Final report for NKFIH PD 125160

The grant identifier is indicated on 11 papers and 1 extended abstract in conference proceedings. Out of the 11 papers, 6 are accepted or published ([6-11]), and 5 are submitted ([1-5]). Out of the 6 accepted or published papers, 5 are in Q1 ranked journals. One of these is SIAM Journal on Computing, an exceptional journal ranked in the top 10% both in Mathematics and in computer science. My coauthors and I are proud to be listed by SIAM among the 20 most cited papers of SICOMP since 2018. My most notable single-author paper [6] is accepted in another journal of SIAM, the prestigious SIAM Journal on Discrete Mathematics. A world-leading expert in coding theory, Ron Roth has congratulated me in a personal e-mail, and as an editor, thanked me for my contribution to the journal. The 5 submitted papers are of similar quality. Based on the feedback obtained so far, I expect that 4 of them will be accepted in Q1 ranked journals. In this (highly probable) case, I added 11 papers to my list of publications during the grant period, 9 of which would appear in journals in the first quartile.

In the past three years, I gave 18 scientific talks (on top of my university courses). The 4 conference talks would have been accompanied by a fifth one; that and an invitation to a research visit never came to fruition due to the pandemic. Beyond writing papers and giving talks, I have been focusing on further scientific activities that are relevant in obtaining the title "Doctor of the Academy". I am an organizer of the conference in honor of Kálmán Győri, János Pintz, and András Sárközi, and an editor of Publicationes Mathematicae. My habilitation was successful with full marks, and I was promoted to associate professor.

1. Homogeneous structures, Thomas' conjecture

A partial order is called semilinear if the set of upper bounds of any element is a chain, and any two elements have a common upper bound. Up to isomorphism, there exists a unique countable, existentially closed semilinear order S. In [10], we characterized the reducts of S up to first-order interdefinability. Moreover, we determined the model-complete cores of all reducts. This is the starting point of the investigation of the complexity of CSPs of the reducts, known as phylogeny constraint satisfaction, a generalization of the scheduling problem.

These results are improved in the newly submitted manuscript [1]. I have classified the reducts of S up to the finer equivalence relation existential interdefinability. It is demonstrated that several parts of the technique work in full generality. An inductive argument is shown which makes it possible to prove general results on reducts of Ramsey structures, and then two such results are verified. One of them reduces the existential version of the Thomas conjecture to a concrete technical problem and the original first-order version. Hence, the paper shows how the classification of reducts up to first-order interdefinability can be improved to existential interdefinability, and moreover, it indicates that the strictly more general latter problem could be easier to handle. One reason is the inductive nature of the proofs: in such arguments, it is not uncommon that a stronger statement is easier to prove. Furthermore, the core idea of the classification technique is to climb up the lattice of algebraic invariants corresponding to the reducts in the appropriate Galois correspondence: that is the automorphism group in case of first-order interdefinable reducts and the self-embedding monoid for existential interdefinability. While it is

unclear to date if there are finitely many minimal automorphism groups above the automorphism group of each reduct, the analogous statement for self-embedding monoids is well-known. The next natural weakening of the logic would be classification up to existential positive interdefinability, however, there are in general a continuum of those equivalence classes. Thus the analysis of existential reducts can lead to the solution of the Thomas conjecture; [1] is the first step in this project.

2. CSPs

In [8] we studied homomorphisms from the polymorphism clone of ω-categorical structures to the trivial clone. The main question is whether such a homomorphism is automatically continuous with respect to the topology of point-wise convergence. In fact, for the applications, only the following less demanding property matters: whenever there is a (discontinuous) homomorphism, there also exists a continuous homomorphism. We have provided a condition that reduces this problem to the binary fragment of the polymorphism clone. Some examples were constructed to polymorphism clones of ω-categorical structures that can be mapped to the trivial clone via a discontinuous homomorphism. Finally, we have shown some general positive results.

We have determined the complexity of the reducts of all Henson graphs and some homogeneous equivalence relations in [9]. Together with earlier results, this paper finishes the proof of the dichotomy conjecture for all reducts of countable homogeneous graphs: all such computational problems are either in P or NP-complete. A finite list L of multivariate functions was provided which can be used to efficiently determine which case occurs for any given reduct. If a function in L is a polymorphism of the reduct, then the corresponding CSP is in P, otherwise, it is NP-complete.

3. Fraïssé classes

A Fraïssé class captures all model theoretical properties of its limit. That is why it is often more convenient to work with this class of finite structures rather than the infinite limiting structure in certain types of problems. One such area is the asymptotic probability of properties. According to a seminal result of Fagin, a closed graph formula holds in the (infinite) random graph if and only if it holds with asymptotic probability 1 in the $G(n, 1/2)$ model. The result can be generalized to further relational languages and Fraïssé classes, and the logic can be weakened from the first-order realm, yielding more interesting problems. Such a problem is the rigidity of structures, or generally speaking, the asymptotic probability of the automorphism group of a graph (or even some other relational structure) being isomorphic to a given group H provided that it contains a given group G. Given the group G, this is nonzero for only finitely many H, and it is always a rational number. In order to determine those finitely many groups H and the positive asymptotic probability they occur with, we need to understand those permutation groups that have a small maximum degree of elements compared to the size of their support. The extremal cases of this combinatorial problem were determined in [4].

Based on the results of [4], the near-extremal cases were classified in [6]. More precisely, I have given a clear description of those binary linear codes whose maximum distance is $n/2 + 1$, where n is the length of the code. The constructions that

appear in the classification are of interest to coding theorists for multiple reasons: these are 2-weight and 3-weight binary linear codes, and besides having a small maximum distance, they also have a large minimum distance. Some of them nearly or precisely meet the Plotkin- or the Griesmer bound.

4. Enumerative combinatorial problems

In [11], we have provided asymptotic and log-asymptotic estimates to the n-element and n-generated models in all varieties of monounary algebras. We obtained infinitely many new examples to varieties with superpolynomial and subexponential spectra: that is, varieties whose spectra grow faster than any polynomial and slower than any exponential function. Only some sporadic examples were known before in the literature.

Voting protocols, such as the push- and pull protocols are mathematical models of the behavior of people prior to an election. However, they are also studied and used in a number of other theoretical and applied areas, such as socio-economic models, peer-to-peer computer networks, and to simulate the spread of gossip or an epidemic. In [7], I developed an elementary linear algebraic technique and used it to provide estimates to the expected runtime and to the probability of each consensus to win for the discordant push- and pull protocols in cycle graphs. These bounds are sharp, provided that the initial state be not too chaotic, and an asymptotically sharp uniform upper bound is given to the expected runtime.

The same method is applied in [2] to obtain similar estimates to the higher moments of the runtime (in cycle graphs). Furthermore, a general bound is presented to the expected runtime of the so-called "gambler's ruin problem" for arbitrary graphs. This is a multiplayer game, where agents are vertices of a given graph, each having a positive integer initial wealth. In each round, an edge is selected uniformly at random, and a fair coin-toss decides which endpoint transfers 1 unit wealth of its money to the other. The process halts when a vertex declares bankruptcy. In the special case when the graph has two vertices linked by an edge, we obtain the classical drunkard's walk, whose expected runtime is the product of the two quantities of wealth. Using the linear algebraic method, I have shown that if the game is played on an arbitrary finite graph, then the expected runtime is at most the harmonic mean of the expected runtimes of the drunkard's walks played on the edges multiplied by the number of edges.

In the manuscript [5], we have extended some of the above results to star graphs. This turned out to be a nonstandard case: in a typical graph, the discordant push protocol is faster than the discordant pull protocol, however, we obtained that it is the other way around for star graphs. In fact, the discordant pull protocol beats the discordant push protocol by a factor of the order of magnitude $\log n$. We have determined the exact asymptotics of the largest possible expected runtimes.

5. Further topics not included in the original work plan

There is one manuscript that falls into this category, the new paper [3] written with a co-author Csaba Vincze. Following two positive referee's reports that found the paper interesting and recommended acceptance, we have revised it according to the referees' suggestions, and we are now waiting for the final decision of the editorial board. The topic of the manuscript originated from a real-life problem. In

order to simulate homogeneous fog on a certain type of photos, it suffices to know the distance of objects on the picture from the camera, a basic problem with many different numerical methods in the literature to solve it. However, if the goal is to simulate inhomogeneous fog - a task in a joint project of mathematicians and computer scientists of the University of Debrecen together with other Hungarian universities - then we need more data, namely the original 3-dimensional coordinates of the objects on the photo. Hence, we need an approximate solution to a variant of a fundamental problem in photogrammetry: photos are produced by a central projection, and we need to determine the coordinates of the center, given the distances and some incidence relations of points on recognizable objects on the picture. Each triplet of points whose real-life pre-images are collinear defines a generalized conic of degree four containing the center. We provided an algebraic condition that holds if and only if the generalized conic is unbounded, and we have proven that the polynomial defining the profile curve is always irreducible. In particular, two such polynomials have at most 16 common roots, effectively reducing the possible locus of the center to a finite set. The journal is ranked in six categories; the relevant one in terms of our paper is computer vision, in which the journal is rated Q1.

Extended abstract in conference proceedings

András Pongrácz: Discordant voting protocols for cyclically linked agents, Proceedings of ICCSE'18, World Congress on Engineering, 2018. http://real.mtak.hu/85192/

Papers

Submitted papers.

[1] András Pongrácz: Existential reducts of the binary branching semilinear order and the Thomas conjecture, submitted, 2020.

[2] András Pongrácz: On the gambler's ruin problem and higher moments of some absorbing Markov chains, submitted, 2020.

[3] András Pongrácz, Csaba Vincze: On the reconstruction of the center of a projection by distances and incidence relations, submitted, 2020.

[4] András Pongrácz: Extremal solutions of an inequality concerning supports of permutation groups and punctured Hadamard codes, submitted, 2019.

[5] Kamilla Kátai-Urbán, András Pongrácz, Csaba Szabó: Voting protocols on the star graph, submitted, 2018.

Accepted or published papers.

[6] András Pongrácz: Binary linear codes with near-extremal maximum distance, SIAM Journal on Discrete Mathematics, 18 pp, to appear, 2020.

[7] András Pongrácz: Discordant voting protocols for cyclically linked agents, The Electronic Journal of Combinatorics 27(1):P1.58 14 pp. An extended abstract appeared in the Proceedings of ICCSE'18, World Congress on Engineering, 2020.

[8] Manuel Bodirsky, Michael Pinsker, András Pongrácz: Projective clone homomorphisms, Journal of Symbolic Logic, to appear, 2019.

[9] Manuel Bodirsky, Barnaby Martin, Michael Pinsker, András Pongrácz: Constraint satisfaction problems for reducts of homogeneous graphs, SIAM Journal on Computing 48:4 1224-1264, 2019.

[10] Manuel Bodirsky, David Bradley-Williams, Michael Pinsker, András Pongrácz: The universal homogeneous binary tree, Journal of Logic and Computation 28:1 133-163, 2018.

[11] Kamilla Kátai-Urbán, András Pongrácz, Csaba Szabó: The fine- and generative spectra of all varieties of monounary algebras, Algebra Universalis 80:22 18 pp, 2018.

All accepted and published papers are uploaded to the REAL repository:

[6]: http://real.mtak.hu/id/eprint/113210

[7]: http://real.mtak.hu/id/eprint/113209

[8]: http://real.mtak.hu/100544/

[9]: http://real.mtak.hu/113102/

[10]: http://real.mtak.hu/85193/

[11]: http://real.mtak.hu/113097/

Their cumulative impact factor is: 0,843+0,762+0,631+2,680+0,860+0,730= 6,506.

Further documents

Habilitation dissertation.
Model theoretic properties and algebraic invariants of omega-categorical structures (42 pp)

Documents not related to the topics of the research project.
Two sets of lecture notes (each approximately 140 pp) were written on group theory, as auxiliary teaching material to the algebra classes at the University of Debrecen (in Hungarian and in English). Furthermore, a similar document is in preparation about Markov chains and their applications.

Talks

Invited talks.
A research invitation was cancelled (to TU Wien) in the summer of 2020 due to the pandemic.

Hadamard-kódok és egy extremális kombinatorikai probléma (90 min)
University of Debrecen, algebra and number theory seminar (Debrecen, 2019/11/08)

Szavazási protokollok hatékonysága
University of Debrecen, seminar of the Institute of Mathematics (Debrecen, 2018/04/12)

Szavazás a kerekasztal mellett
MTA Alfréd Rényi Institute, algebra seminar (Budapest, 2018/02/26)

Markov-láncok és döntési rendszerek
Festival of Hungarian Science (Debrecen, 2017/11/23)

Conference talks.
My participation on a conference in the summer of 2020 was cancelled due to the pandemic. The talk would have summarized my recent results on permutation groups and binary linear codes.

A vetítési centrum rekonstrukciója illeszkedési relációk és távolságadatok alapján - elmélet és gyakorlat
HU-MATHS-IN Researchers' Days (in Hungarian, 2020/06/29-30)

Push and pull protocols on finite graphs
Computability in Europe (Durham, United Kingdom, 2019/07/15-19)

Non-determininstic decision making on finite graphs
9th European Congress of Mathematics (Bled, Slovenia, 2019/06/23-29)

Discordant voting protocols for cyclically linked agents
World Congress on Engineering, ICCSE'18 conference (London, United Kingdom, 2018/07/04-06)

Further scientific talks.
Monoidok és klónok rekonstruálhatósága
Habilitation, scientific talk (in Hungarian, Debrecen, 2019/03/28)

Reconstruction of monoids and clones
Habilitation, scientific talk (Debrecen, 2019/03/28)

Szele Tibor csoport- és gyűrűelméleti munkássága
Tibor Szele Memorial Day (in Hungarian, Debrecen, 2018/06/13)

Scientific talks to the public.
Egy lemma, ami nem Burnside-é
TTK Summer Camp (in Hungarian, Debrecen, 2020/08/27)

Bevezetés az útvesztőkbe - és ki is
Night of the Researchers (in Hungarian, Debrecen, 2019/11/28)

Végtelen halmazelmélet és logika I (90 min)
University of Debrecen, to high school students (in Hungarian, Debrecen, 2019/11/23, 9:00-10:30)

Végtelen halmazelmélet és logika II (90 min)
University of Debrecen, to high school students (in Hungarian, Debrecen, 2019/11/23, 11:00-12:30)

Sétálunk, sétálunk I (90 min)
Verseghy Mathematics Day (in Hungarian, Szolnok, Ferenc Verseghy Secondary School, 2018/12/14, 8:25-9:55)

Sétálunk, sétálunk II (90 min)
Verseghy Mathematics Day (in Hungarian, Szolnok, Ferenc Verseghy Secondary School, 2018/12/14, 10:25-11:55)

Véletlen szavazás
Night of the Researchers (in Hungarian, Debrecen, 2018/09/28)