NKFIH KH 129601 Final Report

Overview

The main objective of the project was the mathematical study of problems in quantum information theory, in particular, a quantitative analysis of the trade-off between various operationally relevant quantities in these problems, mainly in the form of error exponents, and the study of the related information measures (divergences, entropies, channel capacities, entanglement measures, etc.). Of the six project points written in the project proposal, we have obtained important results in all.

The publication output of the project at the end of its three-year duration is 48 research papers, of which 33 have already appeared in peer-reviewed journals, 2 in peer-reviewed conference proceedings, and 13 are currently in online available preprint form, being under review at some journal, or just before submission. On top of that, 3 further papers are close to completion. All our papers are publicly accessible at the online repository arxiv.org. For the complete and up-to-date publication list, please see our group's website: `http://qi.math.bme.hu/publicationsAll.html`. Below we give a detailed account of a selected list of our results that we consider the most important.

The results of the project were presented at 56 conference or seminar talks, of which 21 were invited or plenary conference talks, 2 invited tutorials, and 6 invited seminar talks. For the details, please see `http://qi.math.bme.hu/conferencesAll.html`. We would also like to mention that the largest and most selective conference in (mainly theoretical) quantum information science is the annual Quantum Information Processing (QIP) conference. The PI was invited to give a 3-hour tutorial on entropies at QIP 2019 (`https://jila.colorado.edu/qip2019/program.html`), and Péter Vrana presented a talk at QIP 2021 based on [28] and [5] (`https://www.mcqst.de/qip2021/program/thursday.html`). Moreover, on top of the above 56 talks, a talk based on [7] was presented at QIP 2020 by a co-author not in our group (`https://qipconference.org/2020/#/programDetails`), and two talks based on [24] and [26] will be presented at QIP 2022 by group members Mihály Weiner and Zoltán Zimborás, respectively (`https://web.cvent.com/event/8adf8248-432b-499c-91e2-63b83ba3f69e/summary`).

Following the plans written in the research proposal, we have put a great emphasis on training young researchers by involving them in the project; this resulted in 2 BSc dissertations (Baksa Kolok and Miklós Tóth, supervised by Tamás Tasnádi), and 1 MSc dissertation (Zoltán Kolarovszki, supervised by the PI), while 1 MSc dissertation (Baksa Kolok, supervised by Tamás Tasnádi), and 4 PhD dissertations (Gergely Bunth and Gábor Maróti, supervised by the PI, Zsombor Szilágyi, supervised by Mihály Weiner, and Dávid Bugár, supervised by Péter Vrana) are currently in progress.

As explained in the research proposal, this project and the NKFIH project K 124152 with the same PI are strongly related to each other, and hence there are also overlaps between the respective reports. As forecasted in the research proposal, the extra funding from the present grant KH 129601 enabled the dedication of a considerably larger amount of researcher capacity to making progress in some selected directions than it would have been possible otherwise, which is clearly reflected in the large number of quality publications obtained during the three years of the project. In the same time, we also covered topics different from those in K 124152, e.g., quantum metrology and multi-user information theory.

Results

I. State discrimination and Rényi divergences

1. Strong converse exponent and sandwiched Rényi divergences in infinite dimension: As it was shown in [M. Mosonyi, T. Ogawa, CMP 334(3):1617–1648, 2015] (the publication of the PI with an outstanding number of citations, on which the project proposal was based) the sandwiched Rényi divergences of two finite-dimensional density operators quantify their asymptotic distinguishability in the strong converse domain. This establishes the sandwiched Rényi divergences as the operationally relevant ones among the infinitely many quantum extensions of the classical Rényi divergences for Rényi parameter $\alpha > 1$. The known proof of this goes by showing that the sandwiched Rényi divergence coincides with the regularized measured Rényi divergence, which in turn is proved by asymptotic pinching, a fundamentally finite-dimensional technique. Thus, while the notion of the sandwiched Rényi divergences was extended recently to density operators on an infinite-dimensional Hilbert space (in fact, even for states of an arbitrary von Neumann algebra), these quantities were so far lacking an operational interpretation similar to the finite-dimensional case, and it has also been open whether they coincide with the regularized measured Rényi divergences. In [20] we fill this gap by answering both questions in the positive for density operators on an infinite-dimensional Hilbert space, using a simple finite-dimensional approximation technique. We also initiate the study of the sandwiched Rényi divergences, and the related problems of the strong converse exponent, for pairs of positive semi-definite operators that are not necessarily trace-class (this corresponds to considering weights in a general von Neumann algebra setting). While this latter problem does not have an immediate operational relevance, it might be interesting from the purely mathematical point of view of extending the concept of Rényi (and other) divergences to settings beyond the standard one of positive trace-class operators (positive normal functionals in the von Neumann algebra case). We also discuss the Rényi (α, z)-divergences in this setting.

In [14] we further extend the above results to states of injective (equivalently, approximately finite-dimensional) von Neumann algebras, and more generally, to states of nuclear C^*-algebras.

2. Test-measured Rényi divergences: One possibility of defining a quantum Rényi α-divergence of two quantum states is to optimize the classical Rényi α-divergence of their post-measurement probability distributions over all possible measurements (measured Rényi divergence), and maybe regularize these quantities over multiple copies of the two states (regularized measured Rényi α-divergence). As mentioned above, a key observation behind the theorem for the strong converse exponent of quantum state discrimination is that the latter quantity coincides with the sandwiched Rényi α-divergence when $\alpha > 1$. Moreover, it also follows from the same theorem that to achieve this, it is sufficient to consider 2-outcome measurements (tests) for any number of copies (this is somewhat surprising, as achieving the measured Rényi divergence for n copies might require a number of measurement outcomes that diverges in n, in general). In view of this, it seems natural to expect the same when $\alpha < 1$; however, we show in [22] that this is not the case. In fact, we show that even for commuting states (classical case) the regularized quantity attainable using 2-outcome measurements is strictly smaller than the Rényi α-divergence (which is unique in the classical case). In the general quantum case this shows that the above "regularized test-measured" Rényi α-divergence is not even a quantum extension of the classical Rényi divergence when $\alpha < 1$, in sharp contrast with the $\alpha > 1$ case.

3. Composite state discrimination: The trade-off between the two types of error probabilities in binary state discrimination may be quantified in the asymptotics by various error exponents, depending on the relative weighting of the two types of error probability. In the composite case, where the hypotheses consist of sets of states, any such exponent is upper bounded by the infimum of the corresponding pairwise exponents of discriminating individual members of the two sets. Attainability of this upper bound may depend on the type of exponents considered; whether the problem is classical or quantum; the cardinality and the geometric properties of the sets representing the hypotheses; and also on the dimensionality of the underlying Hilbert space. Our main contribution in [24] is clarifying this landscape considerably. In particular, we show that in the quantum case unattainability of the upper bound is the general behaviour for all the exponents, already in finite dimension, even for a simple null-hypothesis and an alternative hypothesis consisting of only two states. This is in sharp contrast with the classical case, where the upper bound is attainable in all previously studied cases. On the other hand, we show that the upper bound may be strict even in the classical case if the system is infinite-dimensional and the alternative hypothesis contains at least countably infinitely many states. We also prove general attainability results, e.g., for classical adversarial and arbitrarily varying hypothesis testing, where we unify and extend several previous results in the literature, and for the quantum case when all states are pure, or the states commute with every state in the opposite hypothesis.

4. Super-exponential state discrimination: The above discussed results mainly consider the asymptotics in the i.i.d. setting, where the samples are prepared independently and in the same state. In this setting it is not possible to attain a super-exponential asymptotics for both types of error probability, unless the two states have orthogonal supports. In [4] we consider the discrimination problem of two correlated translation-invariant states on an infinite spin chain (equivalently, on the algebra of the canonical commutation relations (CAR)); in this case the asymptotics is considered in taking measurements on increasing portions of the chain. The class of states we study consists of translation-invariant and gauge-invariant quasi-free states, which are uniquely characterized (via Fourier transformation) by their symbol, which is a measurable function on the one-dimensional torus. We show that if there exists a non-trivial interval on which the symbol of one state is constant one, while the symbol of the other state is constant zero, then by the right choice of measurement sequence, both types of error can be made to converge to zero with a super-exponential speed. This is in sharp contrast with the i.i.d. case, and in fact with any previously studied scenarios that we are aware of.

II. Channel coding and divergence radii

1. Strong converse exponent of classical-quantum channel coding: The strong converse exponent of a classical-quantum channel W for a given coding rate R (number of messages transmitted per channel use) gives the optimal exponent with which the probability of successful decoding goes to zero when R is larger than the capacity of the channel. In [23] we consider the constant composition version of this problem, which is a refinement of the general coding problem, where each input symbol x of the channel has to be used with a pre-defined frequency $P(x)$. Our main result is that the strong converse exponent can be expressed as $\mathrm{sc}(W,R,P)=\sup_{\alpha>1}\frac{\alpha-1}{\alpha}[R-\chi_\alpha^*(W,P)]$, where $\chi_\alpha^*(W,P):=\inf_\sigma\sum_x P(x)D_\alpha^*(W(x)\|\sigma)$ is the P-weighted sandwiched Rényi divergence radius of the channel; here $D_\alpha^*(W(x)\|\sigma)$ is the sandwiched Rényi divergence of the output $W(x)$ of the channel on input x and an arbitrary state σ on the output space of the channel. This result further confirms that it is the sandwiched type quantum Rényi divergences that have operational relevance in the strong converse domain of coding problems, and it extends Csiszár's classic result showing that it is the geometric quantity $\chi_\alpha^*(W,P)$ that faithfully quantifies the usefulness of a channel for information transmission with constant composition coding, and not the possibly more intuitive concept of Rényi mutual information. Based on the above result, we also determine the exact strong converse exponent of classical-quantum channel coding with cost constraint.

2. General study of divergence radii: As it turns out, the concept of the weighted Rényi radius is naturally connected to an intensively studied concept in matrix analysis, the barycenter (also called Fréchet mean, or Karcher mean). Motivated by [23], we started a systematic study of this concept for various quantum Rényi divergences and more general quantum divergences. In [29] we give a fixed-point equation characterization of the barycenter of finitely many positive matrices with respect to maximal quantum f-divergences (also called quantum Hellinger divergences).

Continuing this line of study we show in [30] that general symmetric Kubo–Ando means admit a divergence center (barycenter) interpretation, and we use this result to define natural weighted and multivariate versions of these operator means.

The quantum Jensen–Shannon divergence is the (Umegaki) relative entropy radius of two equally weighted quantum states (more generally, positive semi-definite matrices). It was conjectured more than a decade ago by Lamberti et al. that the square root of this quantity is a genuine metric. We prove this conjecture in [34].

3. Barycentric Rényi divergences: Since Rényi divergences and derived information quantities appear in the analysis of most information-theoretic problems, exploring the mathematical properties and potential applications of the plethora of quantum Rényi divergences is an important research line in quantum information theory, and it is also a central theme in our project. In [21] we introduce a systematic way of constructing quantum Rényi α-divergences with desirable mathematical properties for the α values below 1. This is based on a variational formula that allows to express the classical Rényi α-divergences for any α-value using only the relative entropy (Kullback–Leibler divergence, or Rényi α-divergence with $\alpha = 1$). In fact, this can be equivalently stated as the Rényi α-divergence being the α-weighted divergence radius of two (classical) states, when the divergence is the relative entropy with the order of its arguments interchanged. We consider an analogous variational formula with a quantum relative entropy in place of the Kullback–Leibler divergence, e.g., the measured (minimal) or the Belavkin–Staszewski (maximal) relative entropies, as well as a one-parameter family of quantum relative entropies interpolating between the Umegaki and the maximal relative entropies that we introduce in this paper. In this way we obtain continuum many quantum Rényi α-divergences for every α value, which have the important property of being monotone non-increasing under completely positive trace-preserving maps (in other words, they satisfy the data processing inequality). We show that these new Rényi divergences are different from all previously studied quantum Rényi divergences, with the exception of the one derived from the minimal relative entropy.

III. Algebraic complexity theory and quantum information theory

1. Asymptotic entanglement transformations: A central open problem in the theory of multipartite entanglement is to determine the rate at which a given state can be asymptotically transformed into a given target state by local operations and classical communication. Transformations between bipartite pure states are well understood under various error criteria, and the optimal rates are expressed in terms of information quantities – entanglement measures – namely Rényi entropies of entanglement. On the other hand, we are still far from a coherent picture when three or more subsystems are considered. In 2008 Chitambar, Duan and Shi found a connection between tripartite entanglement transformations and notions in algebraic complexity theory, in particular the unsolved problem of determining the asymptotic complexity of matrix multiplication. Several of our results use or are inspired by this connection and the techniques developed in the study of the exponent of matrix multiplication. The current best upper bound on the exponent is based on a method developed by Coppersmith and Winograd to bound the asymptotic subrank of certain order-three tensors. Using a generalization of this method to arbitrary orders, in [2] we determine the optimal rate of distilling Greenberger-Horne-Zeilinger states from balanced Dicke states, a class of multi-partite states arising in quantum optics.

A remarkable result in bilinear complexity is Strassen's characterization theorem that, in its abstract form, states that a commutative semiring equipped with a suitable preorder gives rise to an asymptotic preorder that can be characterized by the asymptotic spectrum, i.e. the set of monotone semiring homomorphisms into the nonnegative reals. In [15] we apply this theorem to entanglement transformations and characterize the trade-off between the rate and the error exponent in the strong converse region. The characterization is in terms of the asymptotic spectrum of LOCC transformations, an axiomatically-defined set of entanglement measures, which in general are not known explicitly. In the bipartite case we completely classify the asymptotic spectrum, identifying its elements as exponentiated Rényi entropies of entanglement with orders between 0 and 1, which leads to an explicit expression for the trade-off curve. In the multi-partite case, in [36] we construct an explicit family of elements of the asymptotic spectrum, which can be viewed as Rényi generalizations of convex combinations of the marginal von Neumann entropies, generalizing recent work by Christandl, Vrana and Zuiddam. The construction involves a multi-partite generalization of the empirical Young diagram measurement, which obeys a large deviation principle. Our formula for the new entanglement measures is given in terms of the rate function, which we determine in [3].

In the bipartite case the earliest and arguably most transparent result is about asymptotic transformations with an asymptotically vanishing error. In this limit the entanglement of a state is characterized by a single number, the entropy of entanglement. However, it is easy to see that in the multi-partite realm a similar uniqueness theorem cannot hold. In [37] we find a multi-partite analogue of the uniqueness theorem, identifying axiomatically the relevant information quantities. We show that the rates of transformations between any pair of states with asymptotically vanishing error are characterized by the set of asymptotically continuous additive entanglement measures. In addition, we show that, assuming LOCC-monotonicity on average and additivity, asymptotic continuity is equivalent to an algebraic condition similar to the chain rule for the Shannon entropy.

The appearance of multiple entanglement measures is closely related to the idea of asymptotic irreversibility of entanglement transformations, a fundamental property of multipartite entanglement, and in contrast with bipartite pure states or thermodynamics. In [8] we introduce this physically motivated, resource-theoretic notion to the abstract setting of relative bilinear complexity. We define the irreversibility of a tensor that quantifies the loss when transforming unit tensors into many copies of the given tensor and then transforming back to unit tensors (equivalently: reducing a bilinear map to independent scalar multiplications and then reducing independent scalar multiplications to the same map). Any nontrivial value of this quantity implies a so-called barrier result, putting a lower bound on any upper bound achievable using a certain starting tensor. In this way we improve on previous barrier results on certain approaches to fast matrix multiplication.

In [7] we focus on a kind of entanglement transformation that is asymptotic in a different sense, which arises in quantum many-body physics. Projected entangled pair states and their generalizations form important ansatz classes that provide good approximations to low-energy states of local many-body Hamiltonians. Such a representation, known as a tensor network state, can be viewed as the result of a local transformation applied to a pattern formed by maximally entangled bipartite states. We show that degeneration, a basic tool in the construction of matrix multiplication algorithms, gives rise to a modified ansatz class that can reach states in the closure of the set of tensor network states with a given bond dimension, with comparable description size (with a linear overhead in the system size, as opposed to the exponential cost of increasing the bond dimension).

2. Zero-error communication: Strassen's theory of asymptotic spectra can be applied to other problems in information theory. One application, due to Zuiddam, is to zero-error communication over classical or classical-quantum channels. The zero-error capacity of a channel is determined by its confusability graph, which led to the notion of the Shannon capacity of a graph. To apply Strassen's theorem, one observes that the set of isomorphism classes of finite simple undirected graphs with disjoint union, strong product and the cohomomorphism preorder form a preordered semiring, and one identifies the abstract asymptotic subrank as the zero-error capacity. It is therefore important to understand the asymptotic spectrum of graphs, i.e. the set of real-valued graph parameters that are in this sense semiring-homomorphic and monotone. These include well-known upper bounds on the Shannon capacity such as the Lovász number and the fractional Haemers bounds. In [39] we make significant progress in this direction, by showing that the asymptotic spectrum of graphs has a convex structure, allowing us to construct uncountably many new elements. Among these we find graph parameters that for certain graphs provide better upper bounds on the Shannon capacity than any of the previously known ones.

3. Relative submajorization: Relative submajorization is a relation defined on the set of pairs of positive operators, and has applications in hypothesis testing and quantum thermodynamics. We studied an asymptotic relaxation of this relation, comparing tensor powers of each component in the pairs. In the setting of hypothesis testing this corresponds to discriminating many independent and identically distributed copies (in particular in the strong converse regime), while in quantum thermodynamics it describes many non-interacting copies of the system, coupled to a thermal bath at a common temperature. While the componentwise direct sum and tensor product together with relative submajorization gives rise to a preordered semiring, Strassen's theorem is not directly applicable because the semiring does not satisfy a crucial Archimedean property. This motivated our work [38], where we proved a generalization of Strassen's characterization theorem to certain non-Archimedean preordered semirings. Under a weaker condition (called polynomial growth), one can still define the asymptotic preorder and we found conditions for the characterization property to hold. In [28] we used this result to characterize asymptotic transformations between pairs of operators in terms of the asymptotic spectrum, and classified its elements, identifying them essentially as sandwiched Rényi entropies of orders greater than 1. This led to a new proof of the strong converse exponent in quantum hypothesis testing, as well as a trade-off between the error exponent and the rate for work-assisted Gibbs-preserving operations in quantum thermodynamics. Subsequently, in [5] and [6] we managed to partially generalize these results to composite hypothesis testing as well as group symmetric hypothesis testing problems and state transformations by time-translation covariant Gibbs-preserving maps (also known as thermal processes). In the latter problem we identified new so-called second laws of thermodynamics that apply to systems of any size, put constraints on the evolution of states with coherence as well as athermality, even in the presence of catalysts. In addition, our partial understanding of the relevant asymptotic spectrum led to a new two-parameter family of monotone quantum Rényi divergences.

IV. Entanglement in metrology and statistical physics

1. Quantum metrology and entanglement: In [33], we study which entangled states are useful for metrology in parameter estimation in a linear interferometer. Here "useful" means that the state is more useful than any separable state at least for a certain local Hamiltonian (these are sums of single-body Hamiltonians, so that they do not contain interactions). It is known that, surprisingly, not all entangled quantum states are useful, but here we show that in the bipartite case any pure entangled state is. Most importantly, we give examples that a quantum state, which is not useful, can be made useful by adding an ancilla or an additional copy of a quantum state; that is, metrological usefulness can be "activated". Moreover, we present a numerical method that looks for the local Hamiltonian, with which a given quantum state is performing the best compared to separable states.

In [27], we show that certain weakly entangled quantum states, called "bound entangled states", can be useful for quantum metrology. Moreover, we present a family of bipartite bound entangled states that are optimal among bound entangled states of a given dimension. For larger dimensions, states of this family are maximally useful. That is, they are as useful as a two-qubit singlet. This is surprising, since it was expected that such quantum states are similar to separable states and cannot be employed for many quantum information processing applications.

In [32] we develop new uncertainty relations with the variance and the quantum Fisher information, following ideas from [Tóth and Petz, Phys. Rev. A 2013]. Some of the new relations are stronger than the Robertson-Schrödinger uncertainty relation, while others give better bounds in certain cases. These ideas can also be used to show that states that violate certain uncertainty relations are more useful than separable states in quantum metrology.

2. Entanglement detection in concrete physical systems: In [16] we develop methods to detect multipartite entanglement close to singlet states in atomic ensembles, and in [35] to detect bipartite entanglement in the vicinity of Dicke states. (Here, our criteria detect only entanglement, not metrological usefulness.) Both criteria have been tested in experiments.

3. Entanglement in fermionic systems: Fermionic systems, such as fermionic ultra-cold atoms or electrons, are often used in current quantum technologies. When examining possible quantum information theory protocols with fermions, it should be noted that the mathematical description of multiparticle fermionic systems differs from that of conventional quantum spin systems. On the one hand, in the case of fermions, the embedding of subsystems into larger systems does not follow the traditional tensor product structure, and on the other hand, the so-called parity superselection rule must also be applied for fermions. Because of these mathematical curiosities and the experimental relevance, the development of quantum information theory in fermionic systems is an area of active research. In [10] we generalize the notion of the relative entropy of entanglement to fermions, taking into account parity superselection, and derive a closed formula for this quantity for subsystems containing two modes. We also consider applications of these findings in quantum chemistry. In [31] we examine the asymptotics of the entanglement negativity generalized to fermions for the ground states of several quasi-periodic and randomly modulated fermion chains. In [1] we use fast oscillating non-analytic matrix functions as Toeplitz symbols to construct families of non-gauge-invariant quasi-free states for which the local Rényi and von Neumann entropies diverge sublogarithmically with the subsystem size. These are the first translation-invariant states presented in the literature that have such slowly diverging local entropies.

V. Miscallenous results

In this point we list results that do not directly belong under any of the six specific project points in the proposal, but which are nevertheless strongly related to the broader subject of the proposal, on the technical or on the conceptual level.

1. Protocols for near-term quantum computers: Quantum sampling is a popular family of protocols for near-term quantum devices. For example, the famous Google quantum supremacy experiment of 2020 and the boson sampling experiment conducted last year at the University of Science and Technology of China fall into this class. Compared to the random quantum gate sampling used in the Google experiment, the random distribution generated by boson sampling has much more structure, and using this structure, the unitary operation used in such an experiment can be verified by a polynomial number of measurements in the number of bosonic modes. However, boson sampling can only be demonstrated using photonic modes, i.e. it cannot be transferred to a quantum computer with a qubit architecture. This problem motivated us to introduce fermion sampling analogous to boson sampling, which may be suitable for the experimental realization of quantum supremacy [26]. Using the Jordan-Wigner transformation, fermion sampling can also be transferred to a qubit based quantum computer and, analogously to boson sampling, can be verified by a number of measurements scaling polynomially with the number of qubits.

In addition to sampling algorithms, we also studied the filtering of errors and noise. We have developed a new method for the characterization and determination of readout errors in quantum computers. We also tested this procedure on IBM and Rigetti online quantum computers. Using our method, we could improve the readout statistics, and thus also improve the efficiency of certain quantum algorithms (Grover search, Bernstein-Vazirani algorithm) on noisy IBM and Rigetti machines [18]. A disadvantage of the method in [18] is that the amount of resources required increases exponentially with the number of qubits. However, in [17], we have shown that by making relatively mild assumptions about the type of errors, our method can be modified so that the required number of measurements scales only polynomially with the number of qubits.

2. Entanglement-assisted classical communication: To quantify the advantage that a non-signaling resource provides to a noiseless classical channel, one might ask how many extra letters should be added to the alphabet of the channel in order to perform equally well without the specified non-signaling resource. As was observed by Cubitt, Leung, Matthews, and Winter in [PRL 104, 230503, (2010)] and [IEEE Trans. Inf. Th. 57, (2011)], there is no upper bound on the number of extra letters required for substituting the assistance of a general non-signaling resource to a noiseless one-bit classical channel. In contrast, we prove in [11] that if the resource is a bipartite quantum system in a maximally entangled state, then an extra classical bit always suffices as a replacement.

3. Error exponents for multiple access channels: In [9] we show a construction of a classical asynchronous multiple access channel where the two senders XOR their binary codewords and the result is sent through a binary Z-channel. In some sense, this two sender channel can be regarded as a single user channel, where, surprisingly, the asynchronous exponent is higher than the single user sphere packing bound. This is not a contradiction, as our system turns out to be a special case of Trellis codes, which can reach an exponent superior to the single user exponent at the expense of a more difficult coding/decoding scheme.

4. Mutually unbiased bases: Mutually unbiased bases (MUBs) arise in quantum information theory related to the problem of state tomography, and determining their maximal number in all finite Hilbert space dimensions has been a long-standing open problem. In [25] we introduce the notion of k-nets over an algbera that provides a common framework for both classical cominatorial designs called k-nets, and for MUBs. Using this framework, we prove a certain rigidity property that was previously shown to hold for k-nets that can be completed to affine planes using a completely different, combinatorial argument. For k-nets that cannot be completed and for MUBs this result is new, and, in particular, it implies that the only vectors unbiased to all but $k \leq \sqrt{d}$ bases of a complete collection of MUBs in $\mathbb{C}^d$ are the elements of the remaining k bases. We show that the $\sqrt{d}$ bound is tight, and apply our result to show that certain large systems of MUBs cannot be extended.

In [19] we prove that if a collection of $d(d+1)$ unit vectors is such that any two elements of this set are either orthogonal or unbiased to each other, then in fact these vectors can be arranged into d orthonormal bases forming a complete system of MUBs. In other words, we give a new characterization of a complete system of MUBs.

5. Isometries of Wasserstein spaces: Wasserstein p-distances feature in the study of optimal transport as distances on probability distributions on a metric space, and our main line of investigation was the study of the isometries of such spaces. In [12] we describe the group of Wasserstein isometries over the unit interval $[0, 1]$ and the real line. As for the real line, we prove isometric rigidity for all positive parameters $p \neq 2$. This is in striking contrast with Kloeckner's result on the quadratic ($p = 2$) Wasserstein space which admits non-trivial and exotic isometries. We find a substantial difference between the real line and the interval as well. Namely, the p-Wasserstein space over the interval is rigid for $p > 1$, but for $p = 1$, it has exotic, moreover, mass-splitting isometries. Using this latter phenomenon we give affirmative answers to the questions posed by Kloeckner in 2010 regarding the existence of exotic and mass-splitting isometries in quadratic Wasserstein spaces. In [13] we show the existence of non-trivial isometries in the quadratic case, and isometric rigidity for every $p \neq 2$, for Wasserstein-Hilbert spaces. The key steps here are a metric characterization of Dirac masses and the use of the Wasserstein potential of measures.

We are currently working on analogous problems in the quantum case, with some preliminary results on the Wasserstein isometries on the space of qubit states [György P. Gehér, József Pitrik, Tamás Titkos, Dániel Virosztek, manuscript in preparation]. Here, the isometry structure depends on the choice of a cost operator, and we show for a certain natural choice that the isometry group of the quantum Wasserstein space coincides with the orthogonal group $O(3)$, which is both a rigidity and a Wigner-type result. Based on this result, we expect that for 2^k-level quantum systems, in the case of a highly symmetric cost operator which involves every possible tensor product of Pauli matrices, the group of quantum Wasserstein isometries will coincide with higher dimensional classical isometry groups.

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