

**CLOSING REPORT**  
**High Performance Computing of Complex  
Delayed Dynamical Systems**

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## 1 BACKGROUND & GOALS

The literature of methods analysing the vibrations occurring during cutting processes has a past of more than 100 years. Nevertheless, we can meet the industrial application of these methods only in very few cases. The commonly used cutting models mathematical descriptions with deterministic constant parameters are used. Bad correlation between repeated measurement of these parameters is denoted to non-appropriate setups, while the deviation of resulting signals is thought to be due to imperfect measurements (noise). Hence only the average values are used in the models. Nevertheless, it is simple to see, that these variables have uncertain average values and they change randomly. This can either have its roots in material inhomogeneities or in unmodelled properties of external systems. Thus the probabilistic and stochastic description of the variables is an important tool for the appropriate characterisation of the system.

## 2 ACHIEVED GOALS

Our main goal was to explore, how the stochastic effects influence delayed dynamical systems. With the help of these methods we are able to predict resulting surface quality, measurable forces and vibration signals for milling with their average values and deviations, too.

Secondly, we investigated how the changing dynamical properties, which depend on the tool position and the removed material rate, modify the stability properties and how the robust stability limit can be determined, which is insensitive to these changes.

### 2.1 Stability of Stochastic Systems

First, we determined the stochastic nature of the cutting force signal, through a series of cutting force measurements. We modelled the cutting force by coloured and equivalent white noises [7,17] (Fig. 1). It is shown that the cutting force is inherently a stochastic process, and the following properties must be considered during the modelling

- An overall relative noise intensity of 4–8% was measured depending on the parameters of the turning.
- Stochastic processes described by models with a small number of parameters can be fitted on the fluctuations of the cutting force.
- A Gaussian white noise estimation has an intensity of 0.1–0.6% of the mean cutting force.
- This small amount of noise in the cutting force does not influence the stability properties of the milling process, but the stationary vibrations due to the noiseinduced resonance. These stationary

vibrations can cause unacceptable surface quality, makes it harder to detect chatter, or it can even cause a transition to chatter in multistable zones near the stability borders.

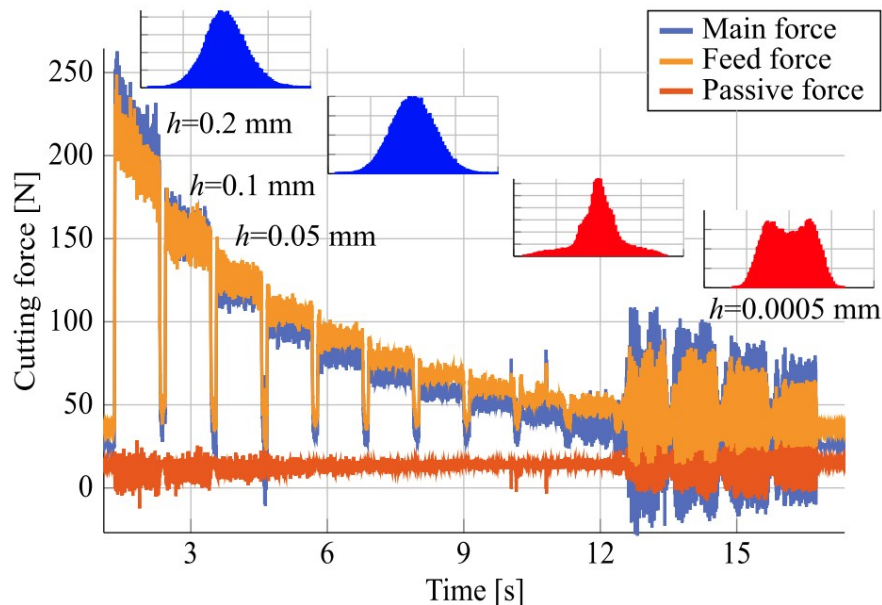


Figure 1. Example of raw measured force signal for exponentially decreasing chip thicknesses  $h$  with the usually occurring force distributions (blue: accepted as Gaussian, red: not accepted as Gaussian)

To be able to analyse the stochastic delay differential equations of the new milling models, we investigated different algorithms and stability definition [5,6,12,21]. It is found that for engineering purpose, the moment stability is the best choice, however, there was no available efficient method for these type of stochastic delayed equations. We established the implementation of the Stochastic semidiscretization method [13,14,19]. It is shown that it can reach higher-order convergence, while its computation complexity is  $\sim O(p^{4.752})$ , which is high, but still the best available.

We determine the stability borders of the milling processes with the Stochastic semidiscretization method and find almost negligible difference compared to the deterministic models, however, we managed to show the stochastic resonance effect close to the stability borders, which presents a hyperbolically increasing random vibration approaching the boundary. With the help of this new stochastic model of milling, the formulation of a chatter peak in the FFT of the signal in the stable machining parameter domains is explained as the manifestation of the noise-induced resonance near the stability borders. This explains properly the practical problems of chatter detection which has high practical importance.

We performed multiple measurements in a milling tool to show this phenomenon which are presented in [15] (see Fig. 2)

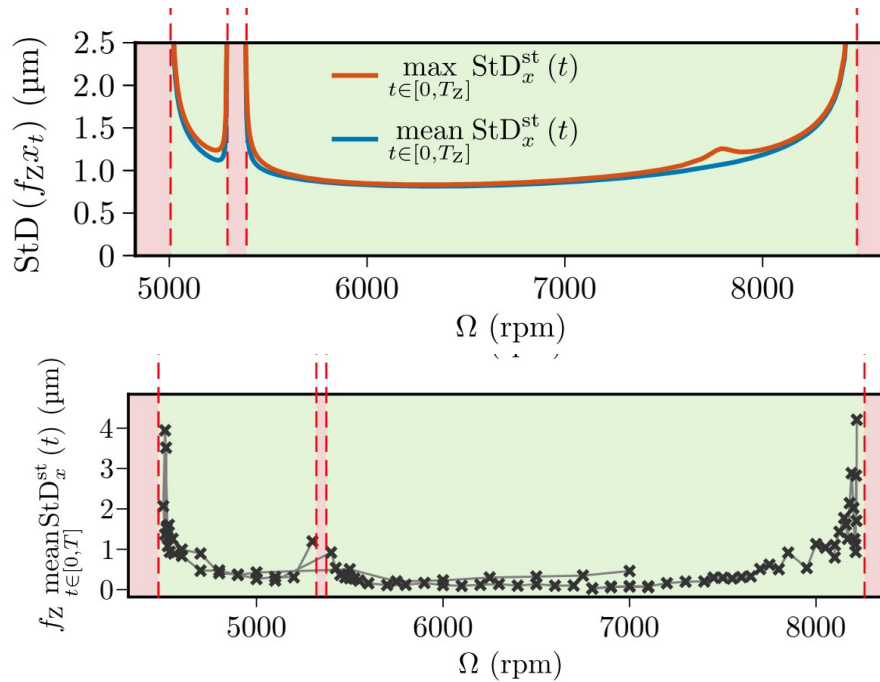


Figure 2. Predicted (top) and the measured (bottom) stochastic resonance effect in the stable (green) domain along the spindle speed.

This method is efficiently used in a collaborative work in a control problem, which is presented in [20].

## 2.2 Effect of changing dynamical properties and robust stability

In the analysis of the changing dynamics, first, a turning process is investigated, in which the effect of the decreasing diameter of a long beam is analysed due to the material removal. It is shown by different analytical models and FEA how the natural frequencies will change [9,10].

The mechanical models are extended to describe the changing properties during the milling processes [16]. By the applied in-process measurement by means of the BallShooter device the changing natural frequencies were followed during the milling process (see Fig. 2 and 3.)

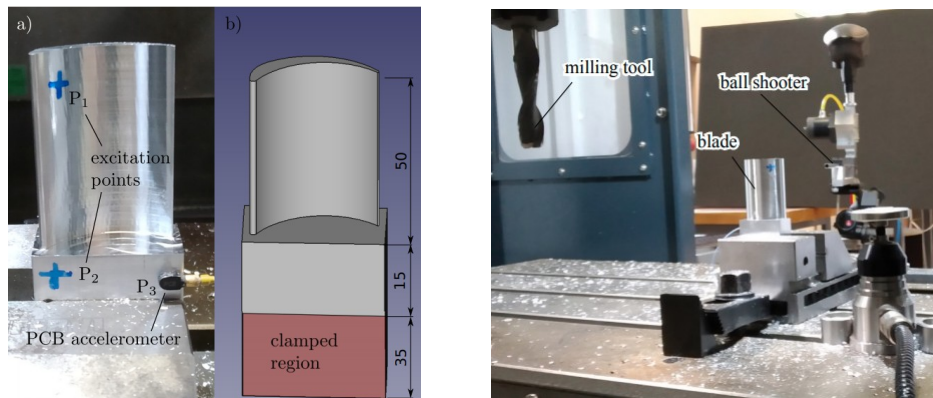


Figure 2. left: (a) The machined final blade profile with the excitation (P1, P2) and sensing point (P3) together with the small sized accelerometer. (b) The CAD model of the blade.: right: the in process measurement setup with the ball shooter.

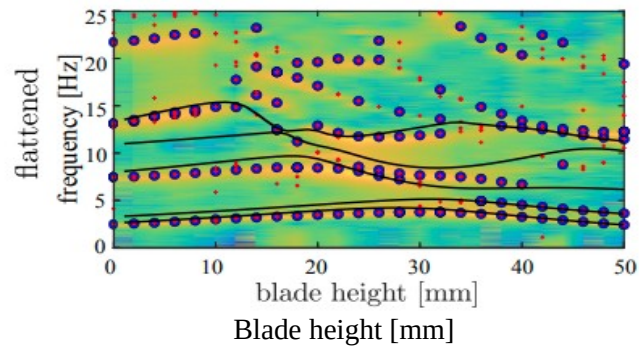


Figure 3. The natural frequencies of the FE model with the artificial elastic layer (black lines) and the measured averaged Frequency Response Function in the function of the blade height presented as a waterfall diagram. The colour scale denotes the logarithmic amplitude of the averaged FRF, the red crosses denote all the fitted natural frequencies by means of the ISD method for each excitation and the blue circles denote the consistent fitted natural frequencies.

To be able to follow the uncertain parameters during the machining, a precise measurement of the spindle speed fluctuation were performed [3] and a contact sensor was created [4]. This way, we could collect input data for the uncertainty analysis and the robustness calculations.

For the validation of the robust stability prediction, we needed precise chatter detection during machining. In [8] we performed a quantitative identification of chatter based on Floquet multipliers in milling operation. A series of measurements were performed, from which we were able to predict the stability boundaries of the stability chart based on the measurements only in the stable domain.

In [18] we applied the impulse dynamic subspace description method to the periodic signals in the milling measurement for precise parameter identification.

### 3 IMPLEMENTED ALGORITHMS

During the project, multiple algorithms were implemented and upgraded in Julia program language which are available for free as a Package of the language.

1. Stochastic differential equation solver - *StochasticDelayDiffEq.jl*
2. First and second moments of stochastic linear delay differential equations - *StochasticSemiDiscretizationMethod.jl*
3. Multi-Dimensional Bisection Method - *MDBM.jl*

### 4 SIDE RESULTS

During the evaluation of the main research directions, many associated problems must have been solved. Most of them related to the measurement of the milling processes. Many of them represent a complete and independence research direction.

- We precisely measured the surface properties using a laser sensor. Furthermore, based on the detected waviness we could determine the frequency of the chatter vibration [1]

- In some measurement, we used a test rig to have a well defined mechanical property (natural frequency and damping ration). In this way, we could measure the vibrations precisely, and using the measured acceleration signals we were able to determine the cutting force and detect some force characteristic. [2]

### 5 CONCLUSION

Overall, it can be said that the targeted efficient algorithms have been developed and have been successfully applied to machining processes. In the research, we developed several numerical algorithms that are suitable for the study of complex delayed stochastic differential equations. Free implementations are available in the Julia program environment. The “Implicit SubSpace Iteration” was used to drastically reduce the complexity of the computational algorithm.

The algorithms have been successfully applied to model cutting processes. The stochastic description was used to predict the uncertainties experienced in the measurement of cutting processes. The phenomenon of stochastic resonance has been quantified, i.e. that the stationer high-amplitude noise process can lead to erroneous identification (stable/unstable) when measured close to the stability limit. With the developed mechanical material removal model, we successfully described the change of the natural frequencies of workpieces with variable geometry.

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