Final report

Quantum renormalization group

Introduction

In this project our goal was to treat the interaction between the physical system and its environment in the framework of the functional renormalization group (RG in short) method. The RG method is one of the most important theoretical tool in quantum field theory. It can be used in every important area of modern physics.

In the RG method we can identify the physical system with the infrared (IR) modes, while the ultraviolet (UV) modes can play the role of the environment. An RG blocking step transforms the IR modes into UV ones. We consider our physical system as an open system, where the traditional RG technique does not work effectively. We need the closed time path (CTP) formalism to get the quantum dynamics of an open system. It is given in Minkowski spacetime, so first we should consider this issue.

The traditional RG formalism is given in Euclidean instead of the original Minkowski one. In Minkowski spacetime the on-shell particles appear in the description, while in the Euclidean formalism we have only virtual, off-shell (and basically non-existing) particles. The real particles are diagonal in the momentum space, which result in nonlocal interactions. It can be taken into account by introducing a bilocal term into the action. A further shortcoming of the Euclidean RG procedure comes from the fact, that it considers only the pure states, and it cannot account for the contribution of the mixed states. Its inclusion in the RG description can be made by treating the problem in the CTP formalism, which should be formulated in Minkowski spacetime, too. The real time evolution is usually called as quantum renormalization group.

The treatment of the open system necessitates the presence of the real particles, furthermore the finite lifetime of the modes. The latter can be taken into account by complexifying the mass coupling. In the Minkowski formalism the Feynman epsilon can play the role of the infinitesimal imaginary part of the mass which belongs to infinite lifetime. It suggests that we should find such an RG evolution, where complex couplings evolve. This result could bring us a step closer to our goals.

Recently the quantum Einstein gravity (QEG) is the most frequently investigated model in the framework of renormalization. Its widebroed interest motivated us to push our investigation into direction of this model. It is nonperturbative, and it possesses a nontrivial fixed point, which makes the model renormalizable. The model is said to be asymptotically safe, which can be the generalization of the concept of asymptotic freedom. The extension of the model, and its connection with more realistic models are extensively investigated, and they also show asymptotically safe behavior.

Results

In this section we collect the results that were obtained in this project.

First, we investigated the model of QEG. We considered such a version of the model, that includes the most general form of the term quadratic in the curvature. The general form of the Einstein-Hilbert action depends on the curvature in a similar way as the potential depends on the field variable in field theories. We chose the quadratic term in the curvature in arbitrary dimensions and containing an arbitrary regulator. We identified the UV non-gaussian fixed point (NGFP) for different forms of regulators in case of dimension 3. We chose this dimension, since in d=4 there are further fixed points

and singularity surfaces, which make the calculations extremely difficult. We also determined the correspondig critical exponent for the correlation length. We looked for such an optimized regulator where the physical quantities show the least regulator parameter dependence. We found that the Litim regulator satisfies this condition. The IR fixed point has also been investigated, and it is found that the exponent of the correlation length is insensitive to the term quadratic in the curvature [1].
As we mentioned, the Minkowski formalism contains real particles, making the action nonlocal. We notice, that the RG blocking transformation itself can generate nonlocal terms in the action. During the RG blocking step we integrate some degrees of freedom, and as a result we get an effective model. Every effective theory is nonlocal due to the regularization. The UV cutoff, for example, serves as a regulator, and since it is the highest momentum in the theory, it defines a smallest value for the distance, and it results in nonloclity in the model. In this sense every effective theory can be considered as nonlocal. In order to understand better the properties of the nonlocal models, we started to investigate a simple O(1) model in d=3. The question has not been clarified even in the Euclidean spacetime, therefore we investigated the simplest possible scalar model first. The nonlocality has been introduced via a bilocal potential. The results were very surprising. We obtained, that the bilocality gives a leading order tree-level contribution to the beta functions. It turned out, that the RG blocking transformation gives a nontrivial saddle point, which contributes to the evolution of the bilocal term in the potential. It serves as a further strong motivation to investigate nonlocal models. The nontrivial saddle point is important, since it is a tree-level contribution. This is the reason, why we expect, that the nonlocality can change the phase structure fundamentally. These results has not been known in the literature yet. We found, that the Wilson-Fisher fixed point dissappears in the model, and a new fixed point appears in the phase space, which can be related to a first order phase transition. The bilocal term in the potential introduces bilocal couplings, which are momentum dependent. It gave us the possibility of finding nontrivial (nonconstant) ground states. We showed, that the phase transition around the new fixed point can be related to a change from constant to periodic ground state field configuration [2]. The novel results suggest, that the inclusion of the bilocal potential changes the physical properties of the model dramatically. It suggests, that we should reinvestigate the field theoretical models, since a leading order term is missing in the treatments. We filled this gap for the case of the sine-Gordon model and for the conformally reduced gravity.
We also obtained, that the evolution of the bilocal couplings can make the model unstable. This result is general, the nonlocality usually leads to instability in the models, because the action becomes nonbounded from below. We can say, that the effective theories are nonlocal, furthermore they can be unstable. We argue, that every realistic model is effective, since in quantum field theory the momentum integrals are infinite, and the regularization makes them nonlocal. We have similar situation in the Standard model, where we have singularities if we remove the high energy UV cutoff due to the triviality problem of the Higgs sector and the Landau pole. We expect new physics at high energy scales. We can conclude, that the Standard model is also unstable. The observations are performed in short times, which makes the instability practically undetectable.
Next we performed the functional RG treatment of the 2-dimensional sine-Gordon (SG) model in the framework of the WH equation. It is also a scalar theory, including a periodic potential. Its additional discrete symmetry in the translation results in a novel phase structure with a topological phase transition. The traditional RG treatment of the model can map out the phase structure of the SG model with the Coleman fixed point. However, there is a strange scaling behavior at the wavenumber $\beta^2 = 4\pi$, which has not been uncovered yet. In order to find the new scaling at this special wavenumber we improved the model by including the evolution of the bilocal term in the action. As we mentioned, the RG blocking steps introduce a nontrivial saddle point, which gives a tree-level contribution to the

RG evolution to the bilocal term of the potential. We showed that the bilocal term can account for the Kosterlitz-Thouless type essential scaling of the correlation length ξ around the Coleman point. This result could only be obtained so far in the framework of the Wetterich equation with smooth cutoff and taking into account the gradient expansion beyond the local potential approximation. We demonstrated that the essential scaling can be found in the framework WH equation, too. The nonlocal interactions modify the phase structure of the models significantly. This has been demonstrated for the SG model, too. We found, that besides the Coleman point, we have a phase separation at $\beta^2 = 4\pi$, where the sine-Gordon model is equivalent to the Thirring model. In summary, the phases, and the essential scalings obtained by gradient expansion could be recovered by the tree-level contribution coming from the bilocal potential [3].

It turned out that there is no such a non-perturbative regulator in Minkowski spacetime, which is Lorentz invariant. It implies, that we cannot derive the beta functions for the QEG model in Minkowski spacetime in a Lorentz invariant way. If we considered the model of QEG at the tree-level, we could not have problem with the regulators, because in this approximation every physical quantity is finite. However this treatment could run into another problem, because the Minkowski spacetime and the CTP formalism inevitably introduces the nonlocality. We can choose a Lorentz invariant renormalization scale, which implies, that we should remove the modes in a hyperbolic shaped shell. Its volume is infinitely large, therefore it needs further regularization, which break the Lorentz invariance. This reason made is us the decision, that we chose the renormalization scale from the 3-dimensional spatial momentum.

In order to simplify the question, which we raised above, we started to treat a simpler scalar model, i.e. the conformally reduced version of the QEG model. We derived the RG equations for the model in Minkowski spacetime. We performed the frequency integral separately. As a result we obtained a cylindrically symmetric theory. We obtained, that there is a NGFP in the phase space. It is an UV attractive fixed point (a focal point), but the scaling exponents are slightly different from the ones obtained in Euclidean spacetime. We have used a Litim type regulator and a sharp cutoff, and both cases gave qualitatively the same phase diagram [4].

We continued the investigation of the bilocal term in the potential in the conformally reduced QEG model. We obtained, that the bilocal coupling can play the role of the gradient expansion. We got the same phase space as earlier, with the NGFP [5].

The conventional RG method in Euclidean spacetime uses such a subtraction point (or renormalization scheme), where the point is close to the origin of the energy plane. It cannot introduce complex couplings, which are necessary to describe the dynamics of an open quantum system, because the imaginary part of the beta functions are proportional to the imaginary parts of the initial complex couplings. However a proper choice of the subtraction point can make the complex evolution finite. As we have shown, the multiplicative renormalization provides the same result. The subtractive renormalization, where the scale of the RG equations is identified by the subtraction point, we can find complex running couplings. To mimic the finite subtraction point in functional renormalization, we should modify the IR field. We used plane wave field variables, with its energy equal the residue of the propagator in the Minkowski case. This choice proved to be fruitful to start the evolution of the complex couplings. We have shown, that the WH equation can be generalized to include the wave function renormalization. We considered the realistic d=4 case. The triviality problem of the scalar field has been considered. We found, that the Landau pole can be smoothed out, however an instability problem prevents us to find the deep UV scaling. We have also found, that there is a strong connection between the singularity due to the spinodal instability and the appearing Landau pole [6]. The results could help us to reach one of our main goal, namely the RG treatment of the open systems. The nontrivial subtraction point could complexify the couplings, and the Minkowski spacetime enabled us to consider real particles. The only ingredient that is missing, is the inclusion of the mixed states, which should be taken into account by the CTP formalism.

In order to get the last issue, we derived the quantum renormalization group equations for the O(1) scalar model by using sharp cutoff. The nonlocality appeared as step function in time, representing the forward time arrow for a particle. We obtained the leading order evolution equation around the Gaussian fixed point. At d=4 we could not find any further fixed points. We found new relevant couplings, which are related nonlocality. The massive couplings can represent the separation of the UV and IR scalings, while the quadratic couplings related to mixing of the time axes can control the separation of the quantum and the classical version of the model. Our investigation is in progress [7].

Final remarks

In our project our main goal was to implement the RG technique into the CTP framework, and investigate the quantum-classical transition. During the project, we had to face the fact, that in order to reach the goal, several preliminary questions had to be answered. As we made progress in the project, several further issues emerged. This is why, the main issue of the work became to understand better how the Minkowski formalism can change the results of the RG method.

The most important result of the project was that the renormalization in Minkowski spacetime can change the properties of the models significantly. We could complexify the couplings by the subtraction point, and then we made significant contribution in the question of triviality in scalar models at dimension 4.

As a whole we think, that the project was successful. The greater part of the elements of the workplan has been successfully worked out. Although the final question could not be answered, but we solved several preliminary problems, and we made significant contribution to the RG method.

The project gave us a great help to achieve our goals. In its framework we wrote 6 articles, and 4 of them was accepted in publication in refereed international journals. They constitute the base of the PhD thesis of Imola Steib, who makes her research under my supervision. She had the preliminary defence on 27th Januray 2020. The title of the thesis is ”Quantum Renormalization Group”.

References

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Conferences, talks

[E1] 9th International Conference on the Exact Renormalization Group (ERG2018), 9-13 July 2018, Paris, France, Sándor Nagy, talk: Real time renormalization.

[E2] 9th International Conference on the Exact Renormalization Group (ERG2018), 9-13 July 2018, Paris, France, Imola Steib, poster: Renormalization of bilocal potentials.

Further talks

[E3] The 18th Debrecen–Katowice Winter Seminar, Hajdúszoboszló (Hungary), January 31 – February 4, 2018, Borbála Fazekas, talk: Double Walsh-Fourier series solutions of second order partial differential equations.

[E4] Numbers, Functions, Equations 2018, Hajdúszoboszló (Hungary), August 26 – September 1, 2018, Borbála Fazekas, talk: Walsh-Fourier series solutions of linear differential equations with error estimation.