Scientific summary

My proposal included several projects aimed at understanding different aspects and different methods of description of the phase structure of strongly interacting matter. Some of the goals of the proposed projects were far-reaching within the general topic, nevertheless connected methodologically by all relying on functional techniques and by the fundamental intent of finding the proper place of chiral effective models within the puzzle of strongly interacting matter.

I will summarize the research carried out reflecting on the original projects proposed, explaining and motivating differences compared to the research plan. In particular Project A was never finished. In short, because the method proposed turned out to be less effective as we hoped while other approximations of the same model are already plenty in the literature.

Project A

This project was aimed at describing the baryochemical potential (μ_B) temperature (T) phase diagram of strongly interacting matter in a very specific model and approximation. The model itself, the Polyakov loop extended quark-meson model, is fairly popular as a low energy effective theory derived from QCD (see e.g. [1] and references therein). We proposed to solve it in the two-loop truncation of the two-particle irreducible (2PI) description. A major factor in deciding against it was that in Project B we discovered certain limitations of the $O(4)$ symmetric ϕ^4 scalar model, which is the mesonic part of the quark-meson model. Firstly, using the model to describe the sigma-pion light meson system requires a parametrization which also sets a very restrictive triviality boundary, the scale of the Landau pole is around 400 MeV. This leaves a very small window for cutoff independence in the effective theory sense, raising questions about the reliability of the description of a phase transition around 150-200 MeV. Secondly, 2PI approximations were compared against lattice results in the $O(4)$ model and it turned out that they have severe limitations: loss of solution in some parameter regions and a limited capacity to reproduce lattice results, possible only at moderate values of the coupling. These two arguments were strong enough deterrents for us not to carry out the originally intended research project as results would be prone to large systematic errors compared to what one could consider an exact solution.

Other functional methods could also be considered, but there are many studies already looking at these possibilities, and therefore we concentrated our attention on other projects.

Project B

The aim of this project was to thoroughly investigate the thermodynamics of the $O(4)$ symmteric ϕ^4 scalar model using lattice Monte Carlo simulations. As a physical system we were interested in describing chiral symmetry breaking, therefore, parametrization was to be done based on the sigma-pion system. This system is also a good candidate for comparing results of continuum functional methods to exact ones obtained in the lattice simulations.

In [2] we obtained the line of constant physics (LCP) of the $O(4)$ symmteric ϕ^4 scalar model in lattice simulations based on the sigma mass (m_σ) to pion decay constant (f_π) and pion mass (m_π) to pion decay constant ratios. The measurments involve scanning the bare parameter space of the model and finding points where the above mentioned ratios are close to the physical and interpolation between these points to obtain estimated points of the LCP. This was particularly cumbersome due to the fact that the two observable ratios depend similarly on the bare parameters therefore errors were hard to keep at an acceptable level. The values of the bare coupling (g_0) as a function of the lattice spacing (a) along the LCP shows divergent behaviour at a non-vanishing a_{min}. This is a sign of the triviality of the model, as the bare coupling indeed has a Landau pole at $\Lambda_{\text{p}} \sim 1/a_{\text{min}}$, in other words the interacting version of the model has no true continuum limit. Using a fit motivated by second order perturbation theory we found $a_{\text{min}} = 0.52(2)$ fm. This is even more restricting than what one could expect based on a calculation by Lüscher and Weisz in the chiral limit [3] which predicts $a_{\text{min}}^{\text{LW}} = 0.40(4)$ fm.

One would expect from an effective, nevertheless perturbatively renormalizable model to express cutoff (Λ) independence in a region where $M \ll \Lambda \ll \Lambda_{\text{p}}$, where M is the largest relevant physical scale in the model (e.g. m_σ). We refer to such a scenario as the continuum limit in the effective sense. The main implications of such a low maximal energy scale ($\Lambda_{\text{p}} \approx 400$ MeV) is that continuum limit, even in the effective sense, is unlikely, especially for temperature dependent quantities. Furthermore the maximal temperature that can in principle be simulated on the lattice is again $1/a_{\text{min}}$ as the temperature is $T = (aN_T)^{-1}$, where N_T is the size of the lattice in the temporal direction and obviously has to be at least one.

Using the points of the LCP, a comparison of results obtained using continuum functional methods to exact lattice results is possible. We use the bare parameters as input values for several continuum approximations and compare the observables defining the LCP to their physical values. We considered three approximations; the two-loop and $\mathcal{O}(g_0^2)$ truncations of the 2PI formalism and the functional renormalization group (FRG) approach in the local potential approximation (LPA). In order to translate the bare parameters of the lattice simulations to continuum ones, we matched the chiral limit critical line of the model (which separates the broken and symmetric phase of the model at vanishing temperature) obtained in [4]. The two-loop 2PI can be matched in a moderate range of the coupling and gives $a\Lambda = 4.9$ (which agrees with the perturbative result). However, when plugging the bare parameters of the LCP into the 2PI equations using the conversion factor, we found that the solution is lost in a fashion described in [5]. The $\mathcal{O}(g_0^2)$ approximation already lost its solution when trying to determine the critical line, therefore no conversion factor was available and any further comparison was made impossible. The LPA-FRG however reproduces the critical line in a huge range of g_0 with the conversion factor $a\Lambda = 6.923$. Furthermore a comparison in the LCP points is also possible. For points where the negative bare mass squared m_0^2 is still larger (that is smaller in absolute value) than Λ^2 a direct FRG calculation is possible and shows that f_π and m_π are practically constants on the LCP however off by $5-10\%$, whereas m_σ is decreasing towards the would-be continuum limit but still $15-20\%$ larger than on the lattice. For points of the LCP with the smallest lattice spacings where $m_0^2/\Lambda^2 < -1$ the FRG equations develop a pole and need an analytic extension to solve. These results are somewhat dubious, as it involves an expansion that changes the functional form of the original differential equation, but still consistent with the behaviours found for the other points.

The results were published in a peer-reviewed journal article [2] and presented in conference [6] (preliminary results) and seminar talks [7].

Project C

The planned project was to develop lattice methods to build new effective models of strongly interacting matter based on expansions of the lattice effective action of quantum chromodynamics (QCD). The idea was to have an extended range of validity for these new models compared to usual chiral effective theories by including more terms, or matching to lattice QCD at different scales.

While this proved to be overly ambitious, a reformulated first step was carried out in [8] where we used magnetic field dependent baryon masses extracted from full lattice QCD simulations to increase the validity range of the Polyakov loop extended Nambu-Jona–Lasinio (PNJL) model when describing the magnetic field (B) temperature (T) phase diagram of strongly interacting matter. It has been shown by continuum extrapolated lattice simulations [9] that the pseudo critical temperature of the chiral phase transition (T_{pc}) decreases with increasing magnetic field. It was also found that the chiral condensate ($\langle\bar{\psi}\psi\rangle$) increases at low temperature as a function of B, while at higher temperatures this trend turns around and higher magnetic field values mean lower values of $\langle\bar{\psi}\psi\rangle$. The former phenomenon is called magnetic catalysis (MC) while the latter is called inverse magnetic catalysis (IMC). Usual chiral effective models cannot reproduce this behaviour (see e.g. [10, 11]), these models have MC at all temperatures and hence T_{pc} always increases with B . In recent papers (e.g. [12]) an *ad hoc* magnetic field dependence of the four-fermion coupling of the NJL model (G) was introduced which seems to correct the qualitative behaviour of the $T_{\mathrm{pc}}(B)$ curve when $G(B)$ is chosen appropriately. This gave us the idea that one could utilise $T = 0$ lattice data to determine the B-dependence of the coupling and see whether the real T_{pc} is reproduced.

While a direct measurement of the NJL coupling seems unlikely using lattice simulations, we deviced a reparametrization of the model at $T = 0$ for every B to define the B-dependence of G. This is a two step procedure, first, measuring the masses of the baryon octet on the lattice as functions of B and extrapolating these results to the continuum limit. Second, we extract constituent quark masses from the baryon masses by using a simple non-relativistic quark model, that takes into account the three light flavours. Then G can be tuned at every B such that these constituent quark masses are reproduced by the NJL calculation. This results in a decreasing $G(B)$ function, which in turn indeed lowers the pseudo critical temperature for increasing magnetic fields. Our results not only show qualitative agreement with the full lattice $T_{\rm pc}$, the lattice-improved model also shows IMC at high enough temperatures.

The detailed procedure was published in a peer-reviewed journal article in [8] and presented at several seminars with international audience [13, 14, 15].

Projects not included in the original research plan

During the 2016-2019 period I also worked on two projects, which were not included in the original research plan however fit well into the agenda of the proposal.

In the first project we studied using multipoint Padé approximants the analytic continuation of self-consistent Euclidean propagators obtained in the 2PI formalism. We showed that the zero momentum part of the self-energy differs only by a few percents from the pole mass, and that the continuation of a function by Padé series can reproduce complex analytic structures (poles and cuts). The first statement was to be used in Project B for the comparison of lattice results with 2PI results, while our experience with Padé approximants was used in the project next described. We published our results in [16].

In the second project we utilised the same method for analytic continuation based on multipoint Padé approximation to continue pseudo critical temperature values measured in QCD lattice simulations at imaginary baryochemical potential (μ_B). The continuation to real baryochemical potential is needed because direct lattice simulations at real μ_B are impossible due to the so-called sign problem. The numerical analytic continuation method was developed and tested using effective theory results, where both the imaginary and the real chemical potential results are known. Using the available lattice data points and the property that even and odd degree Padé approximants converge (with the number of points used) from different directions to the exact result we localised the region in which the $T_{\rm pc}(\mu_B)$ curve is expected for real values of the baryochemical potential. The project is however still incomplete and we will continue working on it.

Personal development

During the three years of the program I also participated in the life of the Department of Theoretical Physics at Eötvös University. In the years 2016-2018 I held the lectures "Differential equations in physics I-II (Differenciálegyenletek a fizikában I-II)". During the second half of the grant I started a very promising and already fruitful collaboration with Dr. Gergely Endrődi at the Goethe University of Frankfurt which we plan to continue.

References

[1] T. K. Herbst, "Polyakov-loops and the QCD phase structure," Ph.D. thesis (2012).

[2] G. Markó and Z. Szép, Phys. Rev. D **100**, 056017 (2019).

[3] M. Lüscher and P. Weisz, Phys. Lett. B **212** 472 (1988).

[4] M. Lüscher and P. Weisz, Nucl. Phys. B **318**, 705 (1989).

[5] G. Markó, U. Reinosa and Z. Szép, Phys. Rev. D **92**, 125035 (2015).

[6] G. Markó, *Linear sigma model on the lattice: a learning example*, ACHT meeting 2017, 20-22. 09. 2017., Zalakaros, Hungary.

[7] G. Markó, *Triviality bounds of the* $O(4)$ ϕ^4 *model as a theory of the sigma-pion system: lattice vs. continuum*, Goethe University of Frankfurt, Nuclear physics seminar, 28. 11. 2019., Frankfurt, Germany.

[8] G. Endrődi and G. Markó, JHEP **1908** 036 (2019).

[9] G. S. Bali, F. Bruckmann, G. Endrődi, Z. Fodor, S. D. Katz, S. Krieg, A. Schafer and K. K. Szabó, JHEP **1202**, 044 (2012).

[10] S. P. Klevansky, Rev. Mod. Phys. **64** 649 (1992).

[11] R. Gatto and M. Ruggieri, Phys. Rev. D **83** 034016 (2011).

[12] R. L. S. Farias, K. P. Gomes, G. I. Krein and M. B. Pinto, Phys. Rev. C **90**, 025203 (2014).

[13] G. Markó, *Magnetic field dependence of the NJL coupling from lattice QCD*, Goethe University of Frankfurt, Palaver seminar, 11. 02. 2019., Frankfurt, Germany

[14] G. Markó, *Magnetic field dependence of the NJL coupling from lattice QCD*, Eötvös University, Particle physics seminar, 27. 03. 2019., Budapest, Hungary

[15] G. Markó, *Magnetic field dependence of the NJL coupling from lattice QCD*, Goethe University of Frankfurt, Lattice group seminar, 15. 11. 2019., Frankfurt, Germany

[16] G. Markó, U. Reinosa and Z. Szép, Phys. Rev. D **96** 036002 (2017).

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Gergely Markó

Principal investigator
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