Summary of the research project NKFIH PD-121223

Recurrence time analysis and decay of colliding and escaping planetary systems

1 Introduction

We completed our study in agreement with the proposed research plan funded by the grant PD-121223. Two main topics are covered by our scientific work

- developing a new and efficient method to analyse exoplanetary systems based on the fact of phase space recurrences and complex network framework;
- theoretical understanding of complex behaviour in the outer solar system and exoplanetary systems by using high performance numerical computations.

The main results and their scientific impacts are described in the corresponding subsections. In addition, some grant related activities are also denoted.

2 Recurrences and their quantification

The continuously growing amount of data and the increasing number and variety of extrasolar planetary systems stimulated various new methods in dynamical astronomy. Although, these schemes are fairly different in nature, one of the essential properties is in common. Namely, they are based on observational data and their uncertainties and use massive statistical analysis to get the best-fitting planetary models. The obtained orbital elements can then be used for further dynamical analysis of the planetary system under study. The extremely precise observations of multiple exoplanetary systems strengthened the link between pure theoretical studies and comprehensive analysis of the signal collected by space based surveys (Kepler, TESS). This synergy allows us to look at the stability analysis and dynamical evolution from a different point of view which does not need the classical n-body simulation, Monte-Carlo analysis, thus, it saves a reasonable amount of computation time.

2.1 Observability and reconstruction of the dynamics

The dynamical analysis of a particular system requires the time evolution of phase space trajectories. Since the observer records the signals in an experimental setting in time domain, which means that not all relevant components of the state vector is known, a reconstruction of multi-dimensional information in an *artificial phase space* is needed. This is possible when the following assumption hold: If the variable corresponding to the observable affects the other variables, then the variables governing the system's dynamics are coupled. In this case a recovery of phase space trajectory can be done by using the *embedding theorem*. It should be noted that the reconstructed trajectory is not identical to that one we would get from numerical integration, i.e. all the components are known. It might differ in shape but preserves mathematical properties such as topology and Lyapunov exponents. Since astronomical observations provide no exact initial conditions, neither in Cartesian coordinates nor in orbital elements, this method is essential in order to have a picture about the dynamics of planetary systems.

As a first stage of our work we showed by using Takens' embedding theorem that the current astronomical observation techniques (radial velocity, astrometry, transit timing) present valuable information about planetary system dynamics. It is however, a highly non-trivial task to find the appropriate time delay parameters that enable us to explore the phase space dynamics of a complex Hamiltonian system. To understand the efficiency of phase space reconstruction a basic planetary model (1 star + 2 planets) has been investigated. The simplified subset of the solar system containing the Sun, Jupiter, and Saturn provides a perfect sample to study even the complex structure in the phase space. The successful comparison of the well-known semimajor axis – eccentricity stability map and the embedding technique indicates the relevance of the method and it delivers a direct application of phase space recurrences to describe planetary dynamics [1].

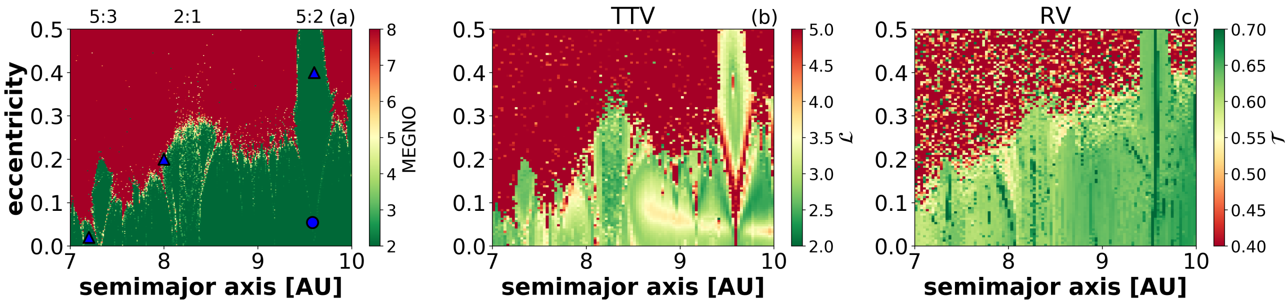

Figure 1: Stability and RN measures for the Sun-Jupiter-Saturn model system. (a) Stability map $(a_{\mathrm{Sat}}, e_{\mathrm{Sat}})$ of two-planet system according to the indicator MEGNO. Semimajor axis of Saturn is measured in astronomical units (au). The green area denotes the stable realm while for larger eccentricities the dynamics is chaotic (red). The dominant low-order mean motion resonances are indicated at the top of the panel. Blue triangles are reference trajectories from different dynamical regimes for further analysis. The blue dot represents the Saturn's current location in the parameter plane. (b) and (c) Two RN measures $\mathcal{L}$, $\mathcal{T}$ are pictured on the same grid as in (a) taking into account two observables TTV of Jupiter and RV of the Sun, respectively. The color bars illuminate the heat map values in each case. For more details see the extra attachment files containing submitted manuscripts in the subject.

2.2 Complex network analysis of time series

In last decades, the successful application of complex networks in various fields stimulated the demand of transforming time series into network representation. A variety of effective methods to construct networks from time series have been proposed (transition networks, cycle networks, visibility graphs, and also recurrence networks) that were rapidly integrated into scientific research. The triumph of this framework is that different dynamical regimes either on local or on global levels can be investigated by means of topological properties of distinct networks.

Applying recurrence network (RN) analysis to a model system (Sun + 2 planets) one can reconstruct reliably the system dynamics from observational time series. The quantitative description of RNs, incorporating classical network measures such as *degree distribution, average path length ($\mathcal{L}$), transitivity $\mathcal{T}$, etc.,* provides relevant geometric information about the underlying dynamics, see the results in Fig. 1(b) and (c). Since the RN-measures do not depend on temporal correlations and on explicite time ordering this approach gives wider applicability to analyse sparse and noisy astronomical data. Furthermore, once we have the appropriate network gained from the observational time series, the treatment of network measures needs much less computational effort than manipulating the equations of motions. We performed nonlinearity test and stability analysis in dynamical astronomy and also demonstrated that complex network description is suitable to explore the dynamical variability.

Thus, we can point out that our basic hypothesis is true [1], that is, even non-uniformly sampled noisy data sets carry important information about the system's original dynamics. The reliability and the sensitivity of recurrence network analysis has also been shown when realistic effects of astronomical observations (seasonality, weather conditions, instrument precision) are taken into account.

2.3 Stability of multiplanet systems

In nonlinear time series analysis one often makes the dynamical analysis of the system through one single scalar time series, called *observation.* Observation in this sense means all the available data points measured, they are practically one or two components of the phase space trajectory or a projection of it that is connected to the dynamics of the entire system. Network analysis gives only one single output which cannot be accepted as a statistically reliable result, hence, it should be tested against a null-hypothesis. In the case of planetary dynamics the null-hypothesis is whether the nonlinear signal comes from a pseudo-periodic or oscillating motion. To have a rigorous answer to this question one needs, paradoxically, more than one observation to perform the statistical significance. The *surrogate data approach* provides exactly what we need. The method is based on *generating an ensemble of time series that are dynamically equivalent to the null-hypothesis.* Then a statistical analysis is accomplished with the observation and the surrogates. By comparing the outcome of the original signal with those acquired

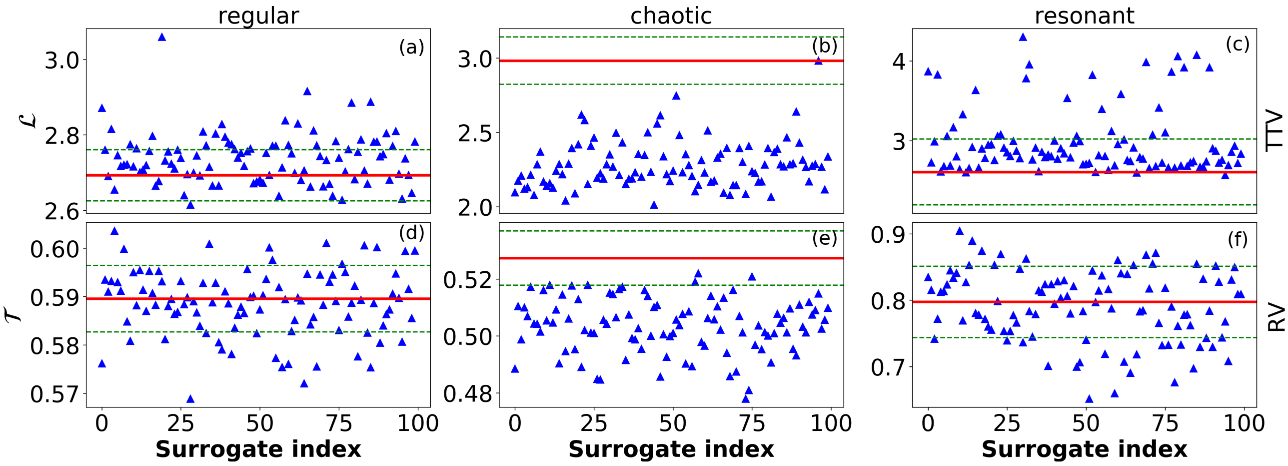

Figure 2: Outcome of hypothesis tests in three different dynamical regimes. Panels (a)-(c) show the results for $\mathcal{L}$ based on TTV signals. Panels (d)-(f) present the measures $\mathcal{T}$ in case of RV data sets. The rank-based statistics involves the noisy and irregularly sampled reference trajectories (red solid line) and 100 PPT surrogates (blue triangles). The green dashed lines mark the ± 1 standard deviation of RN measures determined from the surrogate time series. The regular, chaotic, and resonant trajectories are marked by blue triangles in Figure 1(a) from left to right, respectively.

from the surrogates, one might observe significant difference and in this case the null-hypothesis can be rejected.

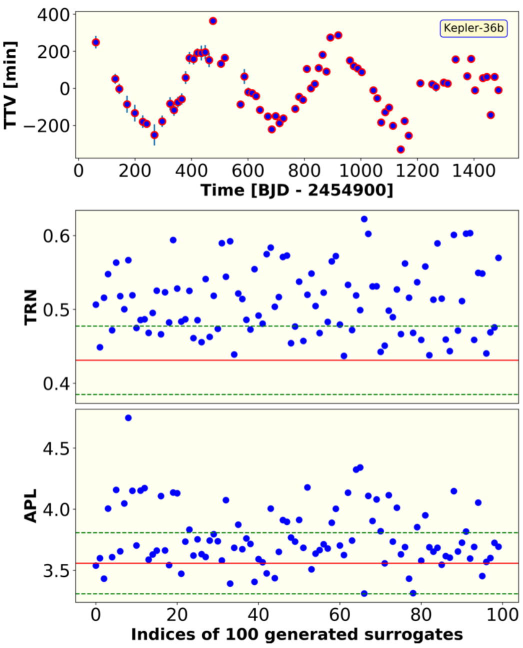

Figure 3: Hypothesis test based on the transit timing variation of Kepler-36b. Upper panel: TTV of Kepler-36b planet which is the the inner planetin the system with smaller mass ($0.0135 M_{\mathrm{Jupiter}}$) and consequently larger TTV amplitude, 72 measurements during 103 epochs yield 30% missing data points.

In order to distinguish chaotic and regular motion in dynamical systems we performed several significance tests based on Pseudo-Periodic Twin Surrogates (PPTS) combined with different observables and RN-measures (Figure 2). We used a one-sided static significance test based on 100 surrogates which yields 99% level.

The so-called noise radius (ρ) is a pivotal parameter in PPTS algorithm. Its value tunes the PPTS which means if ρ is too large the generated surrogate will be a sequence of random values. While, in contrast, if the value of ρ is too small, the produces time series is identical to the original one. What we need, therefore, is a method that gives a suitable noise radius which is not too large and not too small [2]. In one of our papers, we suggested to use the novel method of dynamic time warping (DTW) that is suitable to quantify the similarity of two time series. We found that small DTW characterizes minor difference between the original signal and the generated surrogate while relative large value reveals significant contrast between them. Thus, this method suits well to test various null-hypothesis and therefore it can be used as a part of stability analysis.

Our RN analysis shows that 950 data points either RV or TTV measurements (in case of Jupiter) are enough to carry out the stability investigation of the system. Next we want to apply the whole procedure to real exoplanetary systems that are known in the literature. As always, the amount and quality of the acquired data is extremely important, we try to analyse the best public data sets. Thus, we decide to use only space-based TTV signals, e.g. Kepler data, since the available RV measurements about two-planet systems contain a small number of data points and are very sparse in time.

Beside our own exoplanet observations [3] carried out on SMART telescopes in the US we also used cutting-edge space based measurements. After 17 quarters the Kepler satellite finished its original mission and collected more than 69,000 transits for 779 KOIs with high signal to noise ratio (SNR). The most interesting systems with significant long-term TTVs have been pilled up and published in a catalog (ftp://wise-ftp.tau.ac.il/ pub/tauttv/TTV/ver_112) in order to make the light curves more usable for further research. We limited the choice of possible systems to those presented in and their stability analysis can be found in literature for comparison.

As an example [2], the great success of the exploration of Kepler-36 is the emergence of stable chaos. As one can see, the original measure of TRN appears to be the lowest one in the rank based test middle panel of Figure 3. This means that the null-hypothesis can be rejected, i.e. the original signal comes with 99% level from chaotic dynamics rather than quasi-periodic.

3 Temporal chaos related to escape and collision

3.1 Ring system of dwarf planet Haumea

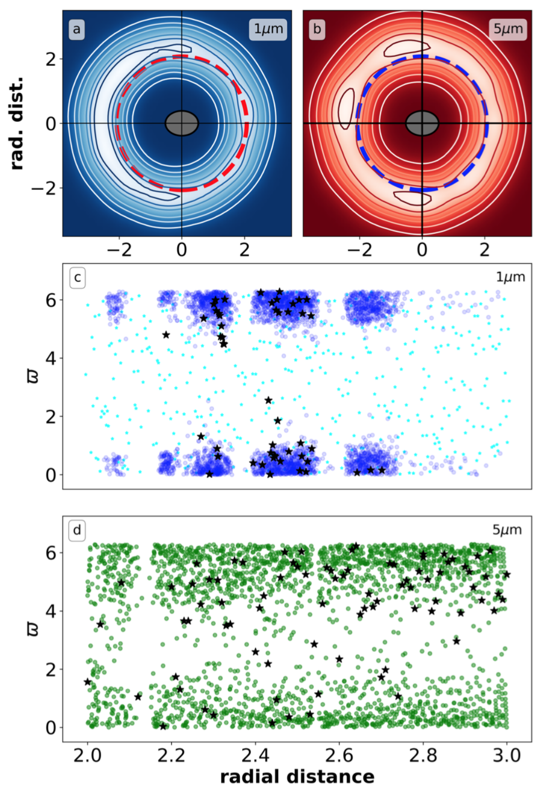

Figure 4: Top: Face-on view of the final stage of the integration. The red dashed line indicates the 3:1 spin-orbit resonance also guiding the eye to eccentric nature of the ring. The gray ellipse represents Haumea to scale. Bottom: The argument of pericentre after the integration (bounded motion only). In the case of $1\mu m$ particles the spin-orbit resonances are well-visible. The accumulation of points around zero suggests an apse-aligned ring. A similar structure has also been found for 5-micron grain size although the azimuthal interval is wider. Black stars are sampling the ring for 1000 year integration.

In the solar system the particles' orbit around a uniformly rotating triaxial object, is affected by various perturbation forces such as oblateness of planet, radiation pressure, tidal forces, or magnetic field. Consequently, the circumplanetary dust grains can have a reach dynamics fairly different from the Keplerian motion even in two-body case. The fate of the particles as well as the shape and structure of the ring depends strongly on the relative strength of these perturbations. We studied a simple dynamical model of the proposed ring system of Haumea a dwarf planet in the outer solar system and investigate the resulting structure under the dominant perturbations. Our results are consistent with the pioneering observations and shed light on the dynamics behind the observed structure of the ring.

Our results show that different grain sizes produce different ring structures: the smaller the size of the particle, the narrower the ring [4]. We also found that a significant apse-alignment cannot be achieved only by considering the oblateness of the planet. However, if the eccentricity is pumped up to a desired value ($\geq$0.1) the periapse ordering occurs. This job is done by the RP, the stronger the contribution of the radiation pressure, the more robust the apse-alignment. Moreover, particles in size of $1\mu m$, initially placed uniformly in wide radial dimension around Haumea, seemingly accumulate circularly near to the 3:1 spin-orbit resonance after surviving a reasonable amount of integration time. We can point out, although it is in good agreement with the proposed radial position of the ring, it is just a coincidence with the maximum accumulation of the particles being on sweeping, individual elliptic orbits trapped into different spin-orbit resonances. (Promotional paper: [7])

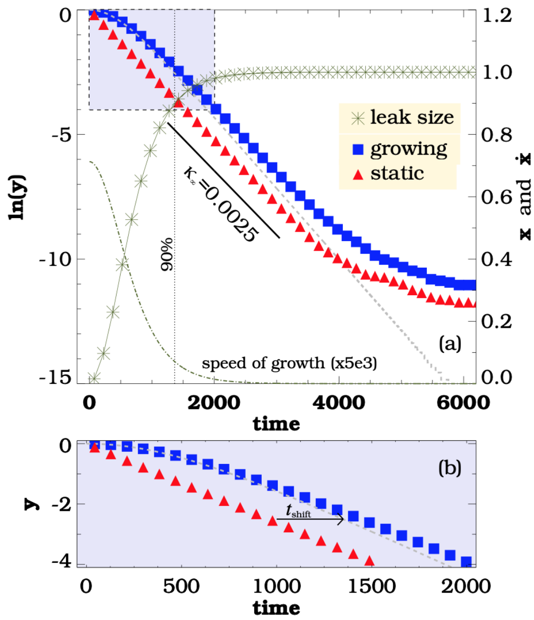

Figure 5: (a) Number of non-escaped particles and the leak size/growth vs. time for K = 2.7, γ = 1/2, m_i = 1. (b) Magnification during growing process.

3.2 Escape and collision in planetary systems

The motivation of the present study [5] is the so-called planetary accretion process which is one of the two competing planet formation scenarios in these days. In this process the forming planetary embryo accretes particles from its vicinity until this region – the feeding zone – becomes empty. The increase of the planet depends on the mass of the particles hitting its surface. Obviously the smaller the embryo at the beginning of this process, the more significant the growth by the ac- cretion. As a very simple model of this process one might consider the gravitational planar circular restricted three body problem (RTBP). In RTBP two point masses (star and planet) orbiting their barycenter on a circle and a third mass-less body (test particle) moves in their gravitational potential in the same plane. Although the planet (and also the star) is considered as a point mass, one can define the Hill radius (r_H) in which its gravitational influence is dominant. The particles entering the Hill radius with an appropriate velocity, i.e. slower than the escape velocity from this domain, can be removed from the dynamics and marked as escaped. In addition, r_H grows with the mass of the forming planet. Therefore, the growth of the planetary embryo can be considered as a growing leak in the phase space.

In addition to the numerical calculations, an analytic solution to the temporal behavior of the model is also derived. We show that in the early phase of the leak expansion, as long as there are enough particles in the system, the number of survivors deviates from the well-known exponential decay. Furthermore, the analytic solution returns the classical result in the limiting case when the number of particles does not affect the leak size [5].

Similarly to leaky chaotic systems we investigated the clearing of the chaotic zone of a giant planet embedded in a debris disc. N-body simulations were performed with a GPU-based high-precision Hermite direct integrator in 3D. We assumed that the debris material consists of collisionless bodies whose dynamics are affected only by the central star and gravitational perturbation of a 1.25–10 MJup giant planet. We modelled dynamically cold and hot discs assuming planetary eccentricity (e_{pl}) in the range of 0–0.9. The outer edge of the cavity was determined by fitting an ellipse to the millimetre wavelength [Fig. 6] (λ = 1.3 mm) thermal emission of the dust [6]. Based on our N-body simulations, we conclude that the chaotic zone and the cavity observed in debris discs are not identical, in general. Therefore, our new empirical formulae for the size of a giant planet carved cavity gives a more realistic values than previous estimations, especially for an eccentric giant planet.

4 Scientific impact and related activities

We have published 7 papers (including popular manuscript) supporting this grant. As of writing this summary we received ~10 [1] independent citations on the above publications. Concerning project-related activities, we note the following. The PI Tamás Kovács received the Bolyai Fellowship and UNKP-2018,2019 Bolyai+ Award. He gave numerous talks and seminars at various international conferences including the invited key note lecture at ICPST2019.

[1]Except for [6] and [7], numbered publications are the ones which acknowledge this grant (NKFIH PD-121223).

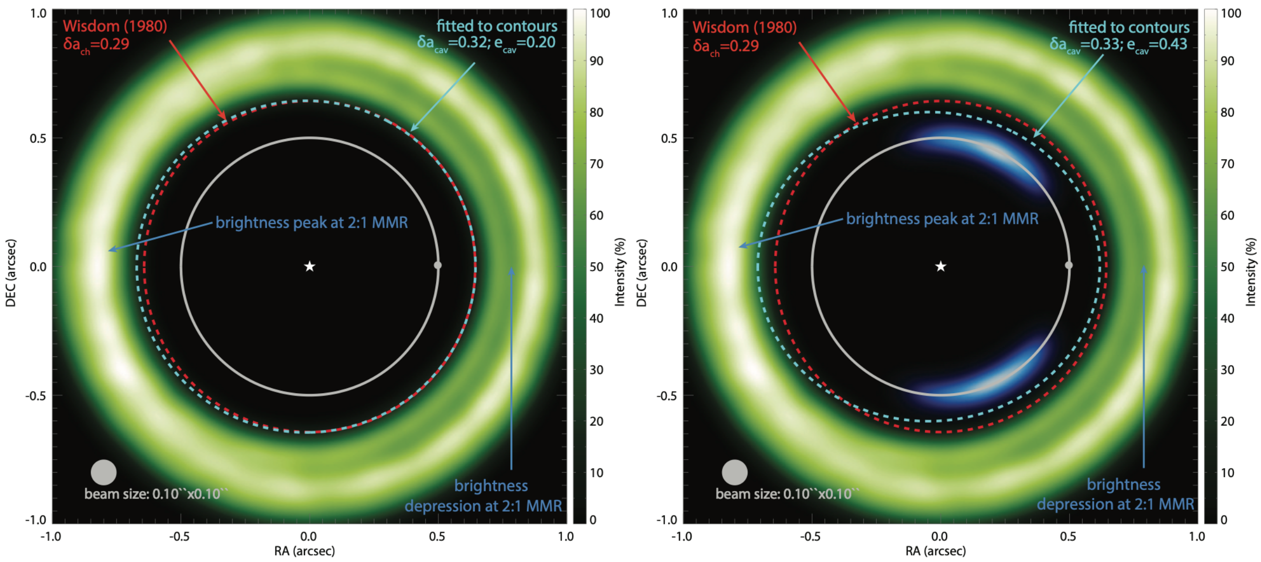

Figure 6: Calibration of the chaotic zone by fitting an ellipse to the dust emission at 1.4mm for $q = 5 \times 10^{-3}$ and e_{pl}=0 cold disc model. Dust particles orbiting in 1:1 (blue) MMR are neglected for the calibration process (left) and included for the general fitting procedure (right). Grey circle represents the planetary orbit. To emphasize particles trapped in 1:1 MMR their emission is artificially strengthened. Note that during the cavity fitting process no artificial intensity strengthening was applied.

References

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