Final Report on PD_19 132118 “Dynamics and entanglement in quantum field theory”

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The published results can be arranged to two groups, given in sections **1.** and **2.** below. Unpublished results and ongoing work is briefly presented in section **3**. Finally, in section **4**. I explain the deviations from the original research and work plans.

1. Spin chains: dynamics and entanglement

1.1. Entanglement oscillations near a quantum critical point

This study is the direct continuation of a work on the transverse field Ising model which was finished after the application to the grant. In this follow up we studied the quantum Ising chain in both transverse and longitudinal magnetic field. Close to the critical point (at critical transverse and zero longitudinal magnetic field) the system can be described by a scaling field theory. When the transverse field is tuned to be critical, but the longitudinal one is small but finite, the related scaling field theory is an integrable field theory with known particle spectrum. We studied the dynamics of entanglement between two infinite subsystems after changing suddenly the longitudinal magnetic field, starting from the ground state (this is a so-called quantum quench). Field theoretical calculations based on twist field field form factors predict oscillations with frequencies given by the masses of particles with amplitudes related to the form factors. We numerically confirmed the presence of these long living oscillations with the predicted frequencies and amplitudes using the infinite system extension of the time evolving block decimation algorithm (iTEBD), with extrapolation to the scaling field theory limit. One also expects linear in time growth of the entanglement. Due to complicacy of the model, we have not been able to calculate this from the field theory, neither we could see any trace of it on the numerically accessible simulation times. This study was done in collaboration with Olalla A. Castro-Alvaredo, István M. Szécsényi and Jacopo Viti. The results were published in Physical Review Letters in 2020.

1.2. Relaxation and entropy generation after quenching quantum spin chains

This project was also triggered by previous results mentioned earlier. In a certain “small quench” limit the relaxation time of the order parameter is related to the proportionality constant of the linear growth of entanglement entropies after a mass quench in the transverse field Ising model. Using symmetry arguments we proposed a formula for the q-state Potts model which is a generalization of the Ising model. We were able to confirm numerically (using iTEBD) that the proposed formula is valid for small mass quenches in the three state Potts model. iTEBD let us study more general quenches in this family of models. We studied

quenches of the longitudinal magnetic field starting from the ground state of the paramagnetic phases. Due to the explicit breaking of the symmetry, the proposed formula is not valid in this case. However we found that the growth rates of Rényi entropies and the relaxation of the order parameter behave qualitatively the same as the entanglement entropy. The curious bahaviour of the entropy growth rate was previously studied by the collaborators and dubbed as the “dynamical Gibbs effect”. The observation that the relaxation rate follows the same qualitative behaviour can lead to an experimental observation the dynamical Gibbs effect. This work was done in collaboration with Octavio Pomponio and Gábor Takács. The results were published in SciPost Physics in 2020.

1.3. Confinement and bound states of bound states in a transverse-field two-leg Ising ladder

In the quantum Ising model in the ferromagnetic phase, the presence of finite longitudinal field leads to the confinement of elementary excitations, forcing them to form bound states. Analogously to quantum chromodynamics, these new states are called mesons. Recently it was found that the presence of confinement severely constrains the propagation of correlations and entanglement. During this work we studied two coupled transverse field Ising chains, which in this way form a ladder. We showed that this system hosts different types of confined excitations without the presence of external longitudinal field: interchain and intrachain mesons, moreover, bound states thereof. It is worth noting that analogue of the interchain meson was found in other spin systems, but the intrachain mesons and the bound states of the mesonic states were not found before. We studied the model numerically with two different approaches: density matrix renormalization group and the extension of the truncated conformal space approach, contrasting results from the spin model and its scaling limit and found good agreement. This work was done in collaboration with Flávia B. Ramos, J. C. Xavier and Rodrigo G. Pereira and it was published in Physical Review B in 2020.

2. Integrability and its breaking in quantum field theories

2.1. On factorizable S-matrices, generalized TTbar, and the Hagedorn transition

In this work we studied the thermodynamic Bethe ansatz of CDD-deformed scattering theories. We found that such a deformation leads to a singular behaviour of thermodynamic quantities similar to the case of TTbar deformations. Such a singularity also appears in string theory and it is called the Hagedorn behaviour. A numerical method was developed to study the continuation of the thermodynamic Bethe ansatz through the singular point, to the secondary branch. We also analyzed an interesting limit called the narrow resonance limit. The dynamics behind the formation of the singularity is still not understood completely and the physics of the secondary branch is also not clear. The mathematical description of the narrow resonance limit is also an open question. It would be also interesting to study behaviour of the form factors under CDD deformations, which could lead to the construction form factors of branch-point twist fields, providing a way

to study entanglement properties of such theories. This work was done in collaboration with Giancarlo Camilo, Thiago Fleury, Stefano Negro and Alexander Zamolodchikov and it was published in the Journal of High Energy Physics in 2021.

2.2. Duality and Form Factors in the Thermally Deformed Two-Dimensional Tricritical Ising Model

In this work we studied the thermal deformation of the tricitical Ising model, which is a generalization of the Ising model. We calculated one and two-particle form factors of the leading and sub-leading magnetization operators and their dual disorder operators. This result is interesting for different reasons. First of all, the set of form factor equations are not sufficient to make distinction between the two magnetic operators, and leaves us with free parameters. In order to overcome this problem, one needs to fully exploit the Kramers—Wannier like duality of the theory, and treat order (magnetic) and the disorder operators on an equal footing. Together with the clustering property, this leads to nonlinear constraints and the delta-theorem provides the physical solution. On the other hand, the form factors can be used to construct the dynamical structure factors, which can be measured in an experimental setting. The results were checked numerically using the truncated conformal space approach. The results of this work immediately generated some interest: right after the publishing the pre-print, we were contacted by an experimental group from Germany, looking for a physical realization of the model. This work was done in collaboration with Axel Cortés-Cubero, Robert Konik, Giuseppe Mussardo, Gábor Takács and it was published in SciPost Physics in 2022.

2.3. Confinement in the tricritical Ising model

Confinement in low dimension appears in many cases and affect the dynamics and the spread of the entanglement through a system drastically. Confinement of kink excitations takes place when ground state degeneracy is lifted by the presence of an external field. We expect this to happen in the thermal perturbation of the tricritical Ising model, when one turns on a small magnetic perturbation (leading or sub-leading) in the ferromagnetic phase of the model. In this work we used a semiclassical approach to predict the mass of the confined particles, which are called mesons. The unperturbed model hosts excited kink particles, that can also get confined in principle. Numerical results from the truncated confromal space show the validity of the semiclassical approach for small fields. We also found that in case of the excited kinks, confinement is present. Moreover, we found a new type of confinement, when it is caused by a thermal perturbation added to the sub-leading deformation of the tricritical point. Surprisingly in this case the semi-classical approach predicts the mass of the mesons for large perturbations as well. This work was done in collaboration with Giuseppe Mussardo and Gábor Takács and it was published Physics Letters B in 2022.

2.4. Variations on vacuum decay: The scaling Ising and tricritical Ising field theories

Another related aspect of the lift of vacuum degeneracy is the decay of the false vacuum. In this case we prepare the system in the false vacuum, and let it evolve. After some very short initial transient, true vacuum bubble nucleation is expected to happen on an exponential time scale. Using the truncated conformal space, we determined the true vacuum state with a given magnetic field. Then we used this state as an initial state to generate quantum dynamics using the Hamiltonian with the opposite sign of the magnetic field, i.e. performed a quantum quench. Using this protocol we prepare a state, which is expected to be very close to the false vacuum of the Hamiltonian used to generate the time evolution. We managed to reproduce the exponential decay of the false vacuum in the Ising model in longitudinal magnetic field and in various perturbations of the tricritical Ising theory, even starting from three degenerate ground states and energy lift generated by an even operator. Moreover, our results on the dependence of the characteristic decay constant on the vacuum energy difference are consistent with theoretical predictions. We point out that the constant in front of the exponential decay has the right dependence on the volume, but there is a non-universal numerical prefactor that deserves further study. We found that if there exist particle excitations above the false vacuum, then oscillations related to the mass of these particles wash out the exponential decay and the truncation method is not powerful enough to study the vacuum decay. This work was done in collaboration with Giuseppe Mussardo and Gábor Takács and it was published in Physical Review D in 2022.

2.5. Multicriticality in Yang—Lee edge singularity

Another aspect of integrability breaking is the analytic structure of the partition function and so the free energy as functions of the couplings of the theory. A well know example of a non-trivial behaviour is the imaginary magnetic perturbation of the thermal Ising model in the paramagnetic phase which leads to the so-called Lee—Yang edge singularity, where the zeroes of the partition function accumulate in a certain way and show critical behaviour. The criticality is captured by a non-unitary minimal conformal field theory, M(2,5). In this work we spotted similar behaviour in the perturbations of the tricritical fixed point. We found that there is a critical surface which realizes M(2,5), furthermore we found non-unitary tricriticality, which is governed by the minimal model M(2,7). We proposed a generalization of this to higher multicriticality and made an attempt to formulate the problem in terms of Landau—Ginzburg effective field theory. This work was done in collaboration with Alessio Miscioscia, Giuseppe Mussardo and Gábor Takács and it is currently under review in the Journal of High Energy Physics.

3. Ongoing work related to the research project

3.1. Quantum quenches in the Blume—Capel model

Based on our results regarding the longitudinal field Ising model (see **1.1.**) and the results on the form factors of the thermally deformed theory (see **2.2.**) I started to work with Csilla Király a physics Bachelor student on a degree project to study similar questions in the Blume—Capel model, which is a spin-one chain. Based

on known results in the literature we were able to determine the couplings realizing the thermally deformed tricritical Ising model on the spin chain and identify the even part of the particle content. Carrying out the scaling limit using iTEBD calculations we managed to get results for the time evolution of the local magnetization compatible with field theoretical calculations based on form factors from our work in **2.2.** The time evolution of entanglement entropy is also similar to the one in the longitudinal Ising model case in **1.1.**, however there are no twist field form factors available to check these results against field theoretical predictions. This project led to a successful BSc defense in 2022, and later evolved to a scientific student conference ("TDK") proposal which earned first price in the Budapest University of Technology and Economics Scientific Student's Conference in 2022 and got into the National one taking place in the Spring 2023. In the near future we would like to carry out more detailed calculations and reproduce the vacuum decay (**see 2.4.)** and publish the results in an international journal. After that, the next step is the determination of the twist field form factors to have analytic predictions for the time evolution of entanglement.

3.2. Truncated (orbifold) confromal space approach to entanglement in field theories

This project aims to extend the truncated conformal space approach to study entanglement properties of quantum field theories numerically. This idea was proposed in the literature a few years ago, but in its form is not suitable for precise numerical study. Based on the twist field approach we aim to solve perturbations of conformal field theories in the replica (orbifold) picture, and get access to the entanglement between different parts of the system. For this to work one needs input data from the orbifold conformal field theory. Much is known about orbifold models in abstract, general terms, but it turned out that the data necessary to implement the numerical method are not readily available in the literature. At the moment we are getting a better understanding of these complicated theories. So far we managed to understand the field content and realize the corresponding representations of the orbifold Virasoro algebra. The next step is to extract the so-called fusion rules and the operator product expansion coefficients. Having these ingredients in hand, we can set up the truncated space method for different models. This project could possibly lead to two publications, one is about the calculation of the necessary input and another about applications. This project is a collaboration with István M. Szécsényi.

3.3. Entanglement spectrum from the Bisognano—Wichmann theorem

There is an important proposal in the literature that connects the entanglement spectrum of the ground state of many-body systems and quantum field theories to the spectrum of a boundary conformal field theory. Exploiting the Bisognano—Wichmann theorem, which states that the so-called entanglement or modular Hamiltonian can be written as the integral of the energy density, weighted by certain space-dependent factors over the subsystem. We are trying to capture this explicitly in 1+1 dimensional quantum field theory using the direct construction of the modular Hamiltonian. As a first step, we studied the entanglement spectrum of the Ising model at criticality, where the theorem seems to apply. However there is an obstacle in this

particular case, namely the large number of modes that have to be included in the construction, which can be overcome exhausting the free nature of this theory. We do not know yet, if this is the general case, making the extension to interacting theories very hard, or this is the result of the theory being conformal. Next step is to extend this result for the thermal perturbation of the Ising fixed point, which is massive free theory, to see whether it is possible to easily extend the idea at least for massive interacting theories. This work is still in progress, together with Márton Kormos.

3.4. Time evolution of the chiral entanglement entropy

The study of the chiral entanglement entropy was the first point of this research project, however due to the unexpected number of results from the study of dynamics in various different models, this study was not prioritized. However, as a by-product of the numerical simulations regarding the vacuum decay problem (i.e. special field theoretical quenches) I managed to collect data on the chiral entanglement entropy. Based on these data I propose that chiral entanglement entropy can be used as a diagnostic tool to put limits on the validity of results from the truncated conformal state approach. In a similar fashion as the bond dimension limits the entanglement can be stored in a matrix product state, the maximal chiral entanglement in a state is limited by the truncation applied. In the cases I studied (vacuum decay quenches) the chiral entanglement initially grows, and after some time it saturates, close to the value dictated by the truncation. So far these are preliminary results, deserving more analysis and analytic treatment. I hope in the near future this can lead to a technical publication that could guide other researchers using truncation methods.

4. Deviations from the research and working plans

The original proposal listed four main objectives of the project, namely:

1. Time evolution of the chiral entanglement entropy in one space and one time dimension
2. Scaling limit of quenches in the Ising model (free fermionic theory)
3. Numerical study of dynamics and entanglement in generic quenches
4. Extension to higher dimensional field theories

Due to collaborations started after the submission of the proposal prior to the starting of this project, most of the research carried out considered point **3**. from the above list. Namely, in the first year I concentrated on the quench dynamics of various spin chains. Later I focused on integrability and its breaking in the tricritical Ising model and studied general dynamical properties such as mass spectrum, form factors and dynamical structure functions, false vacuum decay process. Although only the last problem can be directly cast as a quench problem, the obtained results on generic two dimensional field theories appear in the quench context (see eg. **3.1.**). Moreover integrable TTbar deformations of scattering theories was included to the project. I

believe that these experiences and results provide a firm base to start the study of questions raised in point **4**. especially regarding non-integrable theories.

Unfortunately point **2.**, which is a rather technical problem with certainly less impact than the above mentioned results, was not carried out during this program.

Point **1.** was however partially addressed leading to a conjecture on the usage of the chiral entanglement as a diagnostic tool for the validity of results from the truncated conformal space. I believe this finding will soon lead to a further publication related to this project.

Due to COVID restrictions the originally planned scientific visits and conference participations were limited to the third year. However during the project I presented two seminars in the Theoretical Physics seminars in the Budapest University of Technology and Economics (2020 and 2021); a seminar in NORDITA, Stockholm (2022); short talks in the Budapest Integrability day (2022) and the Statistical Physics Day in Budapest (2022); a workshop talk in the program “Integrability in String, Field and Condensed Matter Theory” in the Kavli Institute for Theoretical Physics, Santa Barbara (2022).

In the working plan I planned to publish six to eight papers during the project; and finally there are seven publications appeared in international journals and one more under review at the time of writing the report. Moreover as summarized in section **3.**, there are several ongoing work related to this project which will hopefully lead to further publications in the near future.