1Final report on the research results of project K124055

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1. Aims and goals of the research

In this section we attempt to give a concise overview of the aims and motivation of the research completed. It must be, however, realised that K124055 was the direct continuation of our research in the past decades, covered by a series of predecessor projects. Thus, the aims and motivations may be summarised as "continuing the research in various interconnected fields of Computational Intelligence, in order to establish new models and algorithms suitable to tackle complex applied problems, and, on the long term, to offer starting points for real life, industrial, logistics, biomedical, management and other solutions which may be utilised in commercial or professional products". In the next, these aims will be explained in some more detail. References will be given to items on the submitted publication list representing the results of this project in parentheses (), while other cited literature gets the usual brackets [].

There are several definitions of complexity, but they agree in that the excessive need of resources (time and space) makes a problem complex (see e.g. [1]). Complex problems often involve nondeterministic behaviour, vagueness and uncertainty. "Nondeterministic" may mean that "too many unknown factors determine the behaviour of the system", and they must be treated as if they were inherently nondeterministic. Complex problems are common in most areas of engineering, logistics, natural sciences, biomedical systems, etc. Problems involving humans, as social systems, management, government decisions, forensics, etc. are full of vague elements. Attempts for modelling, control, decision support, optimization with the classic toolkit of mathematics and traditional computer science in all these areas typically lead to no usable results.

For problems of a given class, it is impossible to give an exact optimal solution if the size of the problem (i.e., the number of components) exceeds an often rather low value. How come that such problems are in an overwhelming majority of cases tackled rather efficiently by human experts, operators, or simple technical solutions? There is the classic example of the "Galactic Computer", an imaginary "upper bound" for the power of any computer system that can ever be created, namely, a machine with as many computing elements as there are atoms in the Galaxy, and that operates with the speed of the light – even since the Big Bang. And it may be surprising, but some real life problems cannot be solved even by this hypothetical construction, although, in the practice, there are reasonable ways to handle them. It must be realised that most real problems do not need an exact solution in practice, but a "good enough" one, satisfying expectations of the problem setter is always sufficient. In general, such problems reflect real phenomena, where the number of components is very (or extremely) high, and/or they contain elements which are non-deterministic from the point of view of the problem solver, see e.g. (51,105).

This latter aspect needs special attention. Uncertainty may root in various phenomena. There is statistical uncertainty, which may be handled by probability and measure theory, but it supposes the existence of a large enough number of occurrences which can be statistically evaluated. In some situations, subjective probabilities may be applied, which are obtained by estimation. However, there is another type of uncertainty that comes from nondeterministic behaviour, or the inexact definition of concepts, or simply, from the subjective elements caused by the participating humans. This is the uncertainty that is often referred to as vagueness, and where an alternative measure theory was worked out, called possibility theory where the main difference from probability is that the measure is non-additive. The classic approach is fuzzy theory, originally proposed by Zadeh [2], and later extended to fuzzy systems suitable for modelling complex and nondeterministic systems with uncertainty [3] offers a convenient tool for dealing with this kind of uncertainty, and later, it lead to an elaborated mathematical theory that goes beyond the scope of this research, where we focussed on new methods and models potentially applicable for real life problems. It is worthwhile mentioning that Goguen proposed a more general concept including the original fuzzy set notion that he called L-fuzzy set [4], which formed the mathematical start for the later introduction of several practically very useful concrete extensions of idea of fuzzy membership degree and fuzzy set.

Let us come back to the question how can it be decided whether a solution is "good enough"? A compromise between the expectations of the problem setter, and the resources needed must be brought in balance. The measure that connects the two sides is the "cost". Here, cost is not considered a financial concept, but it means the amount of resources, such as capacity and speed; and on the other side, the loss of accuracy or adequacy of the solution. When modelling most of these problems, and generating algorithms/procedures to solve them, it is usually sufficient to take an approach, which offers a good sub-optimal solution. While there is a plethora of mathematical tools in approximation theory which help with this to some extent, the need of resources here easily grows beyond tractability, even though it would be silly to neglect the results of this well-established branch of traditional mathematics.

However, we are looking for more and more efficient tools for meeting the challenges of complexity. The first approaches of Artificial Intelligence showed a new way, as they made the first attempt to imitate the ways of humans coping with difficult problems. However, at the beginning, the direction of research could not get away from the assumption that people think in clear symbols, that assumption was supported by the Aristotelian and Boolean logic that very deeply roots in the minds of people, since primary school. It was later recognised that people exclude this way of thinking when they search for efficient solutions for difficult situations, and so, symbolic thinking gave way to a more "layered", sub-symbolic knowledge representation based thinking, which ended in what is called Computational Intelligence (CI) nowadays. CI consists of three main components: fuzzy systems (FS), artificial neural networks (NN) and evolutionary/ population based computing (EC), and also of a series of smaller areas, just to mention some examples, artificial life, some aspects of ubiquitous computing, etc. CI is often applied in hybrid forms, such as fuzzy-neural or neuro-fuzzy, or is combined with classical mathematics, which offer an infinite variety of efficient and efficacious CI models and algorithms. Their common feature is that they all use sub-symbolic information represented by fuzzy membership degrees and function, neurons' excitation functions and genetic, chromosome like representation of solutions.

As a natural "side product" of this ongoing research, we have laid increasing stress on solutions where CI is combined with traditional mathematics, e.g., evolutionary algorithms may be combined with gradient based optimisation or bounded discrete swaps (such as n-opt). Many other combinations have emerged in the past decades, and that was the motivation for starting the conference series European Symposium on Computational Intelligence and Mathematics, which is now an annual conference series where the Chief Investigator is one of the General Chairs. As result of this and the antecedent projects, a series of edited books and special issues in journals emerged, all containing large numbers of research results where CI and classical mathematical approaches merged, just to mention here the ones connected with this project period (27,42,66,111, etc.). All these volumes (and one more, in production) contain several chapters and articles with our new results achieved in the frame of the antecedent project K124055.Problems belonging to the complex class tackled in these volumes by other authors also apply solutions by CI approaches, often combined by traditional mathematics.

It should be mentioned at last that some of the "Editorials" and "Prefaces" written to the special issues and edited volumes are self-contained articles, discussing the general problems treated in this introductory part of the report, see e.g. (9,51,100,105).

2. Backgrounds and preliminaries

Several years ago the Chief Investigator published a study on how to use fuzzy rule based systems as tools in solving what was called (maybe, in a somewhat exaggerating way) the "Key Problem of Engineering", even though "Key Problem of Engineering" is not a term accepted by consensus in the relevant literature. Moreover, the term "Engineering" has many different interpretations, only the narrowest one referring to technological sciences and disciplines only, while in various broader interpretations it includes computer science, agricultural engineering, social engineering and certain aspects of economic models, and so on. In the present special issue, Engineering may be replaced by the concepts "Applied Problems", "Real Life Applications", or similar.

The essential point is that problems arising in various real life contexts have a number of common features, namely, their high complexity and the requirement of solving the problem with "reasonable quality" from the application side. The behavior of this kind of models was addressed in a form reduced to optimizing fuzzy rule based models from the point of view of practical applicability, which was called "Fuzzy Cat and Mouse Problem"; a theoretical experiment where the "fuzzy cat" was the implementation of a simple CI approach (the model and algorithm representing a Mamdani-type fuzzy system, see [5]). In this example, the problem to be solved is to catch a mouse following a very simple and mechanical algorithm:

1. The cat takes a "picture" of the position and dynamic features of the mouse.
2. The rule base in the "head of the cat" is calculating the likely future position of the mouse at the time of the cat grasping it. (Because of the uncertainty involved, this future position may be given only by and estimated area.)
3. The cat searches the estimated future position area of the mouse systematically until it catches is the mouse.

Essentially, two time periods will add together when the sum time of catching the mouse is calculated:

- the time of running the Mamdani-system for the determination of the area containing the future position of the mouse, and
- the time searching that area (exhaustively).

The goal is that this sum time should be minimized. The dilemma is the following: if the model is very refined and precise, its evaluation, i.e. the calculation of the expected future position of the mouse takes a longer time, and thus, the uncertainty of the mouse's position becomes bigger, and so the area to be searched becomes larger. However, within the limits of this uncertainty, the estimated future position (the "area center") will be calculated more precisely, and thus, the resulting uncertainty of the future position area, which is identical with the area to be searched in step 3 of the algorithm will be determined by the delicate balance between the fineness and precision of the prediction model and the speed of calculating the result, as obviously, the smaller will be the area to be searched the faster the prediction model produces a result.

All complex problems are similar in the sense that the balance of resource intensity and solution quality must satisfy a preliminarily fixed goal function (where parameters may set the relative importance of the two goal components). The entirety of CI methods, along with hybrid approaches, where CI is combined with traditional mathematical and statistical methods, and Computer Science techniques, form a nice toolkit from where such solutions satisfying the user, or the presenter of a problem in a high degree.

The research now completed always had in focus the search for good solutions in the above sense. The three main directions may be assigned to the three main "columns" of CI. In all three, in the past decades, we were intensively working on reducing the model and algorithm complexity.

In the fuzzy systems field, there are two main elements which are the lead lines of the present research: Fuzzy rule interpolation (FRI, see e.g. [6,7,8]) and fuzzy signatures (FSig, see [9,10]). Both are techniques for compressing information (although, with some potential loss, in the first case) and thus, reducing the complexity of the model. [8] and [10] have a common feature of being hierarchically structured, but the basic FRI method that was proposed in [7] is directly applicable in itself in a variety of problems. In this last research period, we continued both components, theoretically and in the application area, as well.

The Fuzzy Cognitive Maps (FCM) concept uses signed fuzzy membership degrees for expressing the interactions of the elements, the so called concepts, but it operates as an artificial neural network. Its roots go back to [11]. We started to apply FCM for modelling an environmental management problem (see e.g. [12]), based on expert knowledge. However, it turned out that the model generally applied for such type of systems, to be found widely in the literature, did not match at all with the historical data, which were available from an independent source. Our conclusion was that the problem lies with the model, rather than the experts were basically wrong, and a new, rather more complex model was established as the result of a workshop overviewing all components of the system. As the result, a rather complex FCM was established that was far beyond interpretability, and thus, it could not be verified, even though it could be matched to the historical data. The compromise solution would be a middle size reduction of the model, somewhere between the oversimplified approach in the literature, and the very much refined one established by our own environmental research. Thus, a new idea, a component (concept) reduction algorithm was proposed that was later elaborated in the frame of this project (see among the Results).

At last, the very interesting and challenging topic of evolutionary based optimization algorithms should be mentioned, where we started to study a rather interesting and less known method, the Bacterial Evolutionary Algorithm (BEA) proposed by Furuhashi and group [13]. Following the general idea of Moscato (an up to date overview is in [14]), we proposed a new type of memetic approach based on BEA and the Levenberg-Marquardt second order gradient method (see e.g. [15]). Studying NP-hard discrete problems, the idea was raised whether a discrete version of the memetic BEA could be developed. Indeed, we proposed the Discrete Bacterial Memetic Evolutionary Algorithm (DBMEA), which produced some initial results but had several weak points as well. In this project, the improvement and wide applications of this improved version, and also, some entirely new methods based on the same BEA were worked out.

3. Results

In the next, we try to summarise the results of the research h, divided into three sub-sections. However, the topics cannot always be separated, as sometimes FSigs are combined with fuzzy rule bases, or EC methods, and there are other fusions of various CI approaches.

And also, there are some results, which are clearly part of the CI related research, and still, they cannot be easily ordered to any of the sub-sections, such as e.g., (1) that uses an NN approach different from the method in 3.2, or (7), which belongs in the broader sense to 3.1. There are also a couple of overview papers and some edited volumes, which are partly connected, but contain some work connected with the current research – these are not referred in this report.

3.1. Fuzzy rule bases, rule interpolation and fuzzy signatures

As mentioned already, uncertainty is omnipresent, wherever human elements or nondeterministic factors, which cannot be explicitly modelled are part of the system. For modelling this type of uncertainty, it is reasonable to apply fuzzy approaches. In our previous research, are studies focused on rule based systems, with a stress on interpolation among the rules (FRI); and fuzzy signatures (FSig), a class of multicomponent hierarchical fuzzy descriptors, which has close mathematical relation to hierarchical fuzzy rule bases.

At the beginning of this sub-section, we mention that a novel intelligent traffic signal control algorithm was proposed that is based on fuzzy rule based control, with some innovations in introducing hierarchical levels (109). A software for simulations has been implemented, but the last results still wait for publication.

Now, coming to FRI, in order to explain its importance, it is necessary recall that Zadeh, the founder of fuzzy sets, proposed the method of Compositional Rule of Inference (CRI) for dealing with control and decision making under uncertain conditions [3]. The idea was not practically implementable, because of the high complexity (exponential in terms of the cardinality of the inputs), but nevertheless, this became the base of all fuzzy rule models and control/reasoning algorithms. As the complexity of CRI was too high to be applied, Mamdani’s approach reducing the model to orthogonal projections transformed the idea of CRI into an implementable technique [5]. A huge number of industrial and commercial applications made Mamdani-controllers appearing in cars, household appliances, consumer electronics, industrial control, etc. an integer part of everyday life by now. The tractability of this model is, however, still limited by up to 5-10 state variables. The reason is that this latter also has exponential complexity, where only the geometric representation of the model in the input-output hyperspace was simplified from general fuzzy relation to a fuzzy set over a hyper-oblong. FRI further reduced this to sparse rules representing the model, with "holes" in the geometric image, thus reducing T in the expression T^k. This geometric equivalent of FRI was the key idea behind an entirely new representation and way of calculation of the result: representing all rule elements in pure geometric form, and, instead of doing lengthy calculations with exponentially growing processing time, we directly proposed the application of geometric construction for obtaining the fuzzy conclusion. This approach is not entirely without any antecedents in the literature, as some earlier methods of fuzzy analogous reasoning already applied geometric construction for limited transformations of single rules. The new method was published in (41), in one of the leading journals of the fuzzy literature. The idea may be used after applying some implementation techniques for fast interpolative decision making and control, although, the results will be approximate compared to the calculated ones. The same approach was extended next to inversion in fuzzy rule bases (see (58)), a rather novel idea that is based on the interpretation of fuzzy rule based models as extended input-output mappings. A more advanced paper fusing the two related ideas was published very recently (112).

In the last project period, some new applications in the biomedical image processing area (the recognition of colorectal polyps) were proposed (see (13,21,33,74,94)), where we combined fuzzy rule interpolation with wavelet transform. This led to some partial success, no breakthrough but promising results. However, this method produced very good results on the prediction of the performance of physical telecommunication lines (22). As an alternative to the FRI approach in biomedical images, more recently, we attempted the fuzzy signature (FSig) modelling approach which seems to be rather efficient. We also attempted the application of wavelet analysis in combination of other fuzzy models, especially, fuzzy clustering and classification algorithms, and entropy models as other alternatives to rule based decision (12, 14,69). It must be mentioned here that other alternative recognition methods were also investigated, like the one based on fuzzy Hough-transform [24], thus connected to our earlier work (93). As a side product of this research, we also investigated the behaviour of outliers in the images processed (34).

Image processing tasks are similar with engineering pictures compared to biomedical images, and thus we extended our earlier methods to some mechanical engineering problem, too (71).

In connection with a COST Action mobility project that allowed the funding of several conference participations for members of this research, fuzzy classification was also applied for identifying crime hot spots in Hungary, based on public police data (50,47).

The latter approach was also modified for and applied with success on telecommunication measurement results (20).

Motivated by the interest of some visiting Indonesian Ph.D. students, we also started to model the environment in schools, and the competencies of pupils. A result based on fuzzy rules was published in (36,37,79,95).

The FSig application attempt already leads to the sub-topic of Fuzzy signatures, which are hierarchically structured compact fuzzy descriptors, being not necessarily homogenous in the structure, even within the same model. They were originally proposed by us as an alternative for fuzzy rule based models [22]. (For a more formal description, see [23].) The main difficulty with FSigs was to fit this model in the framework of abstract algebra. After several years of intensive research, in the frame of this project, in collaboration with some Spanish colleagues, it was at last shown that FSigs belonging to the family of a general mother-signature form an algebraic lattice (90), thus being a special case of the L-fuzzy sets defined by Goguen, extending the original fuzzy set concept [4]. Paper (90) is the most important mathematical result of K124055, that opens up a wide gate to further research in connection with FSigs, e.g., towards applying FSigs in rule bases, including FRI.

There is one more theoretical direction that emerged: the extension of the concept of FSigs to linguistic FSigs. As there are no publishable theoretical results here, the first publications will be mentioned in the next sub-section, as this modelling covered both FSigs and FCMs. However, this idea is worth further investigations, where the problem of linguistic aggregations must also be studied in the future.

Besides the theoretical results, a series of applications were proposed during the last project period. As a successful and ongoing application steam, we continued working out the necessary FSig structures for residential building condition evaluation and connected decision making on the necessary actions for the maintenance and operation of the given building (4,19,56). Focus was here laid on the structural condition. As FSigs consist of the tree graph itself and the aggregations in all intermediate nodes (including the root), it is important to work out both features of them. As this problem is very specific in Civil Engineering, besides the graph structure that was based on standard engineering models, entirely new fuzzy aggregations had to be worked out, which reflected the experimental knowledge of structural engineering concerning various types of buildings, especially, historical buildings from the 19th and early 20th century. This application, however, brought a feedback to theory. The problem of the new (civil engineering inspired) aggregations needed further investigation (86,97,111).

At this point, it must be mentioned how important aggregation operators are in all fuzzy related models, especially, rule bases and FSigs. As in the above application stream it emerged as one of the key problems, and as it will be mentioned later concerning linguistic FSigs, the study of aggregations is the foundation of fuzzy modelling. This is why we investigated certain classes of operators and achieved some mathematical results (101,102). Further study of aggregations will be necessary, without doubt.

Returning to applications, a practical company related project induced the use of FSigs for recommender systems (18).

A rather rich application area emerged when questionnaires in management science. where the replies are graded according to the Likert scale were coming in the focus of our interest. Collaborating with a Lithuanian team, we transformed the questionnaires into fuzzy signatures, based on statistical and genetically optimised parameters, and ended up with interesting results on the benchmark questionnaires used in over 50 countries for evaluating employee attitudes. The model itself, evaluation of the results, and some comparisons between Hungarian and Lithuanian employees were the result of this direction of the applied research on FSigs – even though, this work has some novel methodological elements, pointing at the efficiency of combining CI with traditional mathematics (statistics, in this case). See (47,57,65,67,104).

A more recent area where FSigs may be successfully applied is the condition assessment of batteries in hybrid and military vehicles (106). This direction of research will be continued for various batteries and cells.

Another new application field is material handling (see (85,110)).

In a logistics related problem area, how to choose the type of packaging (returnable or disposable, special packaging for dangerous goods, etc.), we proposed the use of a more advanced and complex extension of the one layer FSig model. As the evaluation of rather complicated FSigs, especially, when it may be assumed that under certain conditions, several sub-trees of the FSig will become meaningless or “empty” of information, it seemed to be a necessity to propose a method based on multiple “cascaded” FSigs. We applied three levels of FSigs, and the results were rather good and in accordance with experts’ practice (25,38,64,80,81,99,114).

The idea of multiple layers of FSigs was modified to parallelised sub-FSigs in the diagnosis support of liver lesions and again, it led to very good results on real hospital data (a paper under submission). These latter successes encourage the research of the multiple FSig constructs, also from the mathematical point of view in the future.

At the end of this sub-section, we mention that the above referred COST Action led towards further image processing problems in forensic science. Fingerprint recognition and matching is one of the classical methods where CI, and traditional mathematical approaches may lead to success. Some of our results can be found in (76). In the continuation, some novel way to reduce the processing time of fingerprints were proposed (1ö8). This method only searches for the core point, but is very fast in the case of (partial) prints where there is a core point visible.

3.2. Fuzzy Cognitive Maps

The field of NN deals with models where simple interconnected computational units are arranged in a mess or in layers. A special type of NN, referred to as Fuzzy Cognitive Map (FCM) [11,20] came into the centre of research interest. High complexity FCMs are hard to deal with, and thus, in the antecedent projects, an important methodological result was a new reduction algorithm that decreases the number of components (concepts) while preserving the essential characteristics of the net [21]. FCM reduction is a rather complex and challenging new field that had been proposed by our team. The main questions are: What metrics to use when deciding whether two concepts are close or not close? Should classes of concepts be disjoint or may they overlap? How to evaluate and compare an original FCM with its reduced version(s)? Etc. etc. We are continuously working on these issues, and a more advanced version of the original reduction algorithm, based on fuzzy tolerance relations was given in (29). We have continued this research, investigating other metrics and relations (103), however, there is plenty of space for the continuation of this research in the future.

Another theoretical direction that we followed successfully was the analysis of the existence, number and stability of fix point attractors of FCM, which is essential for modelling the stability, sustainability and sensitivity of interconnected systems (23,24). The importance of this direction of the research lies in the fact that FCMs are rarely based on purely objective data, often, expert estimations (even, if they are mean or median values given by a multitude of experts). There is an unavoidable subjectivity in this, and that brings the necessity to assume the presence of a certain error. Let us take the example when a banking management system is working in a stable and sustainable way, and all values (fuzzy membership degrees) assigned to the states of concepts and the signs and values of the mutual interactions are changing because of some action or unintentional occurrence in the whole management system. It is a very important question then, to what degree the stable state of the whole system will change (if at all), and whether stability will be maintained. E.g., in the case of a bifurcation of the fixed point attractor, the result may become unpredictable. The FCM model itself deals with uncertainty, but there are other related types of uncertainty, vagueness and nondeterministic features in the model, and this is why we extended our investigations to various generalised FCM models, such as Grey FCM, Hyperbolic Tangent FCM, Interval Weight FCM and Fuzzy-Rough CM (23,24,45,44,61,62,63,87). These various mathematical results could be summarised in a global stability statement (88). On the other hand, extensive simulation experiments with FCMs also lead to general recognitions about the stability of FCM behaviour (89). Some remarks on rescaling FCMs were also published as results of this research (60)

Beside the theoretical results, we successfully applied FCM-s for modelling in several areas. Continuing the environmental engineering line that we had followed in the frame of the previous connected project, and applying in the practice the above described FCM reduction algorithm, we proposed a model for circular economy (5).

A more challenging field where we have applied the FCM modelling technique for getting information on the sensitivity of the model, especially, on how an erroneous estimation of the state and interaction degrees by the experts may affect the stability of the existence and number of the fixed point attractors was also studied, motivated by a real life project with data originating from a French bank (11, 30).

FCMs are widely usable, and we are convinced, they have much perspective in the future. One of the good application experiments was on user interface design (32).

Another interesting biomedical application was motivated by two guest Ph.D. students from Mexico: the modelling of plantar foot using FCM (50), and later, the comparison with another NN type approach (72).

In this sub-section, the research about the analysis of traffic systems with uncertainty elements will be discussed, where in the focus is the emergence of congestion phenomena. We did research with fuzzy rule based systems, FSigs and FCM as well and got some pro missing advancement with both (84,92,98).

Based on the previous theoretical and applied research in the FCM field, a monograph is now under production at Springer [25].

3.3. Evolutionary, memetic and hybrid optimisation algorithms

Evolutionary computation (EC) is an extremely interesting part of CI, where rather efficient approximate optimization algorithms for a wide and motley scope of intractable problems. Without doubts, this area, with its extensions, combinations and hybrid versions may be one of the most promising fields of "meta-mathematics" and computer science. In addition to the usual concept of efficiency, we introduced the ideas of predictability and generality ("universality") of an algorithm, and proposed that a method with efficiency, good predictability and generality be defined as "efficacious". In the frame of the research, we attempted to develop new efficacious algorithms.

In the focus of our research was the problem of applying EC type heuristics in (approximately) solving discrete NP-hard problems. First, we started with the classic the symmetric Traveling Salesman Problem (TSP), where for many years, the best solution had been the Lin-Kernighan (LK) heuristic, which worked well for graphs with several 100 to 1,000 nodes, while the best exact algorithm for solving the TSP, called Concorde, is suitable for problems with less than 2000 nodes. Above that, in a few cases, the latter may find the (optimal) solution between 2000 and 10000 nodes, as well, however, it depends on the topology of the graph. Above that limit, Concorde never reaches the goal – the problem is intractable though. Even when finding the optimum, the runtime of Concord is very capricious. In order to compare, in one of our previous projects, K108405, we constructed a real variable version of the originally integer variables Concorde, and we did simulations for obtaining reference runtime data, confirming the above. Later, Helsgaun published a tailor made heuristics extending the LK method (HLK) [17] that is still now the best practice.

In the previous, K124055 project, we developed a bacterial evolutionary based heuristic technique (DBMEA), which although proved not to be more efficient in solving the TSP, but has no capricious behaviour as the HLK, thus, it is more predictable. About the efficaciousness property, the question was whether this algorithm could be applied for solving other similar NP-hard problems.

This new DBMEA consisted of a global and a local search layer. The former is the Bacterial Evolutionary Algorithm proposed by Nawa and Furuhashi [18], while the latter is a combination of the simple 2-opt and 3-opt exhaustive searches. While this method could not defeat the speed of the best practice (cf. [19]), it is rather well predictable and, as it will be shown here, we could demonstrate that it has a rather general applicability feature.

The first step in speeding up the algorithm was to introduce several implementation techniques, which helped avoid repeated checks of options, and other redundant steps. A rather important result of the present research was that after experimenting with various deterministic initial population generation methods (Nearest Neighbour (NN), Second Nearest Neighbour and Alternating Nearest Neighbour), we developed an entirely new method, the Circular Neighbourhood Search (83, 111). The basic idea is that within a suitable radius circular neighbourhood, the NN algorithm is applied, then, outside the circle, the next nearest node, where again the circular neighbourhood is checked, etc. etc., until the starting node (the depot) has been reached. Obviously, the key question is what should be the optimal radius of these neighbourhoods. Two alternative optimisation methods were applied, the latter being an evolutionary (genetic) algorithm, and it worked rather well. A benchmark set of selected cities in various countries was used for testing. With this new algorithm we succeeded in further speeding up the DBMEA, but even so, the Helsgaun-LK algorithm is still faster. Nevertheless, the research of new initial population generation methods is promising, and in the planned continuation of this research, it would be investigated, whether it is applicable for other TSP related problems, too. In (10), a well-established description of the algorithm and its application to TSP was given, and in (26), an even more advanced model and evaluation of the same. (Cf. also 43,59,96).) TSP is a very classic problem and there are many researchers who work on new algorithms, even though Helsgaun's method is so far the most efficient. However, in some cases, speed is much more important than accuracy, and there may be other aspects to motivate the investigation of various methods for solving TSP. An analysis of such possibilities was given in (107), and we continue looking for further methods that excel in some sense.

Experimenting with the DBMEA, we discovered that it was rather generally applicable for the problem class similar to TSP, thus almost "universal" in the sense as it was requested for efficaciousness. We successfully applied the DBMEA to several related, but mathematically more difficult search and optimisation problems.

The first such problem was the TSP with Time Windows (TSP TW), where the arrival and departure times are predefined in each node. In case of earlier arrival to a node means additional waiting time and thus an increase in the cost. DBMEA turned out to be rather efficient in this problem, compared to other approaches in the literature (2). The TSP TW problem is a rather practical one that occurs e.g. in delivering goods to a network of shops, where no delivery before opening the store by the first arriving staff member is possible, while the delivery should happen before the opening for the clientele.

Another similar but still, mathematically considerably different problem is called Travelling Repairman (TRP) or Minimum Latency Problem. This differs from the TSP in the cumulative calculation of the cost, i.e., in the way of adding all waiting times of all nodes, from the starting time of the traveller until his arrival. DBMEA did a good job here, too, and we found the first application problem where we could produce in most larger cases a more efficient method than any other in the literature (3,75).

It should be stressed that exactly the same optimisation algorithm was used for all three problems, while in the literature, all three were tackled more or less efficiently by very different, tailor made approaches. After showing the applicability of the DBMEA for three different problem types, it may be stated that it is rather generally usable, so, we call it now an efficacious method with justification.

The next similar problem was the Time Dependent TSP (TD TSP). This is a very important extension of the original TSP as it allows that in a certain region of the graph (the node network), the cost is changing in time. This is based on the assumption that a part of the road network modelled is within a central city area where there is a rush hour period, where in the rush hours, the cost increases, maybe, tremendously. So far, one of the well-known TSP benchmarks was studied in the literature, with local costs growing with factors between 1.01 and 2000. We applied again DBMEA and the results were very satisfactory (16). Unfortunately, there were no numerous benchmarks in the literature, so, a reliable comparison was impossible, but definitely, we achieved better efficiency, and so, we generated a series of other potential benchmarks, where the results seem also rather promising. WE hope that other researchers will take the challenge and will compare their respective methods with our results. It should be stressed again that all related, but mathematically different tasks were tackled by the same new "universal" approach, the DBMEA rather successfully, in some cases producing best practice (see also 78)).

Two more related problems are called One Commodity Pickup and Delivery Problem (77), and the Cumulative Capacitated Vehicle Routing Problem (115), where DBMEA was also applied with rather good efficiency

When studying the TSP and other related problems, a rather obvious question was asked: how did this group of rather abstract models match the real life. In logistics, these problems play crucial roles, but applying the mathematical abstractions directly was not really beneficial to dispatching at companies, route selection of travellers, etc. In the cost of traveling between two nodes (cities, shops, etc.), namely, a considerable amount of uncertainty emerges – which definitely increases the complexity of the problems. In this project, we introduced various types of fuzzy uncertainty in modelling the TSP with Time Window, for the costs of traveling: the locations of the nodes, and the times of travelling, which all include uncertainty. We extended all three elements towards fuzzy sets of different types (type 2 interval valued, intuitionistic and type 2 intuitionistic fuzzy sets). The initial results show that the models match the reality more adequately than the "crisp" (non-fuzzy) one. Here, the DBMEA algorithm could be applied directly for optimising this rather more complex problem class. This problem group forms a straightforward example for why it is necessary to develop general (limitedly universal) optimisation algorithms, and why efficacy is more informative than just plain efficiency. For the various fuzzy extended models and results of simulations for simple case studies, see (39,40,49,52,53,54,55,73)

The universality and predictability of DBMEA, and partly, its efficiency, encouraged further investigations of another NP-hard problem, the Flow Shop Scheduling Problem. The first attempts for applying DBMEA directly did not give really good results, the results were less efficient than the best practice in the literature. as the n-opt type local search does not fit to this kind of optimisation. Studying other authors' results, it was found that the best result was produced by a population type heuristic, the Simulated Annealing method. So, we attempted to construct a new type of optimisation algorithm, following the structure of the memetic algorithms, but applying another heuristic that also could perform good global search: the local search part was replaced by the SA heuristic. The thus obtained hybrid approach consists of two nested EC methods and it proved to be more efficient than the so far best practice SA, so here, we got the next breakthrough in efficiency in this research (98). It is remarkable, that the new method still uses BEA as the outer hull, and that is a hint for the very general usability of this EC method.

The direction of hybrid EC algorithms seems to be one of the most promising further directions of research, and it is our intention to continue investigating hybrid algorithm, especially with BEA as the outer(global search level) one.
At the end of this section, it should be mentioned that we proposed a new evolutionary type algorithm that was called Fungal Growth Algorithm that was applied to handwritten character recognition (15). This method deserves further research.

And at last, we developed a model for the simulation of higher order proteins' behaviour, entirely based on the principles of EC – the bio-inspired method here modelling a real biological phenomenon, rather adequately (35).

4. Conclusions and future work

The total number of publications resulting from this research project is over 110, among which there are four D1, another 12 Q1, and 14 Q2 journal papers, further one classified as A by the HAS. There is a large number of papers classified as Q3 and Q4 as well.

Besides a number of edited books published by Springer, and several special issues in Q1 and Q2 journals, one monograph (on FCM) was also completed, which is now under publication.

In the frame of the research, one Dr habil. procedure was successfully carried out, and four Ph.D. students received their degrees, further four "sandwich students" also completed their degrees (two in Mexico, two in Indonesia), another two completed the studies and have submitted their theses, and eight more are continuing their work. Some master theses with publication results were also completed in the frame of the project.

One COST Action was also won and completed, of which the Chief Investigator was the Hungarian member of the Management Committee. This helped participate in conferences (also to some students) where there was no other funding available.

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