Report

Élvonal KKP126683 project

by András I. Stipsicz

1 The research team

During the grant period (2018-2023) several members of the research group achieved major advances in their respective careers, including the following examples

- The PI (A. Stipsicz) was elected as regular member of the Hungarian Academy of Sciences
- The PI submitted an ERC proposal, the topic of which grew out of the work of the KKP project, and the ERC proposal has been selected for interview in the second round (which will take place in February 2024)
- A. Némethi was elected as corresponding member of the Hungarian Academy of Sciences
- T. Ágoston (student of A. Némethi) completed his PhD thesis (and will defend soon) and got a position at ELTE
- V. Földvári (student of the PI) completed her PhD thesis, defended and earned her PhD degree at ELTE
- S. Mihajlović (student of the PI) completed his PhD thesis, defended and received his degree and got a postdoctoral position at the University of California, Davis (CA, USA)
- T. László got a permanent position at Babes-Bolyai University in Kolozsvár
- J. Nagy (student of A. Némethi) completed his PhD thesis, defended and got his degree

- A. Sándor (student of A. Némethi) completed and defended his PhD thesis and got his degree
- postdoctoral participant F. Gironella got a postdoctoral position at Humboldt University (Berlin) and then a tenure (CNRS) position in Nantes, France
- A. Cavallo (student of PI) completed his PhD degree, received a 1-year position at Max-Planck-Institute in Bonn, and then a 3-year position at McGill University in Montreal
- I. Matkovic (student of the PI) received her PhD degree, got a 3-year position in Oxford, UK, and then a further postdoctoral position at the University of Uppsala, Sweden.
- A. Alfieri (student of PI) received a three-year position at University of British Columbia, Vancouver, Canada, and then a two-year position at McGill University in Montreal (Canada)

The research group actively participated in building a lively group of low dimensional topologists at the Rényi Institute. In particular, a weekly seminar was organized by the team, and the special semester on low dimensional topology and singularity theory (the very subject of the KKP project) was organized by the PI and by Professor Némethi (with the help of Marco Marengon and Javier Bobadilla). During this event (in the Spring of 2023) probably the Rényi Institute was the (unofficial) center of low dimensional topology and singularity theory in Europe. Our aim is to keep this status — we hope that the potential ERC Grant can be of great help to maintain this ambitious goal. (It is rather unfortunate that the Élvonal grants seem to have changed profile and no longer support projects delivered by this proposal, and were closed for researchers working hard in Hungary.)

2 Mathematical achievements

András Stipsicz, in collaboration with Sz. Szabó and P. Ivanics studied fibrations on the moduli spaces of Higgs bundles using four-dimensional techniques, resulting in a sequence of three papers. In a joint project with Z. Szabó he also examined the cosmetic surgery conjecture and verified it for pretzel knots. In addition, they studied negative spheres in four-manifolds, and found an interesting (but probably very hard) conjecture for them. They also examined the minimal genus problem in various four-manifolds. With Alfieri and Kang they used the connected homology technique for double branched covers of knots and proved novel independence results in the concordance group. In a slightly different direction, they extended the Upsilon invariant (earlier defined by Ozsváth-Stipsicz-Szabó for knots in S^3) to other manifolds and made some interesting calculations concerning the new invariants. In an AIM (American Institute of Mathematics) project (in collaboration with P. Aceto, J. Meier, A. Miller, M. Miller and J. Park) they showed examples of non-slice knots for which all cyclic branched covers are trivial in the respective homology cobordism group. In another AIM project (now with P. Aceto, N. Castro, M. Miller and J. Park) they found a new slice obstruction, which applies to the (2,1) cable of the Figure-8 knot — a question which was open for quite some time. Regarding the minimal genus problem, in collaboration with M. Marengon, A. Miller and A. Ray they found new phenomena in the complex projective plane and its connected sum with itself.

András Némethi continued the construction of the lattice cohomology and the graded root in certain new situations. Together with T. Ágoston they finished the construction in the case of isolated curve singularities, and later he constructed the filtered version of this (with comparisons to knot Floer homology). He also established the theory of filtered lattice cohomology associated with the topological type of normal surface singularities. With Ágoston they also succeeded to construct the homological functor in the curve case induced by analytic deformations. With Romano-Velasquez they classified all the reflexive modules of rank one on rational and elliptic normal surface singularities, in this way they extended results of Artin, Verdier and Esnault. This study also emphasises the deep connections between the analytic properties and the topological invariants of the link.

Szilárd Szabó made progress in the study of the Hitchin WKB-problem, the P=W conjecture and its geometric counterpart, as well as on the behaviour of the quantum connection of surfaces under blow-ups. He proved the P=W conjecture and the geometric P=W conjecture in all Painleve cases. For the proof he studied the large energy limiting behavior (a.k.a. WKB-analysis) of solutions to Hitchin's equations. In a joint work with A. Némethi they gave an independent proof of the same results by using plumbing calculus. He further studied the Hitchin WKB-problem and established the P=W conjecture in highest weight for a complex 4-dimensional moduli space of rank 2 logarithmic parabolic Higgs bundles, by determining corresponding generating cycles up to homotopy. In joint work with Gyenge, they worked out the asymptotic behavior for small quantum parameters of the spectrum of the quantum connection of surfaces under blow-ups, thereby establishing the 2-dimensional special case of a conjecture of Kontsevich.

Agustin Romano-Velasquez is completing a new article in collaboration with Jos Luis Cisneros Molina and Jos Antonio Arciniega Nevrez. In this article, they finish the classification of reflexive modules in quotient singularities of dimension two. These singularities are the singularities with a finite number of indecomposable reflexive modules (called singularities of finite type). Thus, this work finishes the classification problem of reflexive modules in singularities of dimension two of finite type. This work is a continuation of a previous paper (Cheeger-Chern-Simons classes of representations of finite subgroups of SL(2,C) and the spectrum of rational double point singularities, arXiv:2302.02000). Also, it generalizes the classification of rank one reflexive modules, which they did previously in the case of quotient singularities.

The main subject of the research of Tamás László was the correspondence of analytic and topological properties of curve- and surface singularities. This is a rather rich subject, right in the crossroad of algebraic geometry and low-dimensional topology. They mainly focus their study to the interpretation of some analytic invariants of a curve/surface singularity through topological quantities associated with the link (a knot/3-manifold canonically associated with the singularity). The method is to study good topological candidates for analytic invariants, then apply them to understand analytic properties as well. They studied the topological Poincaré series which has many interesting properties and applications to singularity theory and other parts of mathematics. Among others, they developed a unique decomposition of the Poincaré series associated with a surface singularity, which provides a topological polynomial invariant as a "polynomial" categorification of the Seiberg-Witten invariant of the link. Applying these methods, they developed a formula for the delta invariant of a curve singularity which is embedded in a rational normal surface.

Fabio Gironella focused on two different questions. First, he studied the topology of the contactomorphism group in relation to that of the diffeomorphism group, in high dimensions. In this context, he found examples of contact mapping classes of infinite order on some explicit tight contact manifolds and, in joint work with E. Fernandez, on all overtwisted contact spheres. The second direction of research concerned the topological study of an explicit construction of contact structures in high dimensions due to Bourgeois. In joint work with J. Bowden and A. Moreno, using pseudo-holomorphic curves techniques, they proved that Bourgeois contact structures are always tight in dimension 5. They also gave, for some special class of contact manifolds which includes some of those constructed by Bourgeois, results of smooth classification and obstructions to symplectic fillings.

Work of Alberto Cavallo addressed applications of Heegaard Floer and Khovanov homology to knot theory, with focus on link concordance, and contact topology. He was part of the project in 2018 as a PhD student, and in 2021 and 2023 as a visiting researcher. His main result during his PhD was the definition of a link version of the Ozsváth-Szabó τ-invariant, and he applied it to prove a more general version of the Thurston-Bennequin inequality. Moreover, he extended a result of Dymara about Legendrian isotopy of loose knots in overtwisted contact structures. Joint with Collari, he studied slice-torus invariants of links, as τ from HFL and s from Kh. Some computations were made for Whitehead doubles. Joint with A. Stipsicz, he studied the relation between exotic structures on simply connected smooth 4-manifolds and the set of smoothly slice links in them. Joint with I. Matkoviç, he explored applications of sutured Floer homology. They described the nearly fibered links of genus one, extending the work of Baldwin-Sivek. Furthermore, they studied links with non-zero Legendrian invariant, relating non-vanishing in the hat version of HFL with the preservation of tightness by adding boundary parallel half Giroux torsion.

Work of Irena Matkoviç addressed interactions between Heegaard Floer theory and contact topology. She was part of the project in 2018 as a PhD student, and in 2023 as a visiting researcher. She completed the classification of tight contact structures on small Seifert fibered L-spaces, proving that they are completely determined by Heegaard Floer contact invariant. She also specified exactly which of them are (Stein) fillable. Her joint work with Alberto Cavallo is described above.

László Fehér studied the global behaviour of complex singularities: the singularity locus of a given singularity in a family of maps. He determined various motivic invariants of certain singularity types. In particular, he studied the Chern-Schwartz-MacPherson classes of the Σ^i singularity loci, and more generally CSM classes of Schubert varieties. He also continued this study with respect to the extension to the motivic Chern class of these singularities, and a connection with the corresponding stable envelopes of Okunkov and Maulik has been established. He developed an axiomatic approach to study the motivic Chern class of more general singularities, and studied the case of conical singularities and the relation to the projectivization of these conical singularities, generalizing the notion of projective Thom polynomial to the motivic Chern class case. He noticed an unexpected connection between the equivariant methods explored in the above works and additive combinatorics, for example the Erdős-Heilbronn-Hamidoune-Da Silva theorem. In the manuscript on Plücker numbers he studies the coincident root loci of binary forms and proves the polynomial (in the degree of the corresponding plane curve) behaviour of generalized Plücker numbers.

Viktória Földvári examined Legendrian knots and knot invariants with various techniques, she published five research papers and a doctoral dissertation. Under the supervision of András Stipsicz, she used knot Floer homology to give a lower bound on the number of Legendrian realizations of some two-bridge knots, yielding infinitely many new examples of Legendrian and transverse non-simple knot types. She found sufficient conditions of non-simplicity on the continued fraction expansion of the corresponding rational number. With Vera Vértesi, they applied convex surface theory to distinguish tight contact structures in knot complements, and obtained an upper bound on the number of maximal tb Legendrian realizations of double twist knots. With Gábor Damásdi and Márton Naszódi they proved colorful Helly-type theorems for the volume of intersections of convex bodies. She also gave bounds on the cardinality of mu-Minkowski arrangements. In a research collaboration started at ICERM and Brown University, they introduced a contact invariant in the bordered sutured Heegaard Floer homology of a 3-manifold with boundary. They showed that the pairing of these invariants induced by gluing compatible manifolds with boundary recovers the Heegaard Floer contact invariant for closed contact manifolds. In another paper, she introduced Ozsváth-Stipsicz-Szabó's knot invariant Upsilon without the holomorphic theory, using constructions from grid homology, and reproved the most important propositions with combinatorical techniques.

Budapest, 1/12/2024

Stipsicz András
Professor

Rényi Institute of Mathematics
Budapest, Hungary