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Final research report on the NKFI 128090 project entitled

Analytical and numerical models for delaminated composite and sandwich structures

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1. Dynamic stability and parametric excitation of delaminated composite beams

During the research work we considered the dynamic stability problem of laminated composite beams containing a single, through-width delamination using the finite element (FE) discretization. In order to formulate the governing equations three distinct generalized FE models were derived of this structure using the system of exact kinematic conditions (SEKC). Both models contained two special FE types. One for describing the behavior in the intact part (considering the SEKC) and one for modelling the crack tip which is able to connect the displacement field of the intact and the delaminated parts.

The first FE model was based on a general higher-order beam theory with the assumption of higher-order displacement continuity along the delamination plane over the intact part. By means of the continuity we derived additional equations, with those we were able to reduce the number of parameters describing the displacement field of the delaminated structure resulting in significantly shorter calculation time. We determined the usual material stiffness, the consistent mass and the geometric stiffness matrices of the finite elements. Using these matrices, we solved large amount of examples of delaminated composite beams subjected to periodically changing axial (conservative) force. The stability diagrams of the discretized structures under periodic loading were determined using Hill's infinite determinant method also called multi-frequency-method (MFM). The method works in frequency domain unlike the widely used Bolotin's harmonic balance method which operates in time domain. The stability boundaries were determined using a numerical root finding algorithm. The analysis involved the investigation of the effect of the delamination length and position and the beam theory order on the natural frequencies and buckling forces. Furthermore, the effect of order of the truncated infinite determinant was investigated, too. Based on our findings the optimal beam theory order was the first-order theory and we also did recommendations regarding the truncated infinite determinant order. The results were presented during the XIII. MaMek national conference held in 2019, Miskolc (Hungary). We also investigated the same problems using Bolotin's harmonic balance method. The results were summarized in a journal paper published in *Composite Structures* (Elsevier). The title of article is: „*Dynamic stability analysis of delaminated composite beams in frequency domain using a unified higher order shear deformable beam theory with higher order displacement continuity*".

The second FE model assumed a general higher-order beam theory all over the structure. However, instead of assuming higher-order continuity along the delamination plane over the intact part, the static equilibrium equations were imposed to be satisfied by the higher-order displacement fields in order to reduce the unknown functions. The stability diagrams of the discretized structure were obtained using the Chebyshev orthogonal polynomial space. The method utilizes the special properties of the polynomial space in order to calculate the series expansion of the periodic excitation and to the determine the monodromy matrix of the system which is involved in the Floquet-theory. The stability boundaries were determined based on the spectral radius of the monodromy matrix and using a simple root finding algorithm. Based on this model and numerical technique similar quantities were investigated as before.

We also carried out some experiments on the parametric excitation of delaminated beams having one fixed and one free end. The test specimens were mounted on a modal shaker in vertical position and were excited at specific frequencies and amplitudes. This measurement setup is equivalent to a mechanical model with displacement excitation at one end and free at the other one. We measured the system response by accelerometers and determined the points of limit stability based on the physical observation of the system response. The measured points of instability were compared to the results of the theoretical (finite element) models and the agreement was found to be good. However, the theory only considered linear system and the stability of that system the nonlinear effects were present in the measurements.

The results of the former analysis and the experimental results were summarized in the PhD thesis of Tamás Pölöskei entitled „*Dynamics and stability of thin-walled composite structures*”. The thesis is available here, but unfortunately, so far he did not defense his thesis work.

The dynamic stability problem of delaminated composite beams was also captured by using plane finite elements (forming a third solution for the same problem) under plane stress state. Thus, four noded plane elements were developed and the model was simplified using a so-called model reduction technique. This reduction technique is essentially related to the free-vibration analysis, mode shape calculation and the dynamical analysis of such structures. The basic concept of the model is that the left and right parts are intact, on the other hand the middle parts (above and below each other) represent the delaminated region of the beam. The classical Craig-Bampton method incorporates the free vibration mode shapes of the subsystems for the reduction process. The novelty of our work was to complement the method with the buckling eigenshape vectors too, leading to the so-called extended Craig-Bampton method. Similarly to the beam finite element models we solved large amount of problems including the determination of free vibration frequencies, buckling loads and dynamic stability limits, respectively. The dynamic stability limits were determined by using Bolotin's harmonic balance method and the Chebyshev polynomials. The results were summarized and published in an article entitled „*Dynamic stability analysis of reduced delaminated planar beam structures using extended Craig-Bampton method*” in the journal *Applied Mathematical Modelling* (Elsevier, 2022).

2. Mode separation in delaminated beams under I/II conditions

The interlaminar fracture in composite beams was also analyzed in the first year by using the first-, second- and third-order beam theories and an analytical solution. The beam was divided into two equivalent single layers and the classical mixed-mode I/II problem was solved using DE methods. The J-integral was determined and separated into mode-I and mode-II components based on the decomposition of stress and strain fields into symmetric and asymmetric parts. The most important aspect of this calculation method is that it does not involve any tricks and artificial conditions, only well-established concepts. The model was applied to asymmetrically delaminated composite beams involving two distinct lay-ups and four positions of the delamination. The comparison of the developed method to those available in the literature shows that our method gives an average curve between the ones determined by the available methods. The improvement of the method was commenced by using four equivalent layers, as well. Finally, the method could be applied to evaluate results of experimental tests as well. The results were published as an article in the journal *Archive of Applied Mechanics* (Springer) in August, 2019 (*Fracture and mode mixity analysis of shear deformable composite beams*).

3. Models for delaminated shells using the first-, second- and third-order shear deformation theory

During the first year we also overviewed the fundamentals of first-, second- and third-order shell theories based on the book of Prof. Reddy. We extended these shell theories to delaminated composite shells. The system of exact kinematic conditions was applied using four equivalent single layers. The PDE equations of the shells were determined involving two independent (but constant) radii of curvatures. The system of PDEs was solved analytically by the state-space approach for simply supported (Lévy-type) delaminated shells. Essentially, two different geometries were considered: hyperbolic and elliptic shells. Apart from that, the position of the delamination was also varied in accordance with four cases. An important step in the research is that the PDE system was solved by an improved MAPLE code. The code was separated into a pre-processor involving data input and by reading simple text files containing the equations. The analysis was done in a separated sheet, finally, the post-processing of displacements, strains, stresses was also done separately. This makes the solution procedure much faster and effective than before. The J-integral was determined along the delamination front of the shells and was separated into mode-II and mode-III components. The mode mixity and its distribution was also determined. We also introduced some improved continuity conditions to enhance the local accuracy of the solution. The finite element models of the shells were also created in ANSYS environment and a comparison with the analytical results was done. The results showed that the analytical model is very accurate and suitable to be the candidate for some shell finite elements. The results were summarized in a conference paper (*Analytical solution of the boundary value problem of delaminated doubly-curved composite shells*), the conference took place in Rome, in 2019, October.

Using the first-, second- and third-order models of laminated doubly curved shells we solved analytically (using the Lévy method) large number of examples depending on the location of delamination over the shell thickness and the shell geometry. The geometry was either elliptic (having positive radii of curvatures) or hyperbolic (having one positive and one negative radii of curvature). The shells were analyzed by using a MAPLE code and the ANSYS models under simply supported boundary conditions. For the delaminated composite shells, we published an article in 2021 in the journal *Archive of Applied Mechanics* (Springer) entitled „*Higher-order semi-layerwise models for doubly curved delaminated composite shells*”. The results of the delaminated composite shells were also presented in a digital conference held in Porto, Portugal on 1-4 September, 2020. The title of presentation was „*Comparison of some higher-order models for doubly-curved delaminated composite shells*”. In the second year of the research we extended the analytical delaminated shell models to sandwich materials as well. We considered three possible scenarios depending on the location of the delamination: core-core delamination, face-core failure and face-face delamination. For each case we derived the first-, second- and third-order shell models including a core made out of soft material. We introduced an improved condition for the continuity of the stress resultants between the delaminated and undelaminated parts of the model. This condition gives significantly more accurate results for the mechanical model than any of our previous works. Quite similarly to the composite plates we determined the mechanical fields in the whole shells and the J-integral distribution along the delamination front. The comparison of these results was carried out using the 3D ANSYS models of the elliptic and hyperbolic sandwich shells. The material pairs for the three different scenarios were Al-foam, cross-ply composite-balsa wood and cross-ply composite-balsa wood again. The results were summarized in an article entitled „*Mechanics of shear and normal deformable doubly-curved delaminated sandwich shells with soft core*" published in the journal *Composite Structures* (Elsevier) in July, 2021.

4. Application of the differential quadrature to delaminated composite plates

In the second year of the research we studied the so-called differential quadrature (DQ) method to solve simple plate bending problems. The main idea of the method is that the parameters (e.g. the deflections, angle of rotations and higher-order displacement field parameters) are approximated by the Lagrange polynomials and by taking these polynomials back into the governing PDEs we needed to determine the weighting coefficients and function values at the specified grid points. The grid points are created very simply (in contrast with the FE method) using the so-called Gauss-Chebyshev-Lobatto scheme. The solution functions are also continuous functions (again in contrast with the FE method, where it is in general piecewise continuous). Using the DQ method it is possible to define any kind of boundary conditions (free edges, clamped edges and simply-supported edges) and any condition can be imposed along any edge of the rectangular composite plate. Previously, we applied the Lévy-type formulation to solve the PDE system of delaminated composite plates. This method is suitable to solve the problems only if at least two opposite edges of the plate are simply supported. Thus, the DQ method made it possible to solve problems with more versatile boundary conditions. We built-up a MAPLE code for some benchmark problems and we solved a couple of delaminated plate problems using the first-order shear deformation theory. The plates were loaded by a concentrated force, which is moderately difficult to model by using the DQ method. As a matter of fact, by using an integral quadrature equation at the point of action of the force it was possible overcome this difficulty (it was known in the literature). The results of the DQ method were compared to those by previously developed analytical and ANSYS models.

A more significant challenge was to solve the problem of continuity conditions between the delaminated and undelaminated regions of the plate. It is worth mentioning that so far (to the best of our knowledge) the literature presented DQ results only for intact plates and shells. The main problem with the continuity conditions of delaminated plates is that at the corner points we have too many conditions and the number of possible unknowns (grid point function values) is smaller than the actual conditions. A thorough analysis revealed that along the delamination front, and especially at the edges and corners of the delamination front the boundary and continuity conditions can be satisfied only by violating certain dynamic conditions, namely ignoring the normal forces on the one (delaminated) side and the bending moments on the other (intact) side. The second finding of our work was that the delamination results in a significant perturbation in the mechanical fields and thus, the grid should be controlled such that the resonance of the solution near the delamination tip and the boundaries reduces to a reasonable level or vanish entirely. To overcome this feature of the model the plates were divided into five regions. Two regions were assigned over the intact part, the one next to the delaminated part involves a denser grid. The delaminated part was created using three parts: one next to the intact part, one having the concentrated force in the middle and another one forming the side of the plate, respectively. The continuity conditions between these parts were formulated in accordance with the previously mentioned concept. The mechanical fields such as the displacement and stress were plotted along material lines crossing the delamination tip. The deflection function of the plates was also determined and used as a primary indicator for comparison to results by analytical and finite element calculations. The analytical as well as the finite element results were provided from some previous studies. In the final stage the distribution of the mode-II and mode-III J-integrals were plotted along the delamination front and compared to the analytical and finite element solutions. The solution was also provided for some fully clamped delaminated plates. The differential quadrature method is very sensitive to the continuity conditions formulated between the delaminated and intact plate portions. The results showed that the differential quadrature method was very accurate, however a relatively dense and well-controlled grid should be applied because of the delamination (perturbation effect) and the concentrated force.

The results were summarized in an article (*Application of differential quadrature method to delaminated first-order shear deformable composite plates*, *Thin-walled Structures* (Elsevier)). Also, we participated on an online conference with the same title on 5-7 March 2021, (*2nd International Conference on Theoretical, Analytical and Computational Methods for Composite Materials and Composite Structures*).

In the fourth year of the project we performed the mechanical analysis of delaminated plates using the first, second- and third-order plate theories. We solved the problems by the differential quadrature method. In contrast with the analytical solution, the problems were solved under the condition of rigidly fixed edges. Two different problems were analyzed, one with a long and another one with short delamination length. The grid structures for both problems were presented and it was shown that actually, the third-order plate theory approximates the mechanical fields the best way. We also collected the required boundary conditions for each plate theory. The convergence study resulted in the fact that the DQ solution oscillates within an upper and a lower bound. The ANSYS solution for the deflection of the plates in the middle is in general in between the two bounds. We provided a lot of numerical results for the two different plates including the three-dimensional distribution of the shear strain in the vicinity of the delamination front and the J-integrals. The most important advantage of the DQ method over the FE solution is that the approximation is provided by continuous functions for the entire problem domain, in contrast with the elementwise approach of the FE technique. The results were summarized in a journal paper entitled „*Differential quadrature solution for composite flat plates with delamination using higher-order layerwise models”* published in *International Journal of Solids and Structures* (Elsevier) in 2022. It was also shown that the DQ grid density plays an essential role in the accuracy of the solution. Approximately 14000 unknowns were involved by a grid providing the required accuracy of the mechanical fields. We observed that the J-integral being based on the product of the strain and stress field components, is much more influenced by the grid density than the field parameters. This led to a significant waviness of the distribution functions of the mode-II and mode-III J-integrals over the delamination front. The waviness has been observed in plates, too, but it was not as significant as in shells. Nevertheless, we also observed, that at the grid points the J-integral values are accurate compared to the analytical and ANSYS solutions. Thus, the waviness came from the Lagrange interpolation functions that the DQ was based on. This deficiency can be eliminated quite simply, by using only the grid point values of the J-integrals by the DQ method and applying updated interpolation functions, e.g. simple cubic functions to determine the distribution. By means of this technique it is possible to obtain quite smooth distributions for the J-integrals and a very good agreement with the analytical and ANSYS solutions. A conference presentation on the DQ solution for delaminated shells at the European Conference on Composite Materials 2022 (Lausanne, Switzerland) was also performed and a conference paper was published, too („*Numerical solution for second- and third-order composite doubly-curved shells with delamination by differential quadrature method”*).

5. Development of shell finite element types to model delaminated composite plates

We also developed a finite element (FE) model for the problems of delaminated plates by creating an own code. The FE model was implemented in Python programming language and it could be used for several configurations of delamination. Especially its location along the thickness and the lay-up may be widely varied. The model was built up by 9-noded rectangular plate elements. The first-order shear deformation theory (FSDT) was considered for the construction of the elementary stiffness matrix. Hence, every node of the plate element had 5 degrees of freedom (DoF). The delamination was introduced as two FSDT plate elements above each other that had the same size. The delamination opening was not allowed, therefore, the transversal displacement was the same in the top and bottom parts at every node. A special transition element was needed since the nodes of the delaminated area had 9 DoFs, in contrast at the intact area the nodes had only 5 DoFs. This transition element had 5 nodes which was considered as a simple FSDT plate, furthermore it had 4 nodes that were supposed to be got separated. It was also a 9-noded rectangular element with two kinds of nodes. A row of transition elements was applied between the delaminated and intact sections, otherwise, they could not be joined to each other. The Python codes were able to generate the global mathematical model of a delaminated plate for several cases of external loads and boundary conditions, where the delamination had only one closed edge (the so-called delamination front). Thus energy-based fracture mechanical analysis could be accomplished. The further steps were the developments of the model and the Python codes, which made the code more robust and effective. For instance, delamination with four closed edges and other possible geometries were investigated. The basic idea of the model was summarized in a Hungarian conference paper (*Elsőrendű nyírási deformációs elmélet alkalmazása delaminált kompozit lemezekre, OGÉT 2021*).

Later, the developed finte element set for the modelling of delaminated first-order shear deformable plates was named as the FSDPT (based on first-order shear deformable plate theory). Laminated rectangular shape composite plates were investigated containing a through-width delamination. The plates were simply supported, however two variants of this constraint type were investigated. The first variant is based on the imposition of zero in-plane displacement components and rotations at the edges of the plate (apart from the transverse displacements). The second variant did not impose any conditions against the former displacement components, only the transverse displacements were fixed. The ANSYS models of the delaminated plates using SOLID45 type elements were alse created and compared to the developed model based on FSDPT. The agreement was quite good for the mechanical fields, thus even the J-integral was determined. Because of the nature of the FSDPT FE model, the J-integral could be determined by defining a closed contour curve enclosing the delamination tip. A quite lengthy procedure was necessary to determine each required term in the J-integral expression together with the application of some interpolations steps. We worked out a procedure providing reasonably accurate distributions of the mode-II and mode-III J-integrals over the delamination front. We observed that the traditional SOLID45 type elements were too stiff and the improved SOLID185 elements were more appropriate for modelling plate like structures. The results were presented in an international conference (25th International Conference on Composite Structures, *Advanced finite element analysis of composite plates with two delamination fronts*) and an article entitled „*Fracture mechanical finite element analysis for delaminated composite plates applying the first-order shear deformation plate theory”* was also published in the journal *Composite Structures* (Elsevier) in 2023.

The drawback of the previously developed FE models (based on the FSDPT) was the discontinuity of the strain field components and the resulting difficulty for the calculation of the J-integral. Therefore, we continued the development (coding) of finite element models for delaminated composite plates employing the concept of equivalent single layers (ESL). We developed an element family based on the third-order shear deformation theory (TSDT) and the method of two ESLs. The elements were isoparametric involving quadratic interpolation for the displacements, rotations and higher-order parameters. A second element family was also created by applying the first-order shear deformation plate theory (FSDPT) with two ESLs. The main goal of these subparametric elements was that cubic Hermite interpolation was applied for the displacement parameters at the nodes, thus the continuity of the planar derivatives (and so the strain field components) was also satisfied providing a better approximation of the J-integral. Besides, in order to present a comprehensive comparison, an algorithm was also created that is able to compute the J-integral from the 3D nodal solutions of the ANSYS commercial finite element software, since there is no available built-in module for calculating the J-integral with orthotropic or monoclinic material behavior in this software. The results showed that the FSDPT with continuous derivatives over the delamination front are in a very good agreement with the results obtained by our own code processing the 3D ANSYS solution. With the aid of this latter work it became possible to compare directly the J-integral distributions by the plate finite element models and the spatial (3D) solution based on SOLID type elements. We highlight again that so far the calculation of energy release rates in 3D ANSYS models was possible only by using the virtual crack closure technique, but not the J-integral. We submitted a relevant paper recently (2023, March) (*Advanced finite element analyses to compute the J-integral for delaminated composite plates*) to the journal *Composites Part B – Engineering* (Elsevier).

6. Consideration of time delay of periodic follower loads in elastic structures

In a recent paper we captured the problem of composite beams subjected to a periodic retarded follower force. The problems involved the simultaneous spatial and time domain discretization of an elastic composite beam. The spatial discretization was performed by using the finite element method. The structure was described by using a set of laminated Timoshenko (or first-order shear deformable) beams: the intact part, the transition part and the delaminated part, respectively. The equation of motion of the system was derived based on some previous works of the principal investigator and the time delay term was introduced through the so-called load stiffness matrix. This latter matrix had a quite simple structure. The time domain discretization of the equation of motion was done using the shifted Chebyshev polynomials of the first kind. The complete discretization of the problem led to the so-called Floquet transition matrix. The spectral radius of this matrix was required to determine in order to apply the unit circle criterion of stability. The system involved five input parameters: the static force, the dynamic force amplitude, the load direction parameter, the time delay and the frequency of parametric excitation, although the latter two were not entirely independent of each other. The stability diagrams of the systems with and without delamination were determined on the different parameter planes. The results indicated that the time delay has a quite different effect on elastic (multiDoF) systems compared to one or two DoF systems.

In the first stage we did large number of computations for intact composite beams. The relationships of the dimensionless static force and the dimensionless time delay indicated that the stability diagram depends quite strongly on the number of finite elements and the convergence of the diagram is slow, as well. The diagram on the static and dynamic force amplitude (including a constant time delay) was found on the other hand almost independent of the number of finite elements. We showed that the load direction parameter had a quite significant effect on the shape and extension of stability diagrams. The stability diagram on the plane of first eigenfrequency of the statically prestressed system and the load direction parameter was quite similar to that obtained for 1DoF systems. The effect of dynamic force was also investigated. Moreover, even the damping effect was determined for four different damping values on each diagram.

In the second stage we continued the calculations with the delaminated composite beam. Two versions of the applied FE model were used: one with and another one without the consideration of longitudinal vibrations over the delaminated part. The results showed that on some planes the longitudinal vibration had a significant effect. Also, the most important difference between the delaminated and intact beams was discovered on the plane of the static and dynamic forces: we experienced that although the delaminated beam was softer than the intact one, on the mentioned plane the delaminated beam involved much more extended stable domains than the intact beam.

We summarized the results and submitted an article to *Archive of Applied Mechanics* (Springer) entitled „*Stability of delaminated composite beams subjected to retarded periodic follower force*" in February, 2023.

7. Differences compared to the original research plan

Finally, we would like to justify the differences compared to the originally submitted research plan.

- Balázs Kiss participated in the project only in 2019 and he quitted in 2020. He was replaced by Péter Máté in 2020 and later on by Bence Hauck also in 2020.
- The combination of dislocation theory and plate models was found to be a quite lengthy and complex mechanical problem, and therefore this part of the project was abandoned.
- We successfully completed about 93% of the scheduled problems. The analysis of elastic structures including time delay was performed in the last part of the project, however we submitted a relevant article to a journal with IF.

We will continue the research work in the future and finally we are very grateful for the support of our research work during the past four years.

Budapest, 28th March, 2023.

Dr. András Szekrényes