Project closing report – 128789 – Mixed-mass String Integrability

Michael Abbott, June 2021

During this grant I published 3 papers on topics described in the proposal. One of these was on the main idea (with a collaborator in the UK), and two are on a peripheral idea which I (and others at Wigner) got interested in.

This is of course less than I would have liked. I have in fact given up on this direction of research, resigned from my job at Wigner, and left the country. This was not my intention when I arrived, and I imagine it is not the outcome you would have preferred. I am nevertheless grateful for the support I received while there.

The central theme of the grant was to be lower-dimensional AdS/CFT, and in particular, the puzzle of how to incorporate the massless degrees of freedom seen in AdS_3 backgrounds into the integrable model description. These are the principal novel feature not present in the original, better-understood, AdS_5 backgrounds. Their integrable description is organised by the length of the string (or spin chain), starting with a very simple Bethe ansatz regime (1/L effects), then a Luscher regime in which some wrapping effects appear (at orders e^-L), then finally a full TBA regime (any L).

The problem we managed to solve was to find toy models with both massive and massless degrees of freedom. In particular, we managed to invent a double-scaling limit of some homogeneous sine-gordon models, in which some modes become massless without decoupling. And we found a way to handle the analysis of this system, an expansion which still makes sense in this massless limit. The different regimes of expansion in L mentioned above are no longer distinct, as (for example) the relevant factor for Luscher terms is e^-mL, where now m=0; but we found an alternative expansion. Because it is a toy model, we could check numerically that our method of taking this expansion did not miss effects of the same order.

We spent a long time working on the problem of how to translate this back to the much more complicated AdS/CFT context. There are a number of disagreements which, we believe, arise from the assumption that the expansion in 1/L used in AdS_5 still makes sense, and the goal was to repair some of them using our new methods. But we did not finish this work, and now that I have left the field, it is unlikely that it will be published.

What was published, however, was work on another topic mentioned in my original proposal. This was the study of resurgence in perturbative expansions, a set of tools for relating a badly-behaved expansion in a coupling g to non-perturbative effects in e^-1/g. One eventual hope was to relate this via g=1/L to the above regimes of AdS/CFT. But the published work is on the O(N) model, an integrable field theory which is often viewed as a toy model for QCD: it has asymptotic freedom and a dynamical mass gap.

Earlier work by others had shown how to calculate many terms of its perturbative series, up to about 25 loop orders. This was sufficient to see the leading e^-1/g effects, but not to go further. We managed to extend this by calculating some 2000 loop orders, at very high numerical precision. From this we could construct the series multiplying the first (imaginary) instanton action, i e^-1/g(g + g^2 + ...), and study its resurgence, and so on, eventually circling back to obtain the correct, real, corrections. Because the O(N) model is integrable, these real, physical, corrections could be checked by direct numerical evaluation of the TBA. This is to my knowledge by far the highest-order calculation of its kind — other systems are known to 50 loops or so, and there are much simpler systems which can be solved analytically to all loops, but little in between.

These papers were all published during 2021, in good journals: PRD, Physics Letters B, and JHEP. They first appeared on the arxiv during 2020. I presented the work on AdS_3 at a conference in Poland in late 2019, and work leading up to it at other conferences that year. The pandemic meant there were no opportunities for me to present the resurgence work last year. I trust, however, that my collaborators will do so.