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1 Results on Valuations

Following classification results about $\mathrm{SL}(n, \mathbb{R})$ intertwining tensor valued valuations by Ludwig, Parapatits, Haberl, together with Judit Abardia-Evéquoz, Mátyás Domokos and Dávid Kertész, we have completely described the space of continuous, $\mathrm{SL}(m, \mathbb{C})$-equivariant, $m \geq 2$, and translation covariant valuations taking values in the space of real symmetric tensors on $\mathbb{C}^m \cong \mathbb{R}^{2m}$ of rank $r \geq 0$ (see the paper $\mathrm{SL}(m, C)$ equivariant and translation covariant continuous tensor valuations. J. Func. Analysis, 276 (2019), 3325-3362.). The classification involves the moment tensor valuation for $r \geq 1$ and is analogous to the known classification of the corresponding tensor valuations that are $\mathrm{SL}(2m, \mathbb{R})$-equivariant, although the method of proof cannot be adapted.

Together with Matyas Domokos and Gil Solanes (see the preprint Dimension of the space of unitary equivariant translation invariant tensor valuations. arXiv:2003.04649), we have determined the dimension of $U(2m)$ equivariant translation invariant continuous rank r tensor valuations on $\mathbb{C}^m = \mathbb{R}^{2m}$. Earlier, the scalar valued case $r = 0$ was solved by Semyon Alesker, and the vector valued case $r = 1$ was solved by Thomas Wannerer. In the scalar valued case, even a basis was determined by Semyon Alesker. We have found a basis in the vector valued case ($r = 1$) based on the representation of smooth valuations by differential forms.

Together with Christoph Haberl, Jin Li and Monika Ludwig (TU Wien), we have started to work on characterizing exponential valuations on lattice polygons, extending the properties of "discrete Laplace transform". We showed that it is the sum of a singular exponential valuation and a simple exponential valuation. We have completely characterized the singular exponential valuations, and we are still working on simple exponential valuations, .

2 Related results on the L_p dual Minkowski problem

The L_p dual Minkowski surface area with additional parameter q is a typical example of valuations on convex bodies in $\mathbb{R}^n$. For the "classical" $q = n$ case, the NKFIH project managed to strengthen all known results about the L_p Minkowski problem if $-n < p < 1$, see the following papers:

- Bianchi, Gabriele; Böröczky, Károly J.; Colesanti, Andrea: Smoothness in the L_p Minkowski problem for $p < 1$. J. Geom. Anal. 30 (2020), no. 1, 680-705.

- G. Bianchi, K.J. Böröczky, A. Colesanti: The Orlicz version of the L_p dual Minkowski problem on S^n for $-n < p < 0$. Adv. Applied Mathematics, 111 (2019), 101937.

- G. Bianchi, K.J. Böröczky, A. Colesanti, D. Yang: The L_p-Minkowski problem for $-n < p < 1$ according to Chou-Wang. Adv. Math., 341 (2019), 493-535.

A generalization of the L_p Minkowski problem is the recently introduced qth dual L_p Minkowski problem by Lutwak, Yang and Zhang where $q = n$ corresponds to the classical problem. The NKFIH project managed to solve the L_p dual Minkowski problem for $p > 1$ and $q > 0$ among any convex bodies, and obtained results about the regularity of the solution in the paper K.J. Böröczky, F. Fodor: The L_p dual Minkowski problem for $p > 1$ and $q > 0$. Journal of Differential Equations, 266 (2019), 7980-8033. In addition, we have solved the L_p dual Minkowski problem for $p = 0$ and $1 < q < n$ among origin symmetric convex bodies in the paper K.J. Böröczky, E. Lutwak, D. Yang, G. Zhang, Yiming Zhao: The dual Minkowski problem for symmetric convex bodies. Adv. Math., 356 (2019), 106805.