

# Final report of the NKFI project FK 124723: From topologically protected states to topological quantum computation

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Experimental realization of topological quantum computation, whether realized in hardware (e.g., Majorana fermions), or in software (e.g., surface code), faces considerable challenges. In our theoretical project we planned to analyze these challenges and suggest strategies to meet them, and explore new approaches to engineering topological phases of matter.

We explored **1) Novel topological states of matter by periodic driving** - topological phases that can be created or stabilized by periodic driving, in the extreme limit of low frequency and high intensity driving. We studied **2) Topological Quantum Computing via hardware: Majorana fermions** and related topological platforms, exploring mechanisms for qubit initialisation, manipulation, control, and decoherence, and giving predictions for Majorana milestone experiments. We investigated the properties of **3) New materials for Topological Quantum Computing**, through abinitio calculations and tight binding models.

## 1) Novel topological states of matter by periodic driving

We explored - using theoretical models - topological phases that can be created or stabilized by periodic driving, in the extreme limit of low frequency and high intensity driving. We used two kinds of lattice models: quantum walks, sequences of alternating local “coin” and spin-dependent “shift” operations, and quantum kicked models. We explored the role of disorder, and found ways in which it can lead to topological semimetal (critical) phases in periodically driven artificial matter systems. We also participated in an experiment probing periodically driven states in double quantum dots.

*New signatures of topological invariants in periodically driven systems.*

We proposed [Sajid19] a scheme to realize topological insulators with nonvanishing Chern numbers using spin-1/2 particles in a quantum walk. By Floquet engineering the quantum-walk protocol, we realized the analog of a strong magnetic field, with flux per plaquette a sizable fraction of the flux quantum. We found nearly flat bands with Chern numbers, which could be used to explore interaction-driven topological phases such as fractional Floquet Chern insulators. We discussed an implementation using neutral atoms in spin-dependent optical lattices for generation of arbitrary magnetic-field landscapes, including those with sharp boundaries.

We collaborated [Koski18] in an experiment, investigating a strongly driven GaAs double quantum dot charge qubit weakly coupled to a superconducting microwave resonator. The Floquet states emerging from strong driving are probed by tracing the qubit - resonator resonance condition. This way we probed the resonance of a qubit driven in an adiabatic, a non-adiabatic, or an intermediate rate showing distinct quantum features of multi-photon processes and Landau-Zener-Stückelberg interference pattern. Our resonant detection scheme enabled the investigation of novel features when the drive frequency is comparable to the resonator frequency. We used models based on adiabatic approximation, rotating wave approximation, and Floquet theory to explain the experimental observations.

#### *New topological semimetal phases in periodically driven systems - role of disorder*

For two-dimensional two-state quantum walks we have found [Asboth20] that complete spatial disorder does not lead to Anderson localization, but to a diffusive spread instead. This delocalization happens because disorder places the quantum walk to a critical point between different anomalous Floquet-Anderson insulating topological phases. We followed time evolution of the wavefunctions and level spacing statistics, and applied scattering theory to calculate the topological invariants of disordered quantum walks. We found the critical exponent  $\eta \approx 0.52$  as in the integer quantum Hall effect.

We have found anomalous levitation and pair annihilation [Liu20], a process unique to Floquet systems affected by disorder. Here the topological gap increases as a function of disorder strength, resulting in a transition to an anomalous Floquet Anderson insulator (AFAI) phase. We have shown a concrete example, adding disorder via onsite potential "kicks" to a Chern insulator model. By changing the period between kicks, we have tuned which type of levitation and annihilation occurs.

#### *Topological phases in interacting periodically driven systems.*

Even without periodic driving, we explored [Perrin21] a novel type of disordered interaction on the simplest toy model of solid state physics: a tight-binding chain. We took two particles hopping on a chain with a contact interaction between them, whose strength depends on the positions of the particles in a spatially disordered way. We observed how disorder leads to Anderson localization of two-particle bound states, but increasing disorder also leads to dissociation of such bound states due to hybridization with the atomic band. We drew analogies to quantum billiards and quantum chaos, and identified a novel class of eigenstates, "separatrix states", similar to scarred states.

## 2) Topological Quantum Computing via hardware: Majorana fermions

We suggested schemes to boost the effectiveness of quantum computing that are suitable for topological quantum computing via Majorana zero modes. We used models of differing

complexity, at times in close collaboration with experiments to include the most relevant features. We have suggested schemes for high-precision electrical control of Majorana qubits. In a more theoretical line of work, we have explored topologically protected pointlike topological degeneracies in parameter spaces of magnetic and other quantum systems.

#### *Few-level models.*

We described errors in a setup [Maman20] where Majorana qubit readout is performed using gate reflectometry of an auxiliary quantum dot tunnel-coupled to two Majorana zero modes. We considered readout error caused by low-frequency charge noise and overdrive effects, and quantified these errors as the function of model parameters. This work provides guidelines for design and interpretation of future experiments on control and readout of topological Majorana qubits.

We proposed and theoretically investigated a resonant qubit control process [Gyorgy22], which is relevant for any qubit realization where the qubit has a finite energy splitting. In the context of Majorana qubits, it is known that universal quantum computing requires quantum gates that are not topologically protected. The control scheme we proposed in this paper serves as a basis for such Majorana qubit operations.

#### *Free-fermion models.*

We proposed and studied a topologically protected quantum gate [Boross19] based on the Su-Schrieffer-Heeger chain. Our proposed Y gate acts in the two-dimensional zero-energy subspace of a Y junction assembled from three chains, and is based on the spatial exchange of the defects supporting the zero-energy modes. We numerically demonstrated robustness against hopping disorder -- a form of topological protection, a consequence of chiral symmetry, time-reversal symmetry, and spatial separation of the modes bound to defects. The proposal has since been experimentally realized (independently of this grant) in Chen et al., Nature Physics 18, 179 (2022).

We theoretically studied a scheme [Szechenyi20] to distinguish the two ground states of a one-dimensional topological superconductor, serving as a basis for readout of Majorana qubits. The ground-state parity is converted to charge of an auxiliary quantum dot. Errors degrade the quality of this process: (i) leakage due to a strong readout tunnel pulse, (ii) incomplete charge Rabi oscillations due to slow charge noise, and (iii) charge relaxation due to phonon emission and absorption. Effects of error mechanisms can be minimized by choosing an optimal strength for the readout tunnel pulse. In our case study based on InAs heterostructure device parameters, we estimated that parity-to-charge conversion error is mainly due to slow charge noise for weak tunnel pulses and leakage for strong tunnel pulses.

We theoretically studied Majorana-qubit dephasing in a minimal model [Boross22]: a Kitaev chain with quasistatic disorder. We predicted Gaussian dephasing, with a rate that depends on the system parameters in an oscillatory way, out-of-phase with the oscillating minigap of the

clean system. In our model, first-order dephasing sweet spots are absent, a feature that is useful for characterizing the spatial structure of noise in a dephasing experiment.

#### *Effects of Coulomb repulsion.*

We theoretically analyzed a mesoscopic Josephson junction [Bouman20], including a serially coupled double quantum dot, in collaboration with the experimental group of Prof. Attila Geresdi. This setup contains the key building blocks of a quantum-dot-based Kitaev chain: superconducting terminals, tunable quantum dots, and tunnel coupling between those. Our model revealed the importance of Coulomb interaction in the sample. We demonstrated a magnetic-field-induced supercurrent blockade and a spin-to-current conversion mechanism. These effects can be exploited in hybrid quantum architectures coupling the quantum states of spin systems and superconducting circuits.

#### *Topologically protected degeneracies.*

Topological protection of the Majorana qubit is ensured by the robust ground-state degeneracy of one-dimensional p-wave superconductors. This observation inspired further work in this project, exploring the properties of topologically protected degeneracies in parameter-dependent quantum systems. In [Scherubl19], [Frank20], [Frank22], we analysed magnetic Weyl points and more exotic degeneracy patterns in interacting spin systems. We have predicted and analysed the Weyl-point teleportation effect [Frank22b], and the 'birth quota' effect of non-generic degeneracy points [Pinter22].

### 3) New materials for Topological Quantum Computing

We investigated novel topological materials, and magnetic systems that can host non trivial topological excitations due to spin-orbit coupling - an important component of proposals for Majorana devices. We focused mostly on low dimensional nanostructures, in close collaboration with experimental groups. Due to recent developments in the field we altered the aim of the present workpackage to include the nodal loop semimetals. Nodal loop semimetals are an exotic type of topological semimetals, where the degeneracy between valence and conduction bands is along a 1-dimensional manifold, a loop - stabilized by a symmetry. These topological semimetals open the door to creating new types of devices based on subtle details of the Fermi surface.

#### *Model parameters for nanodevices.*

In collaboration with experimental groups at the Budapest University of Technology and Economics and at the ELKH Center for Energy Researches we investigated novel materials, including bismuth-tellurohalide [Fulop18] and  $\text{ZrTe}_5$  [Tajkov22]. These materials are characterized by strong spin-orbit coupling which is drastically influenced by strain fields present in fabricated devices. An important consequence of the sensitivity to mechanical distortions is

the unique capacity of these materials to be mechanically tuned through topological phase transitions. Our ab initio calculations underpinned the first isolation of single layer bismuth–tellurohalide structures [Fulop18], while our subsequent [Tajkov19] tight-binding parametrization of the system allowed us to predict phase transitions in hybrid systems.

We also investigated thin layers of the topological semimetal  $\text{ZrTe}_5$  [Tajkov22][Kovacs22] where our ab initio calculations resolved a long standing issue regarding the groundstate of this material. We showed that strain fields present in these devices due to different fabrication conditions may influence its ground state and yield strong or weak topological insulator or a Weyl semimetal.

In loose connection to these investigations we also developed a numerical method [Oroszlany19] to extract magnetic model parameters from ab initio calculations and applied it [Carracedo22] to novel realization of the topological  $S=1$  chain in triangulene based molecular devices.

### *Proximity effect in nanodevices*

We explore how spin-orbit coupling can be considerably boosted and engineered in graphene based hybrid systems by proximatizing graphene based devices with materials characterized by strong spin-orbit coupling. We have shown through ab initio calculations, [Tajkov19a] that in hybrid bismuth–tellurohalide–graphene heterostructures spin-orbit coupling is considerably increased for the electrons of graphene. We also explored how mechanical distortions can be used to engineer a topological phase transition, turning the sample in to a novel realization of a time reversal invariant topological insulator. We distilled [Tajkov20] a simple tight-binding picture and highlighted that mechanical distortions disproportionately affect different gap generating mechanisms in the system. We show that while the trivial gap, due to Kekule type distortion is sensitive to mechanical distortions, the topological gap caused by Kane-Mele type induced spin-orbit coupling, stays resilient.

### *Nodal loop semimetals in electric and magnetic fields*

We determined [Oroszlany18], using tight-binding models, how the conductivity of nodal loop semimetals depends on electric or magnetic fields, depending on the direction of the fields with respect to the plane of the loop.

We clarified, the source of conflict between experiments using magnetic oscillation measurements on nodal loop materials. We found that whether unambiguously trivial oscillations, or a mixture of topological (with a phase shift of  $\pi$ ) and trivial oscillations should be expected depends on the orientation of the magnetic field with respect to the plane of the nodal loop: trivial for perpendicular field, a mixture for inplane field.

Investigating the non-linear electronic response we found [Okvatovity22] that the current grows monotonically with time for electric fields perpendicular to the nodal loop plane, however, it exhibits non-monotonical behavior for field orientations aligned within the plane, arising from interband processes. The long-time non-linear response represents an intraband effect where a large number of excited quasiparticles respond to the electric field.

## Publications

[Fulop18]

B Fülöp, Z Tajkov, J Pető, P Kun, J Koltai, L Oroszlány, E Tóvári, H Murakawa, Y Tokura, S Bordács, L Tapasztó and Sz Csonka:  
Exfoliation of single layer BiTeI flakes,  
2D Materials 5, 3 (2018)

[Koski18]

JV Koski, AJ Landig, A Pályi, P Scarlino, C Reichl, W Wegscheider, G Burkard, A Wallraff, K Ensslin, T Ihn:  
Floquet spectroscopy of a strongly driven quantum dot charge qubit with a microwave resonator,  
Phys. Rev. Lett. 121, 043603 (2018)

[Oroszlany18]

L Oroszlány, B Dóra, J Cserti, and A Cortijo:  
Topological and trivial magnetic oscillations in nodal loop semimetals,  
Phys. Rev. B 97, 205107 (2018)

[Sajid19]

M Sajid, JK Asbóth, D Meschede, RF Werner, and A Alberti:  
Creating anomalous Floquet Chern insulators with magnetic quantum walks  
Phys. Rev. B 99, 214303 (2019)

[Oroszlany19]

L Oroszlány, J Ferrer, A Deák, L Udvardi, and L Szunyogh:  
Exchange interactions from a nonorthogonal basis set: From bulk ferromagnets to the magnetism in low-dimensional graphene systems,  
Phys. Rev. B 99, 224412 (2019)

[Boross19]

P Boross, JK Asbóth, G Széchenyi, L Oroszlány, and A Pályi:  
Poor man's topological quantum gate based on the Su-Schrieffer-Heeger model,  
Phys. Rev. B 100, 045414 (2019)

[Scherubl19]

Z Scherübl, A Pályi, Gy Frank, I Lukács, G Fülöp, B Fülöp, J Nygård, K Watanabe, T Taniguchi, G Zaránd, Sz Csonka:  
Observation of spin-orbit coupling induced Weyl points in a two-electron double quantum dot,  
Communications Physics 2, 108 (2019)

[Tajkov19]

Z Tajkov, D Visontai, L Oroszlány, J Koltai:  
Uniaxial strain induced topological phase transition in bismuth–tellurohalide–graphene  
heterostructures,  
Nanoscale 11, 12704 (2019)

[Tajkov19b]  
Z Tajkov, D Visontai, L Oroszlány, J Koltai:  
Topological Phase Diagram of BiTeX–Graphene Hybrid Structures,  
Appl. Sci. 9 (20), 4330 (2019)

[Bouman20]  
D Bouman, RJJ van Gulik, G Steffensen, D Pataki, P Boross, P Krogstrup, J Nygård, J Paaske,  
A Pályi, and A Geresdi:  
Triplet-blockaded Josephson supercurrent in double quantum dots,  
Phys. Rev. B 102, 220505(R) (2020)

[Szechenyi20]  
G Széchenyi and A Pályi:  
Parity-to-charge conversion for readout of topological Majorana qubits,  
Phys Rev B 101, 235441 (2020)

[Frank20]  
Gy Frank, Z Scherübl, Sz Csonka, G Zaránd, and A Pályi:  
Magnetic degeneracy points in interacting two-spin systems: Geometrical patterns, topological  
charge distributions, and their stability,  
Phys. Rev. B 101, 245409 (2020)

[Liu20]  
H Liu, IC Fulga, and JK Asbóth:  
Anomalous levitation and annihilation in Floquet topological insulators,  
Phys. Rev. Research 2, 022048(R) (2020)

[Asboth20]  
JK Asbóth, A Mallick:  
Topological delocalization in the completely disordered two-dimensional quantum walk,  
Phys. Rev. B 102, 224202 (2020)

[Maman20]  
VD Maman, MF Gonzalez-Zalba, and A Pályi:  
Charge Noise and Overdrive Errors in Dispersive Readout of Charge, Spin, and Majorana  
Qubits,  
Phys. Rev. Applied 14, 064024 (2020)

[Tajkov20]

Z Tajkov, J Koltai, J Cserti, and L Oroszlány:  
Competition of topological and topologically trivial phases in patterned graphene based heterostructures,  
Phys. Rev. B 101, 235146 (2020)

[Perrin21]  
H Perrin, JK Asbóth, J-N Fuchs, R Mosseri:  
Two particles on a chain with disordered interaction: Localization and dissociation of bound states and mapping to chaotic billiards  
Phys. Rev. B 104 205408 (2021)

[Boross22]  
P Boross, A Pályi:  
Dephasing of Majorana qubits due to quasistatic disorder  
Phys. Rev. B 105, 035413 (2022)

[Gyorgy22]  
Z György, A Pályi, G Széchenyi:  
Electrically driven spin resonance with bichromatic driving  
<https://arxiv.org/abs/2206.00399>  
Accepted to Phys. Rev. B

[Frank22]  
Gy Frank, D Varjas, P Vrana, G Pintér, and A Pályi:  
Topological charge distributions of an interacting two-spin system  
Phys. Rev. B 105, 035414 (2022)

[Frank22b]  
Gy Frank, D Varjas, G Pintér, A Pályi:  
Weyl-point teleportation  
Submitted to Phys. Rev. B, <https://arxiv.org/abs/2112.14556>

[Pinter22]  
G Pintér, Gy Frank, D Varjas, A Pályi:  
Birth Quota of Non-Generic Degeneracy Points  
Submitted to Phys. Rev. Lett., <https://arxiv.org/abs/2202.05825>

[Tajkov22]  
Z Tajkov, D Nagy, K Kandrai, J Koltai, L Oroszlány, P Süle, ZsE Horváth, P Vancsó, L Tapasztó, P Nemes-Incze:  
Revealing the topological phase diagram of ZrTe<sub>5</sub> using the complex strain fields of microbubbles  
npj Comput Mater 8, 177 (2022)



[Okvatovity22]

Z Okvátovity, L Oroszlány, and B Dóra:

Time-dependent electric transport in nodal loop semimetals

Phys. Rev. B 104, 035130 (2021)

[Carracedo22]

G Martínez-Carracedo, L Oroszlány, A García-Fuente, L Szunyogh, J Ferrer:

Electrically driven singlet-triplet transition in triangulene spin-1 chains

Submitted to JACS, <https://arxiv.org/abs/2207.13683>

[Kovacs22]

Z Kovács-Krausz, E Tóvári, D Nagy, A Márffy, B Karpiak, Z Tajkov, L Oroszlány, J Koltai, P

Nemes-Incze, S Dash, P Makk, Sz Csonka:

Revealing the band structure of ZrTe<sub>5</sub> using Multicarrier Transport

Submitted to Phys. Rev. B, <https://arxiv.org/abs/2209.06797>