Project closing report

Title of the proposal: Device-independent quantum protocols for non-distillable quantum systems

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Bound entanglement is a weak form of entanglement. No pure entangled states can be obtained from bound entangled states by means of local operations and classical communication, that is they are non-distillable. All known bound entangled states have a positive partial transpose (PPT). It was supposed that bound entangled states are not useful as resources in quantum information.

Peres conjecture.—Peres proposed a conjecture according to which all such states obey locality. In an earlier work one of us refuted this conjecture by finding a bound entangled state in the 3×3 dimensional space violating a two-party Bell-inequality with three two-outcome (binary) measurements for one party and one three-outcome and one binary measurement for the other one. In the present project we have generated all tight two-party Bell inequalities with four binary measurements for one party and one four-outcome and one binary measurement for the other, and we have found that one of them is violated by a 4×4 dimensional bound entangled state, but not by any lower dimensional one. We have extended these results by finding a whole family of tight two-party Bell-inequalities (d binary measurement for one party and one d-outcome and one binary measurement for the other) and $d \times d$ dimensional states violating them. We have got the states in a semi-analytic form, and we have shown that they violate the corresponding inequality for any dimensions, while the inequalities are not violated by lower-dimensional bound entangled states, hence, they act as dimension witnesses for such states.

Symmetric Bell setups for PPT and infite dimensional states.—We have also generated all tight two-party Bell inequalities with one three-outcome and two binary measurements for each party. Our motivation has been the hope to find that some of them can be violated by bound entangled states that has not been known yet. We have found 76 independent inequalities. Three of them are trivial ones (requiring the non-negativity of the conditional probabilities), four of them are equivalent to the Clauser-Horne-Shimony-Holt (CHSH) inequality, while eight of them are equivalent to the I_{3322} inequality. We have adopted the method of Moroder *et al.* to determine an upper bound for the violation of the inequalities by PPT states. We have found twelve inequalities for which violation is not ruled out at the level we could afford. Despite our efforts, we could only find appropriate PPT states and measurement settings only for three of them, all of them are violated by the already known 3×3 dimensional state, no new state has been found. We plan to make further improvements on our methods, although it is possible that there are no appropriate inequalities in the set. We have studied other properties of the set of inequalities, like their maximum violation by any state. All but two of them can be violated maximally by finite dimensional systems (most of them by qubits or qutrits). We are still studying the two interesting ones that may require infinite dimensional systems for maximum violation. We have also determined maximum violation with qubits and qutrits for all inequalities. It would have been interesting to find cases requiring POVM (positive operator valued measure) measurements, but unfortunately, projective measurements are sufficient for all of them. Nevertheless, the results we are getting on the properties of the complete set will certainly be interesting enough such that they could eventually be published. We have generated several of the inequalities with four binary measurements for one party and two four-outcome measurements for the other one. We have found neither of them to be violated by unknown PPT state.

Useful PPT states for metrology.—We have investigated the question whether there exist bound entangled states which are useful for metrology, that is can the shot-noise limit be overcome with such states when the task is to estimate parameters in linear interferometry. Using separable states one can not do that. We have developed a powerful numerical method to search for such states and we have found both bipartite and multipartite examples. Some bipartite examples have been found to be very robust to noise, and also they outperformed separable states quite significantly. We have also proven that usefulness for metrology and nonlocality are two different notions.

We have found the analytical form of some of our numerically derived metrologically useful $d \times d$ dimensional bipartite bound entangled states. We have also investigated the performance of states constructed earlier by others for other purposes. Among them, we have found a family of states based on private states whose performance exactly matches the performance of the family of states we have found numerically, and then analytically. The two families are distinct. We have shown that for large dimensions the states not only surpass the classical limit significantly, but they approach the quantum limit. This is quite unusual, as in most quantum information processing applications bound entangled states may perform only very slightly better than separable states. The states are also found to be robust to noise.

We investigated further the role of entangled states in quantum metrology. We asked the following question: which multipartite quantum states are more useful than separable states for metrology? We showed that even if an entangled state is not more useful than separable states, it can be made useful by adding ancillas or by providing several copies of the state. Hence, we showed examples of activation of metrological usefulness, analogously to the activation of Bell nonlocality or bound entanglement. To this end, we proposed a powerful numerical method for looking for the optimal Hamiltonian for the best metrological performance of a given multipartite quantum state. In particular, we found a PPT bound entangled state that is not useful metrologically. However, by the addition of an ancilla to the state, it becomes useful.

PPT states with high Schmidt number.—As a bound entangled state is only weakly entangled, it was supposed that it can not be entangled across many degrees of freedom, that is its Schmidt number should be low (separable states have Schmidt number one). Recently, it has been shown that $d \times d$ dimensional bound entangled states with Schmidt number $d/4$ do exist for $d \geq 4$. We have significantly improved this result by constructing $d \times d$ dimensional bound entangled states of positive partial transpose with a Schmidt number of $d/2$ for $d \geq 4$. Our construction has been based on a recent work by other authors, who have proven that certain states of positive partial transpose are entangled if they have some properties. The proof relies on features of the projections onto the two-qudit symmetric and antisymmetric subspaces. We have found conditions to ensure that the state is not only entangled, but also its Schmidt number is higher than a certain value. This made it possible to construct explicitly states of Schmidt number $d/2$. We have also shown that a quantity defined in the article mentioned above, which can be calculated for a state using semidefinite programming, may be used to give a lower bound for the Schmidt number of the state.

Refuting hidden variable models with spin magnitude conservation.—It has been known for quite long that any hidden variable theory compatible with the predictions of quantum mechanics has to be nonlocal and contextual. In a collaboration with Polish colleagues we have shown that in a hidden variable description of spin describing the physical reality, the magnitude of the spin can not be conserved. This is an aspect of non-classicality which is related to the earlier theorems, but distinct from them. We have shown that for many spin values there does not even exist values of spin components allowed by quantum mechanics which are compatible with the required value of the length of the spin vector (no allowed s_x, s_y and s_z such that $s(s+1) = s_x^2 + s_y^2 + s_z^2$). This is true for one half of the half-integer spins and also for some integer spins. We have also shown that even if there exist combinations of component values of the required property, there are values which never occur in such a combination, therefore, their observation experimentally contradicts either the hidden variable model or the conservation of the spin magnitude. We have also constructed experimentally testable Bell-like inequalities whose violation proves the point, too.

Is measurement incompatibility equivalent to Bell nonlocality?—We answered this question in the negative. In the case of a pair of two-outcome measurements, incompatibility is equivalent to Bell nonlocality: any pair of incompatible two-outcome measurements can be used to violate the CHSH Bell inequality. In the case of more than two measurements the equivalence between incompatibility and Bell nonlocality has been an open problem. We showed that the equivalence breaks for a special choice of three measurements. In particular, we presented a set of three incompatible binary outcome measurements, such that if Alice measures this set, independent of the set of measurements chosen by Bob and the state shared by them, the resulting statistics cannot violate any Bell inequality.

Geometry and typicality of nonlocal correlations.—The geometrical shape of the set describing the set of correlations allowed by classical physics between results of measurements is a polytope. We have investigated the shape of the set allowed by quantum mechanics, and found that it has rather counter-intuitive features even in the simplest non-trivial Bell-scenario, and it gets even more complex for more inputs, more outcomes or more parties. We have also shown that the geometrical shape may impose limitations on the task of self-testing. For more than two parties we explored Bell inequalities which can be violated with the use of two efficient detectors, whereas the remaining ones may have arbitrary small efficiencies. In a three-outcome Bell setup, we proposed a Bell-type inequality whose violation has proved experimentally the existence of stronger than binary (i.e. two-outcome) correlations.

We addressed the question of how Bell nonlocality behaves in the case of randomly chosen projective measurements both for multipartite qubit systems and bipartite qudit ($d < 8$) systems. As a figure of merit, we have employed the probability of violation of local realism. In the case of multiqubit states, we find a rapid increase with the number of parties. On the other hand, in the case of bipartite higher dimensional qudit systems, we find that the nonlocality decreases as the dimension d grows. Furthermore we investigated generic quantum nonlocality not only in terms of the probability of violation, but we have also considered the strength of these violations with respect to resistance to white noise admixture. We introduced and studied numerically the concept of typicality of nonlocality for pure states which is defined as the probability to generate a nonlocal behaviour with randomly chosen measurements on random pure states. We find that a modest increase in the number of involved qubits (up to 6 qubits) results in the typicality of nonlocality surpassing 99.99%.

In a further research, we were interested in the nonclassical properties of N-party k-uniform mixed states. Such N-party states feature maximally mixed states for all their k-party reduced states. We found that the states obtained this way are usually highly mixed, still reveal strong nonclassical properties such as large violation of Bell inequalites. We also presented an efficient numerical method to find N-party k-uniform mixed states of high purity, which allowed us to find instances of k-uniform states beyond qubit systems (e.g. for 3 and 4-qutrit systems).