Final report for NKFIH FK 124814

The grant identifier is indicated on 35 papers, 2 book chapters and 3 extended abstract in conference proceedings.

Out of the 40 publications, 33 are accepted or published (mostly the latter); 11 of them are in Q1 ranked journals, and we expect 5 more out of the submitted ones to end up in the first quartile. One of these prestigious journals is SIAM Journal on Computing, an exceptional journal ranked in the top 10% both in mathematics and in computer science. Our paper there is listed by SIAM among the 20 most cited papers of SICOMP since 2018. Before presenting our results in detail, we mention that, as much as the circumstances permitted, we were also active in maintaining scientific relationships and giving talks. The PI alone has given at least 20 scientific talks during the time period of the grant and obtained his habilitation. Several research visits and invitations had to be cancelled due to the pandemic, along with many conference talks. Nevertheless, Csaba Szabó was invited to Prague and to Olomouc twice (and gave dozens of talks at different universities in Hungary and abroad). Gábor Horváth attended the Rhodesfest81 conference in Tel Aviv on behalf of the invitation of John Rhodes himself. András Pongrácz was invited to Oberwolfach to present his recent results on the existential version of the Thomas conjecture. The other two members were less active for different reasons. The fourth senior member Kamilla Kátai-Urbán had ongoing health problems, and our young researcher Attila Földvári obtained a post-doc position in Prague and got involved in a different research group from early on in the period of the grant (he hasn't used this budget at all). Despite these major setbacks, they both contributed some papers to the project.

1. Homogeneous structures, Thomas' conjecture

In [12], András Pongrácz and co-authors have characterized the reducts of the unique countable, existentially closed semilinear order S up to first-order interdefinability. Moreover, we determined the model-complete cores of all reducts. This is the starting point of the investigation of the complexity of CSPs of the reducts, known as phylogeny constraint satisfaction, a generalization of the scheduling problem. These results are improved in the recently submitted manuscript [23]. András Pongrácz has classified the reducts of S up to the finer equivalence relation existential interdefinability. It is demonstrated that several parts of the technique work in full generality. An inductive argument is shown which makes it possible to prove general results on reducts of Ramsey structures, and then two such results are verified. One of them reduces the existential version of the Thomas conjecture to a concrete technical problem and the original first-order version. Hence, the paper shows how the classification of reducts up to first-order interdefinability can be improved to existential interdefinability, and moreover, it indicates that the strictly more general latter problem could be easier to handle. One reason is the inductive nature of the proofs: in such arguments, it is not uncommon that a stronger statement is easier to prove. Furthermore, the core idea of the classification technique is to climb up the lattice of algebraic invariants corresponding to the reducts in the appropriate Galois correspondence: that is the automorphism group in case of first-order interdefinable reducts and the self-embedding monoid for existential

interdefinability. While it is unclear to date if there are finitely many minimal automorphism groups above the automorphism group of each reduct, the analogous statement for self-embedding monoids is well-known. The next natural weakening of the logic would be classification up to existential positive interdefinability, however, there are in general a continuum of those equivalence classes. Thus the analysis of existential reducts can lead to the solution of the Thomas conjecture; [23] is the first step in this project. These results were presented in an invited talk in Oberwolfach.

Csaba Szabó published a joint paper with two co-authors [6]. We provided a counterexample to Thomas's conjecture for finite language structures. It is shown that the countably infinite dimensional pointed vector space (the vector space equipped with a constant) over a finite field has infinitely many first-order definable reducts. The construction also provides an example to an omega-categorical structure with finitely many reducts such that by adding a constant, the structure obtained has infinitely many reducts. Earlier, such examples were not known, and in fact, some results indicated that this is impossible. As a consequence, we showed that the countable homogeneous Boolean algebra has infinitely many reducts. The construction over the 2-element field is related to the Reed-Muller codes. As a preliminary result, the reducts of the countably infinite dimensional vector spaces over prime fields were classified in the yet unpublished manuscript [13]. Csaba Szabó and Kamilla Kátai-Urbán have participated on the AAA 97th conference in Vienna. After his talk, Csaba Szabó was invited for a longer visit to repeat his talk back to Vienna and to the Department of Algebra at Charles University, Prague. In Vienna, he continued the joint project on the reducts of homogeneous vector spaces, and a new member joined the team. A second draft of the results was delivered. The main result is that there are finitely many reducts and they can be classified by the action of the automorphism group on the 1-dimensional subspaces. It turns out that this action is either the full symmetric group or the projective linear group.

2. CSPs

In [39] András Pongrácz with several co-authors studied homomorphisms from the polymorphism clone of ω-categorical structures to the trivial clone. The main question is whether such a homomorphism is automatically continuous with respect to the topology of point-wise convergence. In fact, for the applications, only the following less demanding property matters: whenever there is a (discontinuous) homomorphism, there also exists a continuous homomorphism. We have provided a condition that reduces this problem to the binary fragment of the polymorphism clone. Some examples were constructed to polymorphism clones of ω-categorical structures that can be mapped to the trivial clone via a discontinuous homomorphism. Finally, we have shown some general positive results.

We have determined the complexity of the reducts of all Henson graphs and some homogeneous equivalence relations in [18]. Together with earlier results, this paper finishes the proof of the dichotomy conjecture for all reducts of countable homogeneous graphs: all such computational problems are either in P or NP-complete. A finite list L of multivariate functions was provided which can be used to efficiently determine which case occurs for any given reduct. If a function in L is a polymorphism of the reduct, then the corresponding CSP is in P, otherwise, it is NP-complete.

3. Fraïssé classes

A Fraïssé class captures all model theoretical properties of its limit. That is why it is often more convenient to work with this class of finite structures rather than the infinite limiting structure in certain types of problems. One such area is the asymptotic probability of properties. According to a seminal result of Fagin, a closed graph formula holds in the (infinite) random graph if and only if it holds with asymptotic probability 1 in the $G(n, 1/2)$ model. The result can be generalized to further relational languages and Fraïssé classes, and the logic can be weakened from the first-order realm, yielding more interesting problems. Such a problem is the rigidity of structures, or generally speaking, the asymptotic probability of the automorphism group of a graph (or even some other relational structure) being isomorphic to a given group H provided that it contains a given group G. Given the group G, this is nonzero for only finitely many H, and it is always a rational number. In order to determine those finitely many groups H and the positive asymptotic probability they occur with, we need to understand those permutation groups that have a small maximum degree of elements compared to the size of their support. The extremal cases of this combinatorial problem were determined in [32] by András Pongrácz.

Based on the results of [32], the near-extremal cases were classified in [20]. More precisely, András Pongrácz has given a clear description of those binary linear codes whose maximum distance is $n/2 + 1$, where n is the length of the code. The constructions that appear in the classification are of interest to coding theorists for multiple reasons: these are 2-weight and 3-weight binary linear codes, and besides having a small maximum distance, they also have a large minimum distance. Some of them nearly or precisely meet the Plotkin- or the Griesmer bound.

4. Enumerative combinatorial problems

In [8], three members of the grant have provided asymptotic and log-asymptotic estimates to the n-element and n-generated models in all varieties of monounary algebras. We obtained infinitely many new examples to varieties with superpolynomial and subexponential spectra: that is, varieties whose spectra grow faster than any polynomial and slower than any exponential function. Only some sporadic examples were known before in the literature.

Voting protocols, such as the push- and pull protocols are mathematical models of the behavior of people prior to an election. However, they are also studied and used in a number of other theoretical and applied areas, such as socio-economic models, peer-to-peer computer networks, and to simulate the spread of gossip or an epidemic. In [21], András Pongrácz developed an elementary linear algebraic technique and used it to provide estimates to the expected runtime and to the probability of each consensus to win for the discordant push- and pull protocols in cycle graphs. These bounds are sharp, provided that the initial state be not too chaotic, and an asymptotically sharp uniform upper bound is given to the expected runtime. The conference version [4] appeared in ICCSE'18.

The same method is applied in [22] to obtain similar estimates to the higher moments of the runtime (in cycle graphs). Furthermore, a general bound is presented to the expected runtime of the so-called "gambler's ruin problem" for arbitrary graphs. This is a multiplayer game, where agents are vertices of a given graph, each having

a positive integer initial wealth. In each round, an edge is selected uniformly at random, and a fair coin-toss decides which endpoint transfers 1 unit wealth of its money to the other. The process halts when a vertex declares bankruptcy. In the special case when the graph has two vertices linked by an edge, we obtain the classical drunkard's walk, whose expected runtime is the product of the two quantities of wealth. Using the linear algebraic method, András Pongrácz has shown that if the game is played on an arbitrary finite graph, then the expected runtime is at most the harmonic mean of the expected runtimes of the drunkard's walks played on the edges multiplied by the number of edges.

In the manuscript [9], three members of this grant have extended some of the above results to star graphs. This turned out to be a nonstandard case: in a typical graph, the discordant push protocol is faster than the discordant pull protocol, however, we obtained that it is the other way around for star graphs. In fact, the discordant pull protocol beats the discordant push protocol by a factor of the order of magnitude $\log n$. We have determined the exact asymptotics of the largest possible expected runtimes.

5. Clones, universal algebra

Three members of the grant investigated the composition-closed function classes (clones) containing binary monomial polynomial functions over the q-element field [17]. We gave the description of all clones generated by a single idempotent binary monomial over the $\mathbb{F}_q$.

In [28], Gábor Horváth and a co-author characterized those algebras that have infinitely many polynomially inequivalent expansions among all finite nilpotent algebras of finite type in congruence modular varieties that are direct products of algebras of prime power order.

6. Combinatorics in finite semigroups

Kamilla Kátai-Urbán and a co-author studied the multiplication operation of square matrices over lattices [30]. If the underlying lattice is distributive, then matrices form a semigroup; we investigated idempotent and nilpotent elements and the maximal subgroups of this matrix semigroup. We proved that matrix multiplication over non-distributive lattices is anti-associative, and we determined the invertible matrices in the case when the least or the greatest element of the lattice is irreducible.

Gábor Horváth with two co-authors analysed the maximal subgroups and the complexity of the flow semigroup of finite (di)graphs [3]. The flow semigroup, introduced by the famous group theorist and expert in automata and semigroup theory, John Rhodes, is an invariant for digraphs and a complete invariant for graphs. After collecting together previous partial results, we refine and prove Rhodes's conjecture on the structure of the maximal groups in the flow semigroup for finite, antisymmetric, strongly connected digraphs. Building on this result, we investigate and fully describe the structure and actions of the maximal subgroups of the flow semigroup acting on all but k points for all finite digraphs and graphs for all $k \geq 1$. A linear algorithm (in the number of edges) is presented to determine these so-called 'defect k groups' for any finite (di)graph. Finally, we prove that the complexity of the flow semigroup of a 2-vertex connected (and strongly connected (di)graph with n

vertices is $n-2$, completely confirming Rhodes's conjecture for such (di)graphs. We mention that Gábor Horváth was invited to the Rhodesfest81 conference by John Rhodes himself, along with only a handful of other mathematicians.

7. Functions, polynomials and complexity

In [5], Attila Földvári proved that one can decide in $O(n^{\frac{1}{2}|G|^2 \log |G|})$ time whether an equation of length n has a solution over the finite nilpotent group G. The proof relies on representing group expressions using the polycyclic presentation of p-groups. This result vastly improves on the astronomically large exponent in polynomial bounds known beforehand.

Attila Földvári and Gábor Horváth further improved the bounds on the complexity of the equation solvability and equivalence problems over finite groups in [24]. We provided a polynomial time algorithm for deciding the equation solvability problem over finite groups that are semidirect products of a p-group and an Abelian group. As a consequence, we obtained a polynomial time algorithm for deciding the equivalence problem over semidirect products of a finite nilpotent group and a finite Abelian group. The key ingredient of the proof is to represent group expressions using a special polycyclic presentation of these finite solvable groups.

An exceptionally strong manuscript [38] is under construction on the equation solvability problem over finite rings, written by Gábor Horváth and two great experts of the field. It has been rewritten and improved several times during the past years, and it is in the final stage before submission.

With a co-author, Csaba Szabó wrote a paper on evaluation of polynomials over finite rings [29]. We gave an improved polynomial bound on the complexity of the equation solvability problem, or more generally, of finding the value sets of polynomials over finite nilpotent rings. Our proof depends on a result in additive combinatorics, which may be of independent interest.

Gábor Horváth with his students investigated polynomial functions over commutative finite rings and have two published papers. In [2] we proved a necessary and sufficient condition for a function being a polynomial function over a finite, commutative, unital ring. Furthermore, we provided an algorithm running in quasilinear time that determines whether or not a function given by its function table can be represented by a polynomial, and if the answer is yes then it provides one such polynomial. Horváth's students won the first prize in the final of the Hungarian National Scientific Competition (OTDK) in 2017 for the other paper [7]. Let $PPol(R)$ denote the group of permutation polynomial functions over the finite, commutative, unital ring R under composition. We generalized numerous results about permutation polynomials over Z_{p^n} to local rings by treating them in a unified framework. In particular, we provided a natural wreath product decomposition of permutation polynomial functions over the maximal ideal M and over the finite field R/M. We characterized the group of permutation polynomial functions over M whenever the condition $M^{|R/M|} = \{0\}$ applies. Then we derived the size of $PPol(R)$, thereby generalizing the same size formulas for Z_{p^n}. Finally, we completely characterized when the group $PPol(R)$ is solvable, nilpotent, or abelian.

8. Lie theory

With several co-authors, Gábor Horváth studied Lie symmetry groups of the general Liénard-type equation [19]. We considered the general Liénard-type equation $\ddot{u} = \sum_{k=0}^{n} f_k \dot{u}^k$ for $n \geq 4$. This equation naturally admits the Lie symmetry $\frac{\partial}{\partial t}$. We completely characterized when this equation admits another Lie symmetry, and provided an easily verifiable condition for this on the functions $f_0, \ldots, f_n$. Moreover, we gave an equivalent characterization of this condition. Similar results have already been obtained previously in the cases $n = 1$ or $n = 2$. That is, this paper handles all remaining cases except for $n = 3$.

In [37], Gábor Horváth and a co-author studied the symmetry analysis of differential equations. It can provide a suitable change of variables, i.e., in geometric terms, a suitable diffeomorphism that simplifies the given direction field, which can help significantly in solving or studying differential equations. Roughly speaking, this is the so-called rectification theorem. The local version of this result is a well-known theorem in the field of ordinary differential equations. In this note, we proved a global counterpart when the equation fulfills the Lipschitz condition. Then we used this result to determine the global symmetry group of such an ordinary differential equation. It turns out that, assuming the Lipschitz condition, the full symmetry group is a smooth wreath product of two diffeomorphism groups, and does not depend on the form of the equation, at all.

9. Further topics

The topic of paper [33] of András Pongrácz together with a co-author originated from a real-life problem. In order to simulate homogeneous fog on a certain type of photos, it suffices to know the distance of objects on the picture from the camera, a basic problem with many different numerical methods in the literature to solve it. However, if the goal is to simulate inhomogeneous fog - a task in a joint project of mathematicians and computer scientists of the University of Debrecen together with other Hungarian universities - then we need more data, namely the original 3-dimensional coordinates of the objects on the photo. Hence, we need an approximate solution to a variant of a fundamental problem in photogrammetry: photos are produced by a central projection, and we need to determine the coordinates of the center, given the distances and some incidence relations of points on recognizable objects on the picture. Each triplet of points whose real-life pre-images are collinear defines a generalized conic of degree four containing the center. We provided an algebraic condition that holds if and only if the generalized conic is unbounded, and we have proven that the polynomial defining the profile curve is always irreducible. In particular, two such polynomials have at most 16 common roots, effectively reducing the possible locus of the center to a finite set. The journal is ranked in six categories; the relevant one in terms of our paper is computer vision, in which the journal is rated Q1.

Recently, Csaba Szabó gradually turned his attention towards didactics, and published a series of high-quality papers in several subjects in this area.

In [1], we discuss how modelling and dressed-up problems play an inevitably unavoidable role in mathematics education. In this study we point out how dangerous it is to dress up mathematical problems. We go back to the principle of De Lange: The problem designer is not only dressing up the problem, but he is the solution

designer, as well. We show three examples selected from Hungarian high school textbooks where the intended solution does not solve the problem, because the dressing changes the context and changes the problem itself.

Csaba Szabó has participated on the PME43 conference in Praetoria, where two of his short papers with co-authors have appeared [14,15]. The van Hiele theory based on Vigotsky's work is known and recognized all around the world. The theory distinguishes five levels of geometric understanding. In this paper we show that the theory holds for University level as well, namely those students who do not have enough background for a geometry course and finish it, will lose their knowledge in a short period of time.

The second short paper is on how retrieving information from memory after an initial learning phase enhances long-term retention more than restudying the material. Test-enhanced learning is a method which uses active recall of the information during the learning process. It has been proved to be efficient concerning texts or foreign words, these experiments were principally carried out in laboratory environment. We carried out two experiments on the efficacy of test-enhanced learning in mathematics. There was no significant difference between the midterm scores of the control and experimental groups during these semesters. However, the two after-tests showed a big difference for the experimental group that was increasing with the time. Our results prove the long-term efficacy of testing effect in university-level mathematics.

Csaba Szabó and a co-author were invited to prepare a short and a long paper in the highly prestigious journal of the European Mathematical Society by the editorial board. The short paper [10] sketches the events of Hungarian talent care in mathematics from 1781, the foundation of the Hungarian Scientific Society until nowadays. It includes the important changes in education and the high school mathematics competitions, as well.

The long version of this paper [11] thoroughly discusses the key elements in Hungarian mathematics, concentrating on math education and the care about gifted pupils. It gives detailed account of math competitions, important mathematics events from 1894, the foundation of KÖMAL to the latest open air mathematical activities, the Bear events. An extended version was published in The De Morgan Gazette.

With two co-authors, Csaba Szabó published a paper on the transition problem in Hungary [16]. The curricular background of the transition problem from high school to university is analysed in Hungary. While students finish their mathematical studies successfully at high school, pass their final exams, this knowledge seems to disappear at their first year at university. We investigate the mathematical knowledge expected by the Hungarian universities and compare it to expectations of the National Core Curriculum. Based on the levelling tests of four universities we created a seven problem test for high school students containing very basic problems required both by the universities and the National Core Curriculum. We analysed the results of the test.

Csaba Szabó and his students submitted three papers [25, 26, 27] on the education of mathematics, two of them to Q1 ranked journals. This is exceptional in Hungary; very few Q1 quality papers are published by Hungarian authors. In two of these papers, we investigated the efficacy of the so-called testing effect. Retrieving

information from memory can - under many circumstances - strengthen one's memory of the retrieved information. The strategic use of retrieval to enhance memory is known as retrieval practice. After an initial learning phase, retrieval practice (testing) can be more effective for long-term retention than restudying. However, it is unclear whether this effect also holds true in the case of learning mathematics. The first research [25] is an exploratory case-study, where we test a method - using test-enhanced learning as a special kind of formative assessment - in a high-needs vocational school. The purpose of the case-study was to get a better view of how the above-mentioned method works in real-life circumstances in mathematics and to lay the groundwork for further experiments. At the end of each lesson, students from the experimental group wrote a test on the material learnt on the given day. In these tests, there were two problems: one theoretical and one practical problem to be solved. In the control groups, the solutions of the exact same problems were told by the teacher at the end of the lessons. All the pupils were given the exact same material and wrote the exact same final test. On this final test, members of the experimental group outscored their schoolmates and reached statistically the same scores as the control group from the elite grammar school. Thus we were able to reduce the performance gap between vocational schools and grammar schools in a given Geometry topic.

The second study [27] involved an experiment on the efficacy of test-enhanced learning used for teaching mathematics at a real university with real students in real math courses. The participants were six groups of undergraduate pre-service mathematics teachers. Three groups learned Number Theory using the testing effect, and the other three learned using traditional methods. The experimental and control groups learned the exact same information in the lecture and wrote the same final test. The experimental group performed significantly better than the control group, although their performance on the initial competence exams was considerably worse. The results indicate that test-enhanced learning has a significant advantage in solving complex mathematical problems. To examine the effect of differences in individual competence, we divided the students in both experimental and control groups into low-, middle-, and high-performing groups. The efficacy of test-enhanced learning was demonstrated in all three performance levels. Regarding the three pairs of groups, members of the experimental group using test-enhanced learning performed better than those of the control group.

In the third paper [26], the mathematical skills of high school students are investigated. Earlier findings suggest that there is a gap between the knowledge of students entering the university and the expectations and prerequisites of the universities' curriculum. These prerequisites are based on the National Core Curriculum for high schools. The admission process to the university is based on a final exam that every student has to take at the end of secondary school. This final exam has a high impact on secondary education and on the transition from secondary to tertiary education in Hungary. High schools are ranked based on the average scores of their students on final exams, and also students are ranked and can get admission to universities based on their final exam results. In our research, we investigate this gap in Hungarian mathematics education. In particular, we concentrate on the geometrical knowledge of Hungarian high school students. For measuring their levels of geometrical understanding we used the Usiskin test for the framework of the Van Hieles. The test was filled in by 342 students from five different high schools. The

results show that there is no improvement during the high school years, the average score of the Usiskin test is between 2.03 and 2.17 on all grades.

Csaba Szabó has published the paper [36] with his students concerning the following methodological question: Is it allowed to publish mathematical problems for pupils such that the sample solutions of the problems are wrong or incomplete? The question is a real question, as in the school curriculum it regularly happens, that precision is neglected for the sake of understandability. In the paper Sándor Róka, a well-known math talent teacher is interviewed on this issue. The paper is open access.

Csaba Szabó and his students were invited to write two chapters in the third volume of a series of four books [34, 35]. In the paper [34], the level of geometry education in mathematics education in Hungary is investigated. The relationship between the National Core Curriculum, the Framework Curriculum and the final exam is analyzed from the geometry point of view via the Van Hiele levels as a tool for comparison. It is observed that the geometry problems on the final exams do not follow the level prescribed by the National Core Curriculum. We compare these observations with the results of the Usiskin-test of first year preservice math teacher students.

In nowadays schools most pupils use pocket calculators for numerical calculations, and most teachers allow them to use those. At the same time irrational numbers are present in our everyday life but their exact values cannot be given in a form that students easily understand. In the paper [35] we showed geometrical constructions and calculations in which non-rational numbers naturally arise and gain meaning. They look at numbers which are expressible with at maximum two roots and are present in the Hungarian curriculum. For each number they present how they appear in Hungarian textbooks, and show multiple problems and solutions in which they arise. These solutions differ in their level of mathematical complexity, from elementary geometry to higher algebra. Introducing these solutions to students, shows them that the different areas of mathematics are interrelated. This approach may inspire students to use their mathematical knowledge not only from the area in which the problem was presented.

The impact playing has on the development of thinking is an important topic of psychology of learning, brain research and mathematics didactics. The research is also connected to the aforementioned topic. In [31] we investigated the effects of playing board games on competence motivation and the development of mathematical competencies. In the paper, they present the results of an experiment carried out in a secondary school class. The experimental group spent one of three weekly mathematics lessons playing board games. Apart from the several advantages of playing games in general, we can conclude that, based on the results of the national competence measurement, the mathematical competence of the students developed properly. The readiness and the progress of the pupils were compared on the basis of input and output tests and an initial knowledge measurement and, at the same time, we compared their level of mathematical competence with the results of the national competence measurement.

Csaba Szabó's experiments on the methodology attained high interest in Central Europe. He gave two independent talks at Palacky University, at two different institutes in two different times. These were highly prestigious invitations.

Csaba Szabó also published a paper [40] in a D1 ranked journal with several co-authors. The topic of the paper is molecular spectroscopy. Despite decades of diligent work on their development, line-by-line (LBL) spectroscopic information systems, widely employed by scientists and engineers, may still contain a number of incorrect rovibronic lines. A novel heuristic protocol, relying on cycle bases of spectroscopic networks (SN) and a system of linear constraints, is proposed to unravel incorrect transitions present in the database. Csaba Szabó and his co-authors worked out two algorithms to sieve out the most possible mistakes from the database.

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