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1 On the L_p dual Minkowski problem

The L_p dual Minkowski surface area with additional parameter q is a typical example of valuations on convex bodies in $\mathbb{R}^n$. For the "classical" $q = n$ case, the NKFIH project managed to strengthen all known results about the L_p Minkowski problem if $-n < p < 0$ or $0 < p < 1$, see the following papers:

- G. Bianchi, K.J. Böröczky, A. Colesanti: The Orlicz version of the L_p dual Minkowski problem on S^n for $-n < p < 0$. Adv. Applied Mathematics, 111 (2019), 101937.
- G. Bianchi, K.J. Böröczky, A. Colesanti, D. Yang: The L_p-Minkowski problem for $-n < p < 1$ according to Chou-Wang. Adv. Math., 341 (2019), 493-535.

The paper

- Bianchi, Gabriele; Böröczky, Károly J.; Colesanti, Andrea: Smoothness in the L_p Minkowski problem for $p < 1$. J. Geom. Anal. 30 (2020), no. 1, 680-705

provided additional insight into the smoothness of the solution

A generalization of the L_p Minkowski problem is the recently introduced qth dual L_p Minkowski problem by Lutwak, Yang and Zhang where $q = n$ corresponds to the classical problem. The NKFIH project managed to solve the L_p dual Minkowski problem for $p > 1$ and $q > 0$ among any convex bodies, and obtained results about the regularity of the solution in the paper

- K.J. Böröczky, F. Fodor: The L_p dual Minkowski problem for $p > 1$ and $q > 0$. Journal of Differential Equations, 266 (2019), 7980-8033.

In addition, we have solved the L_p dual Minkowski problem for $p = 0$ and $1 < q < n$ among origin symmetric convex bodies in the paper

- K.J. Böröczky, E. Lutwak, D. Yang, G. Zhang, Yiming Zhao: The dual Minkowski problem for symmetric convex bodies. Adv. Math., 356 (2019), 106805.

Moreover, we have generalized the Alexandrov problem giving a deeper understanding of Gaussian image in the paper

- K.J. Böröczky, E. Lutwak, D. Yang, G. Zhang, Yiming Zhao: The Gauss image problem. Communications on Pure and Applied Mathematics, 73 (2020), 1406-1452.

2 Problems related to the Brunn-Minkowski inequality

The famous Minkowski inequality, a consequence of the Brunn-Minkowski inequality and a far reaching extension of the isoperimetric inequality, provides a sharp lower bound for the mixed volume $V(K, M[n-1])$ of two convex bodies K and M in $\mathbb{R}^n$ in terms of their volume. Generalizing a Betke and Weil's planar result, we have provided a Reverse Minkowski inequality (a sharp upper bound for $V(K, M[n-1])$) in terms of the mean width of K and the surface area of M in the paper

- K.J. Böröczky, D. Hug: A reverse Minkowski-type inequality. Proc. AMS, 148 (2020), no. 11, 4907-4922.

The case of equality is also characterized.

A conjectured "true" discrete version of the planar Brunn-Minkowski inequality in terms of triangulations is stated in the paper

- K.J. Böröczky, M. Matolcsi, I. Ruzsa, P. Santos, O. Serra: Triangulations and a discrete Brunn-Minkowski inequality in the plane. Disc. Comp. Geom., 64 (2020), 396-426,

and many important special cases are verified.

Finally, we proved the isodiametric inequality in the spherical and hiperbolic space in the paper

- K.J. Böröczky, A. Sagmeister: Isodiametric problem on the sphere and in the hyperbolic space. Acta Math. Hung., 160 (2020), 13-32;

more precisely, the statement that among sets of given volume, balls have the minimal diameter. We note that in the Euclidean space, the isodiametric inequality is a direct consequence of the Brunn-Minkowski inequality.