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Viktor Kiss continued to work on a project, joint with Márton Elekes and Hugo Nobrega to define and study ranks associated to games characterizing Baire class functions. During the investigation of the Baire 1 case, they managed to prove a surprising result, stating that the rank of a well-founded subset W of the tree $2^{<\omega}$ depends only on the number of points of W on each infinite path of $2^{<\omega}$. The manuscript containing the results is in preparation.

Márton Elekes, János Flesch, Viktor Kiss, Donát Nagy, Márk Poór and Arkadi Predtetchinski investigated the class of the so-called limsup functions. They constructed a two-player game $\Gamma(f)$ that depends on a function f that characterizes the class, that is, Player II has a winning strategy in $\Gamma(f)$ if and only if f is a limsup function. They showed that the game is determined if f is Borel measurable, but not in general. They also formulated a sufficient condition for Player I to have a winning strategy in $\Gamma(f)$, that turns out to be necessary if f is Borel, or if the range of f contains no infinite strictly increasing sequence. Using the game $\Gamma(f)$, they also constructed a game $\Gamma'(f)$ that characterizes functions of Baire class 1. The manuscript containing the results was submitted.

Viktor Kiss, with Márton Elekes, János Flesch and Arkadi Predtetchinski, continued the project by investigating the complexity of the value of several parameterized games. They already have several results, and the manuscript containing them is in preparation.

Kiss, with Richárd Balka, Márton Elekes, Donát Nagy and Márk Poór managed to answer a question of Mycielski by showing that in the sense of Christensen's Haar null sets, almost every self-homeomorphism of the unit interval is singular. They also proved an analogous statement in higher dimensions (where defining singularity is not straightforward), and constructed an example of a homeomorphism of the unit square with a 4 Hausdorff-dimensional graph. The results are published in the Advances in Mathematics.

Viktor Kiss, with Richárd Balka, Márton Elekes, Donát Nagy and Márk Poór, continued to work on a project related to a question of Banakh, Jablonska and Jablonski. They previously constructed a compact subset K of the plane such that $K + K$ is nowhere dense, but the projection of K to any line contains a non-empty open set. They proved generalized versions of this statement in higher dimension, as well as investigated the case of more than two summands. A manuscript containing the results has been submitted.

Viktor Kiss and Sławomir Solecki completed a project started a couple of years ago concerning a question of Bing. In 1988, Bing hypothesized that the pseudoarc may be obtained as the intersection of a nested sequence of chains such that each chain is picked as a random refinement of the previous chain in a way similar to a random walk. Kiss and Solecki came up with a precise probabilistic model in which a random chainable continuum is constructed, allowing Bing's question to be investigated. They showed that the random continuum is indecomposable with probability 1. They left open the question whether it is the pseudoarc with probability 1. The paper containing the results is to be published in the Bulletin of the London Mathematical Society.

Richárd Balka, Márton Elekes and Viktor Kiss investigated the modified lower dimension introduced by Fraser and Yu to transform the lower dimension into a dimension notion that is monotonic. They answered two questions of Fraser and Yu. First, they showed that the modified lower dimension is finitely stable. Second, they proved that given a compact metric space X, considering the modified lower dimension as a map $\dim_{ML} : \mathcal{K}(X) \to \mathbb{R}$, it is Borel measurable. In particular, the map is of Baire class 2, but not necessarily of Baire class 1. The manuscript containing the results has been submitted.

We say that E is a *microset* of the compact set $K \subset \mathbb{R}^d$ if there exist sequences $\lambda_n \geq 1$ and $u_n \in \mathbb{R}^d$ such that $(\lambda_n K + u_n) \cap [0,1]^d$ converges to E in the Hausdorff metric, and moreover, $E \cap (0,1)^d \neq \emptyset$. Richárd Balka, Márton Elekes and Viktor Kiss proved that for a non-empty set $A \subset [0,d]$ there is a compact set $K \subset \mathbb{R}^d$ such that the set of Hausdorff dimensions attained by the microsets of K equals A if and only if A is analytic and contains its infimum and supremum. This answers a question of Fraser, Howroyd, Käenmäki, and Yu. They also showed that for every compact set $K \subset \mathbb{R}^d$ and non-empty analytic set $A \subset [0, \dim_H K]$ there is a set $\mathcal{C}$ of compact subsets of K which is compact in the Hausdorff metric and $\{\dim_H C : C \in \mathcal{C}\} = A$. The manuscript containing the results is in preparation.

Richárd Balka and Viktor Kiss showed that given a compact metric space K and $P \subseteq [0, \dim_P K]$, where $\dim_P K$ is the packing dimension of K, there exists a compact family $\mathcal{C} \subseteq \mathcal{K}(K)$ of compacts subsets of K such that $P = \{\dim_P C : C \in \mathcal{C}\}$ if and only if $P \in \mathbf{\Sigma}^1_2$. They also showed that for a non-empty set $P \subset [0,1]$ there is a compact set $K \subset \mathbb{R}$ such that the set of packing dimensions attained by the microsets of K equals P if and only if $P \in \mathbf{\Sigma}^1_2$ and P contains its infimum and supremum. The manuscript containing the results is in preparation.

Viktor Kiss, with Lionel Levine and Lilla Tóthmérész completed a project started a couple of years ago concerning the parallel chip-firing game. Previous numerical experiments in the literature suggested that the activity increases as a Devil's staircase as one increases the number of chips in a configuration. Kiss, Levine and Tóthmérész managed to show that this will indeed be the case for a sequence of random graphs. In order to prove their result, they extended the parallel chip-firing game from graphs to graphons. A manuscript containing the results has been submitted.