Final report of NKFIH project K124002

Title: Stability analysis of mechanical systems with contact and friction

Principal Investigator: Peter Varkonyi

1 Short summary

In the research project we investigated the stability and the stabilization of rigid and elastic objects subject to *unilateral contact*, *impact*, and/or *friction* interactions.

The first part of the project concerned contact systems within the framework of *rigid body theory*. Even is one assumes point-like contacts and simple impact and friction models, the emerging dynamics is complex, and many aspects of it are poorly understood . Our first aim was to make progress in relation to a fundamental question of dynamical systems theory with many engineering applications: how to evaluate stability of static equilibria of contact systems. In the project we successfully developed new conditions of local *Lyapunov stability* for a simple model system as well as a general framework for computational stability tests. These results are highly valuable for robotic applications.

Stability against large pertubations, and system response to large perturbations are even more difficult questions with important engineering applications. These aspects of contact dynamics were also successfully investigated through several specific examples, including rocking structures subject to earthquake excitation, impacting structural networks subject to dynamic excitation by wind-induced vibration, and shock protection of falling electronic devices.

The second part of the project concerned elastic fibers subject to contact and friction. Even when impacts are not considered, frictional contact may create peculiar phenomena of shape dynamics. Our main goal was to explore theoretical limits of performance of compliant structures under contact (i.e. maximal reachout of a soft manipulator) as well as the role and benefits of controlled intrinsic curvatures. These questions have natural applications in soft robotics. At the same time, our results also lead to a new explanation of growth patterns of soft, elongated plant roots by combining contact dynamics with the theory of morphoelasticity.

In addition to the application-oriented research program outlined above, we also made significant efforts to develop new models of basic contact-related phenomena, which are used as building blocks in other parts of the project.

In the research plan, 6-7 journal papers and 1-2 conference papers were predicted. The actual publication count exceeded these expectations. 9 journal papers and 1 conference paper have been published, the majority of which were accepted by leading journals (Scopus Q1 or D1 category) of the field. One additional paper has been published as pre-print, and it is currently under review.

2 Detailed report

Below the main results of each one of the four work packages of the project are described in more detail.

Local stability analysis of contact systems with impact and friction

Despite the amazing development of robotics and object manipulation techniques in recent decades, the fundamental question of Lyapunov stability of a rigid multibody system in static equilibrium remains unsolved. Prior work of the Principal Investigator uncovered sufficient stability conditions of Lyapunov stability for an important model system: a planar rigid body with 2 frictional point contacts. In the current research project, the Poincaré map-based techniques delivering these preliminary results were adapted to 'non-persistent' equilibria of the model system, which show some special forms of dynamic behavior. The extended stability theory was published in a leading journal of the field [11]. We also designed and built an experimental test rig to investigate the instability mechanisms predicted by the theory. Systematic investigations verified the main predictions, and the first published experimental demonstration of instability via the 'inverse chatter' mechanism was also reported in [11].

As next step, we also developed new tools to test the stability of other systems. The research group of Prof. Russ Tedrake at MIT recently found that sums-of-squares (SOS) programming techniques can be used to search for Lyapunov functions, which verify the stability of contact systems algorithmically by virtue of Lyapunov's direct method. During the present project we applied their method to some multi-contact systems for the first time and we found that their method was not successful in these cases. In order to improve the verification method, we developed two generalizations of Lyapunov's direct method:

- we allowed the simultaneous use of multiple Lyapunov functions
- we pointed out that stability can be verified with the aid of Lyapunov functions that may temporarily increase along solution trajectories, relaxing one of the basic restrictions of Lyapunov's direct method.

The generalized theory remained compatible with SOS programming techniques. A MatLab-based implementation of the SOS-based stability test was developed. It was found that the improved theory is able to verify Lyapunov stability for many configurations of the previously introduced model system. A preprint of the paper summarizing these results was made available through the arXiv open-access repository of electronic preprints [9]. Publication in a peer-reviewed journal is in progress.

Non-local stability and response under dynamic excitation

Stability against large perturbation is a key question of the earthquake protection of large-scale engineering structures. The remarkable earthquake resistance of rocking block-like structures is caused by energy dissipation during impact at unilateral contact surfaces. In the current research project, three-dimensional rocking motion of rigid blocks was investigated. It was pointed out that impacts of rocking structures are extremely unpredictable due to the fact that these impacts occur at extended contact surfaces. To address this challenge, a new universal impact model of three dimensional rocking impact was developed. The new model determines all feasible outcomes of an impact in parametric form. In addition, we proposed to model impacts as a random process, and to perform statistical investigation of rocking dynamics in order to estimate overturning probabilities. We also proposed that extreme sensitivity of impacts is closely related to geometric imperfections. These results have been published in [10]. An experimental device has been designed and built to verify this hypothesis. The device is a rocking block with adjustable geometric and inertial properties, and with a movable small 'bump' representing geometric imperfections of the base facet. Preliminary observation of this device confirms the dominant role of imperfections in the variability of rocking impacts. Systematic investigation of the device will be continued after the end of the current project. We also implemented the new impact model in a MatLab-based numerical simulation code. Overturning curves of rocking blocks have been created. Our results confirmed that rocking impacts have strong influence on the three-dimensional motion and overturning of blocks for some (but not all) types of ground excitation. This finding suggests a novel way of earthquake protection through designed geometric imperfection. A journal publication of the last findings is under preparation.

Rigid objects dropped onto a hard surface undergo a complicated sequence of motion, which includes episodes of rocking motion as well as other types of bouncing motion. As before, motion trajectories are unpredictable mainly due to spatially extended impacts, and a statistical approach can be used to describe their dynamics. We extended our approach to this problem, and successfully demonstrated that shock protection of sensitive components can be improved by careful placement within the device, and by careful design of geometric and inertial properties of the device [1].

The investigations were also extended to another type of dynamic excitation: wind-induced vibration of large scale building structures. Here, unilateral contact interaction and impacts are introduced via heat expansion gaps of the structure, which are needed for structures above a certain critical size. As before, unpredictability of important motion trajectories plays an important role if there are many points of potential contact. Using a statistical approach, we proposed and investigated the structural behavior of a novel type of structural system, in which soft, high-rise buildings are connected by urban-scale networks of horizontal bridges. A low degree-of-freedom conceptual model of an individual building was adopted, which enabled us to use the approach of rigid body dynamics. It was demonstrated that the nonlinearity and energy dissipation associated with impacts reduces vibrations of such networks and improves user comfort [8]. A preliminary study of the architectural consequences of such a development strategy was also published in [4].

Friction-induced instability of expanding and moving rods

In the next part of the project, elastic rods subject to contact interaction and dry friction were investigated. First, the quasi-static behavior of an elastic rod, which is constrained to grow or move was investigated. Numerical simulation revealed a certain behavior that resembled 'buckling' of columns. Yet, we found that the underlying mechanism is not analogous to buckling. These systems are time-dependent, and do not display time scale separation. Hence stability analysis requires a concept of *stability over finite time horizon*. We developed such a new concept of stability, identified the critical point associated with loss of stability, and the post-critical behavior was also described by numerical simulation [2]. It was pointed out that a potential application of our results concerns extremely soft robotic arms, which exploit contact with an underlying surface, to deliver heavy loads to a large distance. To put these results into context, we also determined the theoretical maximum of the reach-out of an initially curved, soft robotic manipulator carrying a heavy load *without contact*. The existence of a previously unknown theoretical limit distance was demonstrated, and its value was estimated by solving non-linear boundary problems capturing all types of elastic instability phenomena [3]. The main conclusions of these works were demonstrating that that appropriate choice of initial curvature as well as appropriate use of slipping contact may both improve the performance of a soft arm significantly.

The previous conclusion also lead us towards the investigation of biological systems, in which initial curvature is modulated in response to environmental clues. The tropic behavior (directional movement) of elongated plant organs in response to gravity, light, and touch has been known for a long time. The interaction of tropism with frictional contact interaction in the case of plant roots has been studied experimentally by many authors, but the underlying mechanical processes have not been described. In order to fill this gap, we developed a morphoelastic model of root growth combined with models of plant tropism and with simple contact models. Our model successfully explained a rich variety of experimentally observed morphological patterns of *Arabidopsis thaliana* including wavy, coiled and skewed root morphology. Hence, we were able to explain for the first time how elastic deformation interacts with internal control of initial curvatures [6].

Modeling contact dynamics

The aim of this work package was to provide all other application-oriented research activities with necessary basic models of contact-induced phenomena.

One such phenomenon concerns simultaneous impacts at several contact points. First, we described a model suitable for numerical simulation in [1], which treats simultaneous impacts as possibly infinite sequences of single-point impacts. Then, the same model was reused in [9], where we also showed that the proposed approach is consistent with SOS-based stability verification techniques.

If two objects establish a spatially extended contact surface, both impacts and dry friction give rise to peculiar dynamic phenomena. Impacts show extreme sensitivity, which is at the heart of modeling rocking motion. Therefore, the development of new impact models was a crucial part of paper [10]. Friction along extended surfaces also tends to be quite unpredictable, in addition it also gives rise to novel types of non-smooth behavior. In collaboration with an external partner, Dr. Mate Antali, we described the underlying mathematical structure of this phenomenon, and we introduced the concept of 'Generalized Filippov systems of codimension-n' [5]. Then, special features of slip motion induced by extended friction surfaces were also studied. Among other, we described an unusual property of slip-stick transitions, namely that they occur through a small set of predictable, nontrivial combinations of translational and rotational motion [7].

3 Publications

[1] T Baranyai, PL Várkonyi: ***Optimal mechanical design of electronic devices for shock protection***, IEEE Transactions on Components, Packaging and Manufacturing Technology 8, 1533 - 1543, 2018

[2] MG Horváth. AA. Sipos, PL Várkonyi: ***Shape of an elastica under growth restricted by friction***, International Journal of Solids and Structures, vol. 156–157, pp. 137-147., 2019

[3] AÁ Sipos, PL Várkonyi: ***The longest soft robotic arm***, International Journal of Non-Linear Mechanics 119, 103354, 2020

[4] M Köhserli, PL Várkonyi: ***Vibration of structurally connected high-rise buildings under wind force.***, Proc. IABSE Congress Ghent, 2021 - Structural Engineering for Future Societal Needs (22-24 Sept. 2021), 2021

[5] PL Varkonyi, M Antali: ***On Differential Equations with Codimension-n Discontinuity Sets***, SIAM Journal on Applied Dynamical Systems 20 (3), 1348-1381, 2021

[6] AA Sipos, PL Varkonyi: ***A unified morphoelastic rod model with application to growth-induced coiling, waving, and skewing of plant roots***, Journal of the Mechanics and Physics of Solids 160, 104789, 2022

[7] M Antali, PL Várkonyi: ***The Nonsmooth Dynamics of Combined Slip and Spin Motion Under Dry Friction***, Journal of Nonlinear Science 32 (4), 1-43, 2022

[8] M Köhserli, PL Varkonyi: ***The effect of connecting bridges on vortex-induced vibration of skyscrapers***, Architecture and Architectonics (Epites- es epiteszettudomany) 50 (1-2), 1-16, 2022

[9] PL Várkonyi: ***Stability analysis of rigid body systems with multiple contacts by sums-of-squares programming***, ArXiV preprint: http://arxiv.org/abs/2209.13908

[10] PL Várkonyi, T Ther, M Kocsis: ***Rigid Impacts of Three-dimensional Rocking Structures***, Nonlinear Dynamics 107 (3), 1839-1858, 2022

[11] Y Or, PL Várkonyi: ***Experimental Verification of Stability Theory for a Planar Rigid Body With Two Unilateral Frictional Contacts***, IEEE Transactions on Robotics 37 (5), 1634-1648, 2022

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Peter Varkonyi

Principal investgator