving desirable outcomes. During the project other reasonable protocols have been devised which give rise to generalized CE's which may provide better outcomes for each player than any WCE.

Forgó (2010) introduces for n-person finite games in normal form a new correlation scheme, leading to a special equilibrium called soft correlated equilibrium (SCE). This scheme can lead to Pareto-better outcomes than Moulin and Vial's WCE extension. The informational and interpretational aspects of soft correlated equilibria are discussed in detail. The power of the SCE generalization is illustrated for dichotomous games in general and for some 2-by-2 games (prisoners' dilemma and the game of chicken).

In another paper, Forgó (2011) applies the concept of soft correlated equilibrium for two-person finite games in extensive form with perfect information. Again, this scheme can lead to Pareto-improved outcomes of other correlated equilibria. Computational issues of maximizing a linear function over the set of soft correlated equilibria are considered and a linear-time algorithm in terms of the number of edges in the game tree is given for a special procedure called "subgame perfect optimization".

3 Sensitivity of core allocations in assignment markets

Assignment markets are special two-sided markets with indivisible goods and money, where each buyer and seller places a monetary value on each of the objects. Shapley and Shubik ([33]) showed that in these markets the set of competitive equilibrium payoffs coincides with the core of the related assignment games. Moreover, there always exist two efficiently computable special core allocations, one is simultaneously the best for all buyers (and the worst for all sellers), the other one is seller-optimal. Several papers (e.g. [18], [5]) address the importance of the minimum equilibrium prices related to the buyer-optimal core allocation in mechanism design for these markets. If, for example, a sealed-bid multi-item auction ([6]) is used to determine these prices, the buyers have no incentives to falsify their stated valuations. The sellers, however, can manipulate the process to their benefits, some of them might even realize the full amounts by which they falsified their reservation prices. We aimed at investigating the individual manipulability of certain equilibrium price mechanisms in assignment markets, which are related to efficiently computable, well-known core allocations of the corresponding assignment games.

Solymosi (2010) considers the "fair equilibrium prices" ([39]), i.e. the average of the buyer-optimal and the seller-optimal prices, which are related to the tau-value of the corresponding assignment game ([26]). It is shown that this midpoint is "linewise monotonic", that is, if we change each entry in a row or a column of the profit matrix by the same amount but keep all other entries fixed, the payoff of the corresponding player cannot decrease. Moreover, a sharp upper bound is established for the extent the payoff of an agent can increase, if he unilaterally falsifies his stated valuation(s). The proof relies on a new characterization of the buyer-/seller-optimal allocations, which can also be used for their efficient computation.

Solymosi (2011) proves "linewise monotonicity" for the nucleolus in assignment games and also establishes a sharp upper bound for the extent the payoff of an agent can increase, if he unilaterally falsifies his stated valuation(s). The nucleolus lies in the "lexicographic center" of the core and it is also efficiently computable ([37]). Another kind of monotonicity, "pairwise monotonicity" of the nucleolus is proved in the paper (Solymosi et al., 2012). It means that if we increase one entry in the pairwise profit matrix but keep all other entries fixed, the nucleolus payoff of neither of the two involved players can decrease. The first draft of this paper was prepared during our previous OTKA project, the current revised version contains a simplified (yet not too compact) proof.

Solymosi (2012) discusses the computability of extreme core points in assigment games. It is shown that all extreme points (not just the seller- and buyer-optimal ones) can be determined as the outcome of an efficient lexicographic optimization procedure.

4 Regression games

The question of measuring the "relative importance" of explanatory variables in linear regression models is very important, but has not been settled in the applied statistics mainstream ([10]). The efforts to avoid the ad-hoc nature of stepwise methods, but to maintain the idea that assessing relative importance can be based on the marginal impacts of the explanatory variables along sequences of regressors have lead authors (sometimes unaware of the connection) to suggest metrics which are in fact the Shapley values in suitable coalitional games ([4], [38], [34], [19]).

Pintér ([29]) has already formulated coalitional games which can meaningfully correspond to regression models. He has succeeded in characterizing the Shapley value on this class of games, called regression games, by the potential approach of Hart and Mas-Colell ([12],[13]), and has concluded that the axioms involved make good sense in the given context.

Continuing these investigations, Pintér (2011a) provides another solid theoretical background for the use of the Shapley value in measuring the relative importance of regressor variables. He shows that also Young's axiomatization ([40]) of the Shapley value works on the class of regression games, and provides solid statistical interpretations of the used axioms (efficiency, symmetry, and strong monotonicity). The main task of settling uniqueness (namely, whether or not only the Shapley value satisfies this set of axioms on the class of regression games) turns out to be non-trivial, since the family of regression games is a proper, but neither open nor closed subset (in the Euclidean metrics) of the class of monotonic games. (The interrelations between the sets of axioms and the structures of their domain had to be studied on a purely game theoretic level, see the separate research topic below).

5 On axiomatizations of the Shapley value

Cost or benefit allocation is a tough problem in managerial accounting. It occurs whenever joint overhead costs or the benefits of cooperation have to be allocated to the participating divisions of a firm. The use of coalitional game theory, and especially of the Shapley value because of its marginalistic nature, to such problems was advocated first by Shubik [35]. A "fair" allocation can be obtained by computing the value of an appropriate transferable utility coalitional game, provided the allocation principles relevant to the given problem are captured by the Shapley value. This necessitates to have various characterizations of the Shapley value for various subclasses of games.

In this research topic, Pintér has examined several characterizations of the Shapley value, including the three best known ones: (1) the potential approach of Hart and Mas-Colell [12]; (2) Shapley's [32] original axiomatization improved later by Dubey [7], and by Peleg and Sudhölter [27]; and (3) Young's [40] characterization purified later by Neyman [25]. Pintér (2012) obtains a new proof for Young's characterization of the Shapley value. This new proof makes it possible to show that the given characterization is valid on some further subclasses of transferable utility games not yet discussed in the literature (e.g., the class of regression games, see the topic above).

Pintér (2009) considers several characterizations on sixteen classes of games (essential / (strictly) convex / (strictly) superadditive / (strictly) weakly superadditive / (strictly) monotonic / additive / (strictly) subadditive / (strictly) weakly subadditive / (strictly) concave games) and verifies or falsifies the mentioned characterizations on each of these classes.

Kóczy and Pintér (2011) introduce generalized weighted voting games and investigate axiomatizations of the Shapley value for this class of games. The difference between generalized and ordinary weighted voting games is that in the former some players might be absent. Being absent is not strategic but stochastic, so the generalized weighted voting games are the "mix" of ordinary weighted voting games and the zero games (where too many players are missing, so the legislation cannot work). Based on Young's axiomatization of the Shapley value, the Shapley-Shubik index is characterized on this class of games.

Another application of the axiomatization results is to the class of risk allocation games (closely related to exact and totally balanced games). Balog et al. (2010) discuss financial applications of the Shapley value. In a related paper, Balog et al. (2010) apply the tools of cooperative game theory in finance. They compare some common risk allocation methods and well-known TU cooperative games solutions. The conclusion is that the Shapley value and the nucleolus perform better then the other risk allocation methods.

6 Reasonable universal complete type space

The universal type space in a fixed category of type spaces is a type space (1) which is in the given category, and (2) into which every type space in the given category can be mapped in a unique way. A type space is complete, if every relevant hierarchy of beliefs is a type in that space. The existence of a purely measurable universal type space was proved by Heifetz and Samet [15]. The same authors, however, gave an example ([16]) which demonstrates that a purely measurable universal type space is not complete in the sense of coherency. On the other hand, several papers ([22], [3], [14], [21], [28]) use compact regular probability measures and obtain, in this stricter sense, complete type spaces. These type spaces, however, are neither purely measurable nor universal in the category used by Heifetz and Samet in [15].

In this research topic Pintér has examined the existence of a universal complete type space in a category of type spaces which is "reasonable" in the sense that it somehow takes into account the players' cognitive abilities. This requirement becomes clear if one compares the categories of topological type spaces (see e.g. [14]) to the categories of purely measurable type spaces (see [15]). The concept of measurability itself is closer to the cognitive constraints of human beings than that of topology.

Pintér (2010a) constructs a counterexample, which shows that the Harsanyi program does not work in topological type spaces, therefore no universal topological type space exists.

Pintér (2010b) proves the existence of an inverse limit of an inverse system of measure spaces in a special case. This result gives the mathematical foundation of a positive result on measurable type spaces. Combined with results of Heifetz and Samet ([15]) and of Meier ([20]), these results imply that the Harsanyi program works in the purely measurable framework.

7 Other results

In this section we summarize the results achieved during this project but which were originally not proposed.

Bednay (2012a) characterizes all von Neumann – Morgenstern stable sets in assignment games with one seller and many buyers as those sets of imputations which are the graphs of a certain type of continuous and monotone functions. The standards of behavior encompassed by the various stable sets can then naturally be interpreted as possible outcomes of well-known auction procedures when groups of buyers may form bidding rings. It is also shown that the union of all stable sets can be described as the union of convex polytopes all of whose vertices are marginal contribution payoff vectors. Consequently, each stable set is contained in the Weber set. The Shapley value, however, typically falls outside the union of all stable sets.

Bednay (2012b) considers stable sets from a bargaining perspective. The stable set of a cooperative game is a set of imputations that satisfies the internal- and external stability properties. Harsanyi ([11]) criticised this concept because it neglects the effect of indirect dominance. He introduced the class of absolutely stable games in which indirect dominance is irrelevant. He showed that in this class we can get a stable set as the fixed point of an equilibrium strategy profile in a bargaining game. In the class of assignment games (which is not the subset of absolutely stable games) the same results are obtained: a bargaining game is defined in which the fixed points of the equilibrium strategy profiles are stable sets, and vice versa, we can get every stable set as the set of fixed points.

Csóka et al. (2011) introduce the notion of exactness for non-transferable utility (NTU) games and relate it to various notions of convexity known for NTU games. The conclusion is that except cardinal convexity, all other considered notions of convexity (e.g., ordinal, coalition merge, individual merge, and marginal convexity) imply exactness.

Pintér (2011b) considers TU games with infinite many players and generalizes to this setting the concepts of the core and of balancedness. He extends the classical Bondareva-Shapley theorem by showing that the core of a TU game with arbitrary many players is not empty if and only if the game is balanced. This result is obtained by applying a new purely algebraic strong duality theorem.

Papers related to OTKA-72856

1. Balog D, Bátyi TL, Csóka P, Pintér M (2011): Tőkeallokációs módszerek és tulajdonságaik a gyakorlatban (Risk allocation methods and their properties in practice). Közgazdasági Szemle, LVIII: 619-632.

2. Balog D, Csóka P, Pintér M (2010): Tőkeallokáció nem likvid portfoliók esetén. Hitelintézeti Szemle, 9: 604-616.

3. Bednay D (2012a): Stable sets in one-seller assignment games. Annals of Operations Research (forthcoming).

4. Bednay D (2012b): Alkuegyensúlyok és stabil halmazok. In: Solymosi T - Temesi J (eds): Egyensúly és optimum, Tanulmányok Forgó Ferenc 70. születésnapjára. Aula Kiadó, Budapest, pp. 3-12.

5. Csóka P, Kóczy ÁL, Pintér M, Herings J-J (2011): Convex and exact games with non-transferable utility. European Journal of Operational Research, 209, 57-62.

6. Forgó F, Fülöp J (2008): On the implementation of the L-Nash bargaining solution in two-person bargaining games. Central European Journal of Operations Research, 16: 359-378.

7. Forgó F (2010): A generalization of correlated equilibrium: A new protocol. Mathematical Social Sciences, 60(3): 186-190.

8. Forgó F (2011): Generalized correlated equilibrium for two-person games in extensive form with perfect information. Central European Journal of Operations Research, 19 (2): 201-213.

9. Kóczy ÁL, Pintér M (2011): Az ellenzék ereje - általánosított súlyozott szavazási játékok (The power of opposition – generalized weighted voting games). Közgazdasági Szemle, LVIII: 543-551.

10. Pintér M (2009): A Shapley-érték axiomatizálásai (Axiomatizations of the Shapley value). Alkalmazott Matematikai Lapok, 26: 289-315.

11. Pintér M (2010a): The non-existence of a universal topological type space. Journal of Mathematical Economics, 46: 223-229.

12. Pintér M (2010b): The existence of an inverse limit of an inverse system of measure spaces - A purely measurable case. Acta Mathematica Hungarica, 126: 65-77.

13. Pintér M (2011a): Regression games. Annals of Operations Research, 186: 263-274.

14. Pintér M (2011b): Algebraic duality theorems for infinite LP problems. Linear Algebra and its Applications, 434, 688-693.

15. Pintér M (2012): Young's axiomatization of the Shapley value – a new proof (version 3) (working paper). http://arxiv.org/abs/0805.2797v3

16. Solymosi T (2010): Sensitivity of fair equilibrium prices in assignment markets (manuscript).

17. Solymosi T (2011): Sensitivity of the nucleolus in assignment games (manuscript).

18. Solymosi T (2012): Lexikografikus allokációk a hozzárendelési játékokban. In: Solymosi T - Temesi J (eds): Egyensúly és optimum, Tanulmányok Forgó Ferenc 70. születésnapjára. Aula Kiadó, Budapest, pp. 33-48.

19. Solymosi T, Brugueras J, Raghavan TES (2012): Pairwise monotonicity of the nucleolus in assignment games (manuscript).

References

[1] Aumann RJ (1974): Subjectivity and correlation in randomized strategies. Journal of Mathematical Economics, 1: 67-96.

[2] Brandenburger A, Dekel E (1993): Hierarchies of beliefs and common knowledge. Journal of Economic Theory, 59: 189-198.

[3] Chevan A, Sutherland M (1991): Hierarchical Partitioning. The American Statistician, 45: 90-96.

[4] Demange G, Gale D (1985): The strategy structure of two-sided matching markets. Econometrica, 53: 873-888.

[5] Demange G, Gale D, Sotomayor M (1986): Multi-item auctions. Journal of Political Economy, 94: 863-872.

[6] Dubey P (1975): On the uniqueness of the Shapley value. International Journal of Game Theory, 4: 131-139.

[7] Forgó F (1983): A game theoretic approach for multicriteria decision making. In: Wierzbicky J (ed): Interactive Decisions in Lecture Notes in Economics and Mathematical Systems, 229. Springer Verlag, Berlin, 1-15.

[8] Forgó F, Pintér M, Simonovits A, Solymosi T: Játékelmélet (elektronikus jegyzet), (Game Theory, electronic lecture notes, in Hungarian). http://www.uni-corvinus.hu/opkut/tantargyak/jatekelmelet.html

[9] Grömping U (2007): Estimators of Relative Importance in Linear Regression Based on Variance Decomposition. The American Statistician, 61: 139-146.

[10] Harsányi J (1974): An equilibrium-point interpretation of stable sets and a proposed alternative definition. Management Science 20: 1472-1495.

[11] Hart S, Mas-Colell A (1988): The Potential of the Shapley Value. In: The Shapley Value, Alvin E. Roth (ed), Cambridge University Press (1988), 127-137.

[12] Hart S, Mas-Colell A (1989): Potential, value, and consistency. Econometrica, 57: 589-614.

[13] Heifetz A (1993): The bayesian formulation of incomplete information - the non-compact case. International Journal of Game Theory, 21: 329-338.

[14] Heifetz A, Samet D (1998): Topology-free typology of beliefs. Journal of Economic Theory, 82: 324-341.

[15] Heifetz A, Samet D (1999): Coherent beliefs are not always types. Journal of Mathematical Economics, 32: 475-488.

[16] Howard JV (1992): A social choice rule and its implementation in perfect equilibrium. Journal of Economic Theory, 56: 142-159.

[17] Leonard HB (1983): Elicitation of honest preferences for the assignment of individuals to positions. Journal of Political Economy, 91: 461-479.

[18] Lipovetsky S, Conklin M (2001): Analysis of Regression in Game Theory Approach. Applied Stochastic Models in Business and Industry, 17: 319-330.

[19] Meier M (2001) An infinitary probability logic for type spaces. CORE Discussion paper No. 0161.

[20] Mertens JF, Sorin S, Zamir S (1994): Repeated games, part A. CORE Discussion Paper No. 9420.

[21] Mertens JF, Zamir S (1985): Formulations of bayesian analysis for games with incomplete informations. International Journal of Game Theory, 14: 1-29.

[22] Moulin H, Vial J-P (1978): Strategically zero-sum games: The class whose completely mixed equilibria cannot be improved upon. International Journal of Game Theory, 7: 201-221.

[23] Nash JF Jr. (1953): Two-person cooperative games. Econometrica, 21: 128-140.

[24] Neyman A (1989): Uniqueness of the Shapley value. Games and Economic Behavior, 1: 116-118.

[25] Núñez M, Rafels C (2002): The assignment game: the τ-value. International Journal of Game Theory, 31: 411-422.

[26] Peleg B, Sudhölter P (2003): Introduction to the Theory of Cooperative Games. Kluwer Academic Publishers, Boston/Dordrecht/London.

[27] Pintér M (2005): Type space on a purely measurable parameter space. Economic Theory, 26: 129-139.

[28] Pintér M (2007): Regressziós játékok. (Regression games, in Hungarian) Szigma, XXXVIII (3-4): 131-148.

[29] Pour-El MB, Richards JI (1989): Computability in Analysis and Physics, Springer-Verlag.

[30] Rubinstein A (1982): Perfect equilibrium in a bargaining model. Econometrica, 50: 97-109.

[31] Shapley LS (1953): A Value for n-Person Games. In: Contributions to the Theory of Games, Volume II (Annals of Mathematical Studies 28), Kuhn HW–Tucker AW (eds), 307-317.

[32] Shapley LS, Shubik M (1972): The assignment game I: The core. International Journal of Game Theory, 1: 111-130.

[33] Shorrocks AF (1999): Decomposition Procedures for Distributional Analysis: A Unified Framework Based on the Shapley Value. working paper.

[34] Shubik M (1962): Incentives, decentralized control, the assingnment of joint costs, and internal pricing. Management Science, 8: 325-343.

[35] Solymosi T, Raghavan TES (1994): An algorithm for finding the nucleolus of assignment games. International Journal of Game Theory, 23: 119-143.

[36] Stufken J (1992): On Hierarchical Partitioning. The American Statistician, 46: 70-71.

[37] Thompson GL (1981): Auctions and market games. In: Essays in Game Theory and Mathematical Economics in Honor of Oskar Morgenstern, Aumann RJ (ed), Bibliographisches Institut, Wissenschaftsverlag, Mannheim, 181-196.

[38] Young HP (1985): Monotonic Solutions of Cooperative Games. International Journal of Game Theory, 14: 65-72.