Final report for the NKFI project PD 120975

Hybrid quantum systems based on interacting quantum objects may have important applications in quantum sensing and quantum information processing. The prerequisite for such quantum interfaces is the precise control of the constituents, of their coupling, of their initial state, and the possibility to measure parts of their output. Atomic ensembles or single atoms can be coupled to another quantum system (e.g. a cavity or a current-carrying nanowire) through their interaction with the electromagnetic field of the other system. Previously, we considered the coupling between an ultracold atomic cloud (a Bose-Einstein condensate) and a mechanically oscillating nanowire and showed that in this hybrid system, the atoms can sense quantum features of the nanowire current noise spectrum [O Kálmán, et al., Nano letters **12**, 435 (2011)] and can amplify the mechanical oscillation [Z Darázs, et al., Phys. Rev. Lett. **112**, 133603 (2014)]. Furthermore, we showed that the Bose-Einstein condensate (BEC) can be used to measure magnetic noise in the radio-frequency range in general [O. Kálmán et al. Phys. Rev. A **94**, 033626 (2016)].

In this project, we investigated whether there are other frequency ranges where the magnetically trapped Bose-Einstein condensate of Rubidium atoms can be used for the same purpose, i.e., as a sensitive measurement device to sense features of another quantum system in a hybrid setup. We found that there are certain magnetic dipole transitions between the hyperfine levels of Rubidium in the microwave range (~ 6.8 Ghz) which make it possible to couple the BEC to a microwave cavity, e.g., a coplanar waveguide resonator (CPW). We considered the interaction of the BEC with the stationary microwave radiation field sustained by the CPW, in a geometry where the BEC is situated at the antinode of the amplitude of the magnetic field. We showed that the coupling between the two quantum systems allows for the measurement of the magnetic field of the resonator by means of counting the atoms that fall out of the condensate due to hyperfine transitions from the trapped state into a non-trapped atomic state. Using our previous results [O. Kálmán et al. Phys. Rev. A **94**, 033626 (2016)] for the three-dimensional description of the outcoupling process in the presence of gravity based on the Green's functions of the quantum mechanical free fall problem, we carried out a numerical determination of the number of outcoupled atoms below the cloud in a given detection volume. We showed that weak microwave fields at the single-photon level can be sensed with this scheme [1].

In the first year of the project, I extended my planned research on coupled quantum systems by the investigation of the nontrivial evolution of two two-level atoms resonantly interacting with an optical cavity. In a collaboration with the group of Prof. Gernot Alber at the Technical University of Darmstadt, we showed that when a post-selection scheme is applied that is based on measurement results obtained on parts of the system (more specifically, on the cavity field and on one of the atoms) then the initial state of the unmeasured atom is transformed nonlinearly. By taking pairs of atoms from an ensemble, and iteratively applying the scheme, the evolution can be described by the dynamical properties of complex quadratic rational functions. We showed that the dynamics is very sensitive to the initial state of the ensemble and the nonlinear dynamics can be diverse: in certain parameter ranges the transformation is chaotic for all initial states, while in others, there is convergence to some attractive fixed cycle(s). I found a case when the transformation can be used to make initially close quantum states nearly orthogonal in a few steps [2]. This operation may be considered a protocol for quantum state discrimination, an intensively studied subject of quantum information processing.

Focusing on the abstract description of the above mentioned probabilistic quantum evolution, I aimed at determining other nonlinear transformations that can lead to the same orthogonalizing effect after a few iterations already. I showed that there is a whole class of such transformations which may be implementable with a corresponding entangling unitary. We also found that a subclass of these transformations could be used to decide whether an unknown quantum state is in the predefined neighborhood of a reference state. This means that if the overlap of the unknown state with the reference state is larger than a given value, then the unknown state after a few steps of the protocol will get very close to the reference state (otherwise close to the state orthogonal to the reference state). Thereby, the unknown state can be matched with the reference state. I examined the protocol for noisy (i.e. mixed) initial states and I found that for small initial noise the protocol purifies the state and its performance is similar to the pure-state case [3].

I became interested in the question whether other nonlinear protocols could also preserve their characterisic pure-state performance for noisy initial states. We considered a special iterated nonlinear quantum protocol on qubits (the so-called Lattes-type of transformation), where all pure initial states on the Bloch sphere can be considered chaotic as the corresponding map does not have any attractive fixed cycle, and the dynamics is thus ergodic. We showed that initial noise radically changes this behavior, since the completely mixed state appears as an attractive fixed point of the dynamics induced by the protocol. Our numerical simulations indicated that initially mixed states all converge to the completely mixed state. This however, does not rule out the possibility of the presence of repelling fixed cycles in the case of the mixed-state dynamics. I came upon the idea to investigate this with the inverse (i.e., backwards iterated) map, which gives an exponentially growing number of points due to the multiple pre-images in every step. With this method, I gave a numerical proof that the dynamics converges to the maximally mixed state for any small initial noise, i.e., that chaoticity is destroyed by noise [4].

In our collaboration with the Czech Technical University in Prague, we found yet another type of behavior of a specific nonlinear transformation. In this case the characteristic features of the pure-state dynamics (including the chaotic regime) survive also for noisy initial inputs, however, when increasing the mixedness (or decreasing the purity) of the initial states, at a given initial purity they suddenly disappear, indicating a phase transition in the system. The dynamics is described by a fractal structure which appears at the border of the different convergence regions in the parameter space of the initial states. By determining the fractal dimension of the fractals corresponding to the convergence properties of the surfaces with different initial purity we showed that when decreasing the initial purity the fractal dimension remains constant up to a critical initial purity, where it suddenly drops as the border of the convergence regions becomes regular. We found that the value of the critical purity coincides with that of a repelling fixed point of the dynamics. My above mentioned idea to study backwards iterations to reveal complementer properties of the dynamics, such as repelling fixed cycles, proved to be a decisive tool. I proved that all pre-images of states from the close neighborhood of pure chaotic initial states have purity larger than the critical purity. I also noticed that the pre-images of these points converge stably to certain fixed points while coinciding with a type of border points of the fractal structures of purity lower than one. These pre-images which unstably converge to the pure chaotic states under the forward iteration of the map, can be considered as quasi-chaotic [5].

We proposed a probabilistic quantum protocol to realize a nonlinear transformation of qutrit states as well, which by iterative applications on ensembles can be used to distinguish two types of pure states. The protocol involves single-qutrit and two-qutrit unitary operations as well as post-selection according to the results obtained in intermediate measurements. We utilized the nonlinear transformation in an algorithm to identify a quantum state provided it belongs to an arbitrary known finite set. The algorithm is based on dividing the known set of states into two appropriately designed subsets which can be distinguished by the nonlinear protocol. In most cases this is accompanied by the application of some properly defined physical (unitary) operation on the unknown state. Then, by the application of the nonlinear protocol one can decide which of the two subsets the unknown state belongs to thus reducing the number of possible candidates. By iteratively continuing this procedure until a single possible candidate remains, one can identify the unknown state [6].

Recently, we have been collaborating with the experimental group of Prof. Peng Xue at Beijing Computational Science Research Center. They were able to experimentally realize for the first time one of our nonlinear quantum protocols on single-photon qubits with linear optical elements and appropriate measurements. The quantum nonlinearity in this case is induced by post-selecting the polarization qubit based on a measurement result obtained on the spatial degree of freedom of the single photon which plays the role of a second qubit. Initially, both qubits are prepared in the same quantum state and an appropriate two-qubit unitary transformation entangles them before the measurement on the spatial part. The result was analyzed by quantum state tomography on the polarization degree of freedom. We demonstrated the usefulness of the protocol for quantum state discrimination by iteratively applying it on either one of two slightly different quantum states which rapidly converge to different orthogonal states by the iterative dynamics [7]. Currently, we are working together on the photonic experimental realization of the Lattes-type of chaotic protocol which we investigated in [4].

Note: In late 2017, a proposal was published in Physical Review A [D. Xu and F. Xue, PRA **96**, 063813 (2017)] to use a BEC of ^{87}Rb atoms to cool the mechanical vibrations of a nanomechanical oscillator in a hybrid scheme, similar to a setup we considered before. I pointed out using our previous results [Z. Darázs et al, PRL **112**, 133603 (2014)] that the calculations in the paper were fundamentally wrong, as the direction of energy transfer was in opposite direction to what they assumed. I sent my findings in the form of a Comment to the journal. The authors as well as an independent referee agreed with my proof and admitted the error. The Comment was not published though, because the authors retracted the paper.

Publications in the reporting period:

[1] O. Kálmán, P. Domokos, *Sensing microwave photons with a Bose-Einstein condensate*, arXiv:1811.09459, submitted to Eur. Phys. J Quant. Technol.

[2] J. M. Torres, J. Z. Bernád, G. Alber, O. Kálmán, and T. Kiss, *Measurement-induced chaos and quantum state discrimination in an iterated Tavis-Cummings scheme*, Phys. Rev. A **95**, 023828 (2017).

[3] O. Kálmán, T. Kiss, *Quantum state matching of qubits via measurement-induced nonlinear transformations*, Phys. Rev. A **97**, 032125 (2018).

[4] O. Kálmán, T. Kiss, I. Jex, *Sensitivity to initial noise in measurement-induced nonlinear quantum dynamics*, J. Russ. Laser Res. **39**, 382-388 (2018).

[5] M. Malachov, I. Jex, O. Kálmán, T. Kiss, *Phase transition in iterated quantum protocols for noisy inputs*, Chaos **29**, 033107 (2019).

[6] P. V. Pyshkin, A. Gábris, O. Kálmán, I. Jex, T. Kiss, *Quantum state identification of qutrits via a nonlinear protocol*, J Russ. Laser Res. **39** (5), 456-464 (2018)

[7] G. Zhu, O. Kálmán, K. Wang, L. Xiao, D. Qu, X. Zhan, Z. Bian, T. Kiss, P Xue, *Experimental orthogonalization of highly overlapping quantum states with single photons*, arXiv:1910:10377, accepted for publication by Phys. Rev. A

Talks:

1. O. Kálmán: *Nemlineáris kvantumos protokollok kezdeti zajra való érzékenysége*, XXX. Magyar Fizikus Vándorgyűlés, Sopron, 22 August 2019.
2. O. Kálmán: *Measurement-induced nonlinear dynamics in quantum protocols*, Kvantumelektronika Szimpózium 2018, Budapesti Műszaki Egyetem, 15 June 2018.
3. O. Kálmán: *Measurement-induced nonlinear transformations for quantum information processing tasks*, RMI Seminar, Wigner RCP, Budapest, 13 April, 2018.
4. O. Kálmán: *Mérések által indukált nemlineáris transzformációk kvantuminformatikai alkalmazási lehetőségei*, SZFI Seminar, Wigner RCP, Budapest, 24 October, 2017.
5. O. Kálmán: *Mérések által indukált nemlineáris transzformációk kvantuminformatikai alkalmazási lehetőségei,* Seminar of Department of Theoretical Physics, University of Szeged, 21 September, 2017.
6. O. Kálmán: *Magnetic-noise-spectrum measurement by an atom laser in gravity*, Seminar of the Exotic Quantum Phases Group, Technical University of Budapest, 25 October 2016.

Posters:

1. *Sensitivity to initial noise in measurement-induced nonlinear quantum protocols*, 26th Central European Workshop on Quantum Optics, CEWQO 2019, Paderborn (Germany), June 3–7, 2019.
2. *Quantum state matching of qubits via measurement-induced nonlinear transformations*, 672. WE-Heraeus Seminar "Search and Problem Solving by Random Walks", Bad Honnef (Germany), May 28 - June 1, 2018.
3. *Measurement-induced chaos and quantum state discrimination in an iterated Tavis-Cummings scheme*, IQIS 2017 - 10th Italian Quantum Information Science conference, Florence (Italy), 12-15 Sept. 2017.
4. *Measurement-induced chaos and quantum state discrimination in an iterated Tavis-Cummings scheme*, Winter school on Complex Networks: From Classical to Quantum, Obergurgl, (Austria), 3-7 April 2017.
5. *Measurement-induced chaos and quantum state discrimination in an iterated Tavis-Cummings scheme,* Workshop of Quantum Simulation and Quantum Walks 2016, Prague, (Czech Republic), 17-20 November 2016.

Awards:

Gombás Pál award of the Roland Eötvös Physical Society (2019).