Final report of NF-72610

"The Strong, the Weak and the Cunning: Power and Strategy in Voting Games"

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Introduction

The aim of this project has been to develop and refine methods to measure the allocation of power in voting situations. In our research plan we have outlined three particular areas. While many of the results are yet to be published, we are proud to say that the project has been successful both in producing interesting results in all these areas and to produce high quality international publications.

In the following we outline the results per area. In the text we omit citations to the general literature – for that we refer the reader to our papers.

1 Strategic voting games

Despite the sound mathematical foundations of power indices, all the widely used power indices exhibit properties considered strange or counter-intuitive. The strange features are known as the paradoxes of voting power in the literature. This line of research has focussed on the paradox of quarrelling members: the observation that two voters may mutually benefit from refusing to cooperate.

We have introduced strategic voting games, where the individual players unilaterally initiate a quarrel to increase their power: At first it may sound paradoxical that someone's power can increase by refusing to participate in majority coalitions. While this general observation is not so surprising, the exact specification of the model including the existence of the corresponding noncooperative equilibrium is less straightforward. We have introduced two alternative models (Kóczy, 2008b, 2010c), where, despite relatively minor differences the conclusions are surprisingly different. In either case we present sufficient conditions for the existence of a unique strategic modification of a power index.

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We find that only minimal winning coalitions might sustain, surplus coalitions are subject to quarrelling (while some minimal winning coalitions might be, too).

We were also interested in other paradoxes and in the possibility to modify power indices in such a way that the mathematical foundations are still there, but their predictions are less surprising. Looking into this problem Kóczy (2009b) has found that the paradox of new members contradicts the null player property. This paradox states that a voter's power may increase when new voters are admitted into the voting body and so each member's voting share decreases. The null player property states that a voter must have zero power if its addition to a subset of the voters never turns the coalition into a winning coalition and therefore never contributes to the decision making. Interestingly the "power index" where power equals to the share of voting weights does not exhibit any of the paradoxical properties (Kóczy, 2008a). This index gives the natural, instinctive allocation of power. When we talk about paradoxes, then perhaps we only criticise that the predictions differ from this allocation.

The effect of quarrelling works via the externalities and so the game can be seen as a coalition formation (or rather: a coalition destruction) game with externalities. For these games we had already defined the core and now we have been working on its implementation. Kóczy (2009c) provide results for totally balanced games, while Kóczy (2009e) promises a completely general characterisation on the set of games in partition function form. A better understanding of coalitional games with externalities will allow us to define the equilibrium of the quarrelling game in a more direct and more general way. This, however remains an open question.

We have also looked at applying the model to particular voting situations where the quarrelling could make sense. It turned out, however, that, apart from fairly small games, the problem is very complex. Lange and Kóczy (2010) introduced a way to compute power indices directly from the minimal winning coalitions, skipping the step of generating the whole set of winning coalitions thereby conceptually and usually also computationally simplifying the calculations.

Tanvé (2010a,b) studies the strategic aspects of programme and communication strategies of parties: what and how do parties communicate in a two-party system. Under what conditions can we expect the convergence of campaign issues, and to what extent do parties wish to make their positions clear.

The field of 'bargaining committees' can be seen as extension of voting games. The theory on voting games aims to answer the question what is the extent of power each individual voter holds. The theory on committees takes this question one step further in asking how voters actually use this power. If a number of voters is to agree on a certain division of a perfectly divisible good, then logically one would expect a dictator to take the entire

good and a voter without power to be left empty-handed. The question what happens when the case is less clear cut can be answered to some extent by the concept of "solution/value", a mathematical object that integrates the Shapley-Shubik power index and the Nash bargaining solution, on a conceptual as well as axiomatic level.

The Nash bargaining solution might be unacceptably robust against changes in the bargaining situation, reveals that the solution/value is not without problems. In particular, in the setting of repeated play, it is possible that abstention (that is: refusal to play) in a number of voting rounds leaves the share a player can secure unaffected. This problem could be mended by integrating the Shapley-Shubik power index with a 'monotonic' bargaining solution. However, the ideal candidate, Dubra's asymmetric Kalai-Smorodinsky solution, has only been formalised for two-person games, which implies that the implied solution/value would only be meaningful when the number of committee members is restricted to two.

An extension to bargaining committees with any finite number of voting members thus necessitates a similar extension of Dubra's bargaining solution. Driesen, Perea, and Peters (2011) obtained a generalised definition and axiomatic characterisation of this asymmetric, monotonic, multi-player solution. Driesen (2011) provided an alternative axiomatisation of Imai's lexicographic maximin solution.

2 Spatial voting games and power over communication networks

Unlike in games with quarrelling, where the set of feasible coalitions is determined endogenously, in this subproject we assume that there is an underlying structure given by geography or the political positions of voters that determines which coalitions may form. For instance, over the linear political spectrum we often presume connected coalitions, that is, the (moderate) left and right will not for a coalition unless centre is also included.

Reynaud, Lange, Gatarek, and Thimann (2007) study spatial voting via the example of voting in the IMF. The interests of countries with intensive trade relations are similar even if their structure or level of development is different. The trade volume between two countries can therefore be seen as a measure of proximity. The game thereby defined can be solved, but the general problem remains open.

Ghintran, Gonzalez-Arangüena, and Manuel (2010); Ghintran (2012) study situations where the power of a player is partly given by its location on a network. While in theory this could depend on the entire network, perhaps the most interesting part is how to share the value added by a new link or arc.

In a forthcoming paper Ghintran (2012) considers situations where agents

use their negotiation power to affect the distribution of the worth on their behalf and defines an allocation rule that describes the impact of the negotiation power of agents on the distribution of the worth among them. This allocation rule satisfies a set of attractive properties.

Ghintran, Gonzalez-Arangüena, and Manuel (2010) considers situations where the network structure is not given and fixed, that is, several alternative networks can form and thus players are able to form occasional alliances as well as long term relationships. We model such situations by using probabilistic networks. We define an allocation rule for the class of probabilistic communication situation and provide two characterisations for this solution.

Béal, Ghintran, Rémila, and Solal (2011) overview the literature of allocation problems in a special network: a river. A river differs from other networks in that it has a direction, strategies affect other players only downstream and there are no (directed) circles.

Béal, Chakrabarti, Ghintran, and Solal (2010) study the solution concepts of partial cooperative Cournot-Nash equilibria and partial cooperative Stackelberg equilibria and establish sufficient conditions for which partial cooperative Cournot-Nash equilibria and partial cooperative Stackelberg equilibria exist in supermodular games. Such solutions are useful in strategic network formation games as highlighted by the application in the paper.

Tournaments are directed graphs, where the the direction of an arc tells us the winner of a particular pairwise comparison. Here the objective is not to determine an allocation, but a winner, or more generally the ranking of the nodes. Kóczy and Strobel (2010) introduced a method to rank academic journals using a tournament approach and provide a ranking of the main academic journals in economics. Kóczy and Strobel (2008) show that the acclaimed invariant method can be manipulated: a journal can improve its position in the ranking by making a citation to another journal. Kóczy, Nichifor, and Strobel (2009) investigate the property of article splitting – the length of a typical paper should not influence the quality ranking – and introduce a modified version of the invariant method that is invariant to article splitting too.

3   Gradual coalitions

In the third and last subproject we are interested in the same type of results, but in a more general setting where the membership of players in the coalitions is not certain and/or the value of coalitions is a random variable.

Kóczy and Pintér (2011b) introduced generalised weighted voting games. These games are motivated by weighted voting situations via representatives, where, due to illness or other engagement a voting block, such as a party cannot always rely on the full number of representatives and hence the full weight in the voting game. Such a model fits very well voting in a national
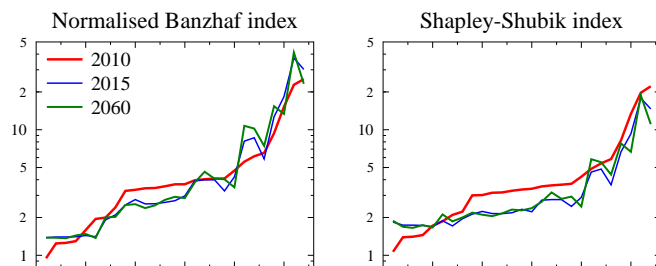
Figure 1: Representation per citizen from Germany to Cyprus (Kóczy, 2011).

parliament (Kóczy and Pintér, 2011a).

Lange and Grabisch (2008, 2011) respectively generalised the Shapley-value to multichoice games and bi-capacities. Lange (2010) took a somewhat different approach by introducing fuzzy voting games and studying the power distribution in these games. The application of this fuzzy approach to network games is still underway with Ghintran.

4 Other results

We also mention a number of results that do not form an integral part of the original research proposal, but are closely related.

4.1 Applications

Kóczy (2011, 2010b) combines the theory of power indices with demographic data from the European Statistical Office to find that Lisbon Treaty does not only increase the decision probability in the EU Council of Ministers, but the benefits are not shared equally. The biggest losses are suffered by medium-sized countries with shrinking populations, with Hungary in the bottom 3 when ranked by the change in power.

Kóczy (2010e) calculates power distributions in international organisations, focussing on the Bretton Woods institutions and finds that if switching between country groups were permitted, Hungary would be better off joining the Scandinavian group.

Still on the topic of international organisations, Kóczy (2010d) provides a foresighted model to explain strategic differences between the EFTAn and Eastern extensions of the European Union.

4.2 Stability

In the theory of values it is simply assumed that the grand coalition forms due to efficiency considerations and the only task for the social planner is to

find a fair allocation of the resource or costs. In many of the applications, however a coalition of the players, unhappy with the allocation, has the possibility to leave the grand coalition and thereby provide an improvement to all its members. Such a departure is never profitable if the allocation belongs to the core, however this can only be guaranteed for convex games. Csóka, Herings, Kóczy, and Pintér (2011); Csóka, Herings, and Kóczy (2011) provide expressions for TU and NTU games to test convexity.

In matching problems we look for solutions that are stable and fair. Kóczy (2009a) provides a general introduction to general matching problems, while Kóczy (2010a) evaluates the Hungarian high school and university admission systems from a matching theoretic point of view.

Kóczy (2009d,e) implements the pessimistic recursive core by means of a sequential noncooperative process. The first paper has more limited results to partition function form games where all the residual cores are nonempty, while the second paper generalises these results to all partition function form games.

5   Dissemination of the results

Besides the publications referenced at the end of the paper, the members of the group have presented their work at numerous conferences and visits including the following:

2008 MKE2008, Budapest; Corvinus, Budapest;

2009 SING5, Amsterdam; EURO XXIII, Bonn; Coalition Theory Workshop, Maastricht; UECE Meeting, Lisbon; New Topics on Game Theory; Seville; Economics, Management and Finance, Monpellier; MOK XVIII, Balatonőszöd; Budapest Tech, Jubilee Conference, Budapest; Tinbergen Workshop, Amsterdam; GREQAM, Marseille; Corvinus, Budapest

2010 SCW10 Moscow; SING6, Palermo; CTN, Marseille; MINT1, Lyon; ComSoc Workshop; Dagstuhl; UECE Meeting, Lisbon; Chinese Game Theory and Experimental Economics Association Conference, Beijing; Bolyai Memorial Conference, Budapest; MKE2010, Budapest; IEHAS Seminar; Budapest; Corvinus, Budapest;

2011 SING7, Paris; MINT2, St-Étienne; SAET, Faro; LGS6, Bucharest; PET11, Bloomington, IN; International Workshop of Game Theory, Tel Aviv; Ostrava University, Ostrava; Corvinus, Budapest

The game theory blogs (koczy.blog.hu in Hungarian; laszlokoczy.blogspot.com in English) of the principal investigator have also proved to be a very efficient way to communicate the results to the broader public.

References

BÉAL, S., S. CHAKRABARTI, A. GHINTRAN, AND P. SOLAL (2010): "Partial Cooperative Equilibria: Existence and Characterization," *Games*, 1(3), 338–356.

BÉAL, S., A. GHINTRAN, E. RÉMILA, AND P. SOLAL (2011): "The river sharing problem: a survey," .

CSÓKA, P., P. J.-J. HERINGS, AND L. A. KÓCZY (2011): "Balancedness conditions for exact games," *Mathematical Methods of Operations Research*, 74(1), 41–52.

CSÓKA, P., P. J.-J. HERINGS, L. A. KÓCZY, AND M. PINTÉR (2011): "Convex and exact games with non-transferable utility," *European Journal Of Operational Research*, 209(1), 57–62.

DRIESEN, B. (2011): "Proportional concessions and the leximin solution," Discussion Paper 1106, Keleti Faculty of Economics, Óbuda University, Budapest.

DRIESEN, B., A. PEREA, AND H. PETERS (2011): "The Kalai-Smorodinsky bargaining solution with loss aversion," *Mathematical Social Sciences*, 61(1), 58–64.

GHINTRAN, A. (2012): "Weighted position values," *Mathematical Social Sciences*.

GHINTRAN, A., E. GONZALEZ-ARANGÜENA, AND C. MANUEL (2010): "A probabilistic position value," Discussion Paper 1006, Keleti Faculty of Economics, Óbuda University, Budapest.

KÓCZY, L. A. (2008a): "Proportional power is free from paradoxes," Working Paper 0806, Budapest Tech, Keleti Faculty of Economics, Budapest.

——— (2008b): "Strategic power indices: Quarrelling in coalitions," Working paper, Budapest Tech, Keleti Faculty of Economics, Budapest.

——— (2009a): "Központi felvételi rendszerek: Taktikázás és stabilitás," *Közgazdasági Szemle*, 56(5), 422–442.

——— (2009b): "Measuring Voting Power: The paradox of new members vs. the null player axiom," in *Towards Intelligent Engineering and Information Technology*, ed. by I. J. Rudas, J. Fodor, and J. Kacprzyk, pp. 67–78. Springer, Berlin.

——— (2009c): "Sequential Coalition Formation and the Core in the Presence of Externalities," *Games and Economic Behavior*, 66, 559–565.

——— (2009d): "Sequential Coalition Formation and the Core in the Presence of Externalities," *Games and Economic Behavior*, 66, 559–565.

——— (2009e): "Stationary consistent equilibrium coalition structures constitute the recursive core," Discussion Paper 0905, Keleti Faculty of Economics, Óbuda University, Budapest.

——— (2010a): "A magyarországi felvételi rendszerek sajátosságai," *Közgazdasági Szemle*, 57(2), 142–164.

——— (2010b): "Lisszaboni kilátások: Magyarország befolyása az Európai Unióban a lisszaboni reform után," .

——— (2010c): "Power indices with endogenously selected infeasible coalitions," Unpublished work 1001, Óbuda University, Budapest.

——— (2010d): "Strategic Aspects of the 1995 and 2004 EU Enlargements," *Group Decision and Negotiation*, 19(3), 267–277.

——— (2010e): "Voting power in international organisations," .

——— (2011): "Beyond Lisbon: Demographic trends and voting power in the European Union Council of Ministers," *Mathematical Social Sciences*.

Kóczy, L. A., A. Nichifor, and M. Strobel (2009): "Article length bias in journal rankings," .

Kóczy, L. A., and M. Pintér (2011a): "Az ellenzék ereje - általánosított súlyozott szavazási játékok," *Közgazdasági Szemle*, 58(6), 543–551.

——— (2011b): "The men who weren't even there: Legislative voting with absentees," Discussion Paper 2011/29, Institute of Economics, Hungarian Academy of Sciences, Budapest.

Kóczy, L. A., and M. Strobel (2008): "The Invariant Method can be Manipulated," *Scientometrics*, 81(1), 291–293.

——— (2010): "The World Cup of Economics Journals: A Ranking by a Tournament Method," .

Lange, F. (2010): "Fuzzy weighted voting games and power indices," .

Lange, F., and M. Grabisch (2008): "A Recursive Solution Concept for Multichoice Games," *Acta Polytechnica Hungarica*, 5(1), 47–57.

——— (2011): "New axiomatizations of the Shapley interaction index for bi-capacities," *Fuzzy Sets and Systems*, 176(1), 64–75.

Lange, F., and L. A. Kóczy (2010): "Power indices expressed in terms of minimal winning coalitions," Discussion Paper 1002, Keleti Faculty of Economics, Óbuda University, Budapest.

Reynaud, J., F. Lange, L. Gatarek, and C. Thimann (2007): "Proximity in coalition building," Technical report 0808, Budapest Tech, Keleti Faculty of Economics.

Tanvé, M. (2010a): "Ambiguity and abstention," .

——— (2010b): "Issue Convergence in the Electoral Campaigns," .