The present project is a straightforward continuation of our previous studies (OTKA K-73449, K-101490) focused on the systematic investigations of social dilemmas under different conditions within the framework of evolutionary games. Using the concepts and tools of statistical physics we have systematically analyzed a wide scale of mechanisms and phenomena which can support the maintenance of cooperative behavior in social systems where the individual rationality dictates the choice of selfish behavior resulting in the state called 'the tragedy of the community'. Additionally, we have continued the exploration of general features related to the compositions of elementary games determining the interactions among the participants of human societies, ecological and biological systems.

In the mathematical models of evolutionary games players are located on the sites of a lattice (or a more complex connectivity network). The income of each player depends on the strategies chosen by the focal player and her neighborhood. The evolution of strategy distribution can be controlled via various microscopic rules inspired by real life observations (e.g., stochastic imitation of a neighboring strategy or noisy version of unilateral best response). Furthermore, in the so-called co-evolutionary games the players can also modify their connections, personal features, they can inform each other, etc., as it frequently happens in real systems. In the simplest social dilemma situations the pair interaction can be described by two-strategy donation game, prisoner's dilemma or snowdrift games. For more complicated systems the players can choose additional strategies, representing punishments, voluntary participation, but other aspects, like memory or information transfer can also be considered. After a transient process the macroscopic behavior of these systems is generally characterized by the strategy portions depending on the payoff matrix, the topological features of connectivity structure(s), the evolutionary processes (including the noise level), additional personal features/capabilities and sometimes the initial state, too.

During the project we have identified several realistic behaviors which could be generally important to maintain a significant level of cooperation in the population. One of these is the introduction of tolerant strategy. Here, depending on the number of defectors within a group, a tolerant player can either cooperate in or abstain from a particular instance of the game. We show that the diversity of tolerance can give rise to synergistic effects, wherein players with a different threshold in terms of the tolerated number of defectors in a group compete most effectively against defection and default abstinence. Such synergistic associations can stabilize states of full cooperation where otherwise defection would dominate. Due to a complex pattern formation an intricate phase diagram was calculated which underlined the possible importance of strategy alliances.

The diversity of players was already identified as a key feature of evolving cooperation more than a decade ago. Since then there are still intensive research efforts to explore different aspects of the mentioned concept. For example in the traditional public goods game all players are involved in every available groups and the mutual benefit is shared among competing cooperator and defector strategies equally. But in real life situations not all groups are attractive enough to players to join. Within the framework of a modified protocol we demonstrated that a carefully chosen threshold to establish joint venture could efficiently improve the cooperation level even if the synergy factor would suggest a full defector state otherwise. Conceptually identical behavior was detected in a model where we allowed cooperator players to decide which neighboring group to prefer instead of supporting all available groups uniformly. Importantly, such a modification resulted in improvement only if cooperators select their most successful neighbor and focus external investments exclusively into the related group independently of the strategy of the leader player. The microscopic mechanism which is responsible for the mentioned effects is based on a blocking mechanism which affects the propagations of competing strategies in a biased way. Our results, which remain intact by using different interaction topologies, reveal that it could be beneficial to concentrate individual efforts to reach a higher global wellbeing.

Beside the large variety of strategies the way of strategy update could also be a crucial way to reach a desired solution. For example learning from a more successful partner is a frequently used working hypothesis. An alternative dynamical rule could be when the focal player prefers to follow the strategy choice of the majority in the local neighbourhood (conformity-driven update). We have introduced a co-evolutionary model in which we assumed that both strategy learning methods are present and compete for space. Our results reveal that the presence of payoff-driven strategy learning method becomes exclusive for high sucker's payoff and/or high temptation values that represent a snowdrift game dilemma situation. In general, however, the competition of the mentioned strategy learning methods could be useful to enlarge the parameter space where only cooperators prevail. This success of cooperation is based on the enforced coordination of cooperator players which reveals the benefit of the latter strategy. Interestingly, the payoff-based and the conformity-based cooperator players can form an effective alliance against defectors that can also extend the parameter space of full cooperator solution in the stag-hunt game region. A general conclusion of the mentioned work was to emphasize that the co-evolution of strategies and individual features such as learning method can provide novel type of pattern formation mechanism that cannot be observed in a static model, hence remains hidden in traditional models.

Global warming, or the depletion of our natural resources are fundamental problems which cannot be solved without high level of cooperation of participants. In a model we considered not only the fundamental dilemma of individual and collective benefits but also focused on their impacts on the environmental state. In general, there is a strong interdependence between individual actions and the actual shape of environment where a natural theoretical framework is a co-evolutionary model. Such approach recognizes the fact that even if our common-pool resources are partly renewable, they have limited growth capacities hence a depleted environment is unable to recover and reach a sustainable level again. This scenario would have a dramatic consequence on our whole society, therefore we should avoid it by punishing those who are not exercising restrain. By means of analytical and numerical evidences we pointed out that punishment alone may not necessarily be a powerful tool to maintain a healthy shape of environment for the benefit of future generations. Cooperator actors, who are believed to take care of present state of our environment, should also consider carefully the growth capacity of renewable resources.

In many realistic systems the coexistence of three or more strategies (called biodiversity in biological systems) is maintained by the presence of cyclic dominance that results in self-organizing patterns in the spatial models, global oscillations if the mean-field type behavior is supported by the model, or formation of competing strategy associations. These features had motivated the investigations for clarifying the effect of zealots or heterogeneity in the invasion rates on the maintenance of biodiversity. We observed that mobility jeopardizes biodiversity in the spatial rock-paper-scissors game if only a small fraction of links of the square lattice is randomly rewired. Furthermore, we demonstrated that zealots are very effective in taming the amplitude of oscillations that emerge due to mobility and/or interaction randomness, and this regardless of whether the later is quenched or annealed. Motivated by the fact that environment is often not uniform but rather spatially heterogeneous, we have studied the possible consequences on the local microscopic dynamics. We found that although heterogeneous species-specific invasion rates fail to have a noticeable impact on species coexistence, randomness in site-specific invasion rates successfully hinders the emergence of global oscillations and thus preserves biodiversity. In our other work we have observed burst events (avalanches in the spatial strategy distribution) caused by the paradoxical effect of the rock-paper-scissors type cyclic dominance if one of the strategies is supported by a self-dependent component corresponding to external field in physical systems.

It is worth stressing that the paradoxical phenomenon observed in rock-scissors-paper game, or in more general in extended Lotka-Volterra systems are more robust and may be observed in other kind of evolutionary game models where the cyclic dominance is induced differently. For example, the paradox effect in the former systems (that is lowering the invasion rate between a source and target species) promotes the growth of former population. Interestingly, conceptually similar effect can be observed in broad range of evolutionary game models where a reduced strategy learning activity will primarily elevate the stationary fraction of another strategy who is the virtual predator of the one with reduced learning activity. We note that the research of pattern formations based on cyclically dominated strategies is a very active field and it was our honor to review recent developments in a Perspective paper invited by the Chief Editor of EPL.

The classification of elementary games into four types (games with self- and cross-dependent payoffs, coordination, and games with cyclic dominance) indicates clearly that in the absence of cyclic dominance we can derive a potential (resembling the negative potential energy in physics). The corresponding multi-agent potential games evolve into the Boltzmann distribution if the so-called logit rule controls the evolution of strategy distribution. Using the standard approximations of statistical physics we could describe the order-disorder phase transitions - as a function of noise level - if the two-strategy coordination game is extended by (n-2) neutral strategies. When analyzing the linear combination of elementary coordination components we have found a system where the higher entropy (the larger number of microscopic states) drives the system into a social trap with reduced average payoff. It is found furthermore that a large portion of potential games represent social dilemmas caused by the presence of the anti-symmetric part of the self- and cross-dependent components. In our latest paper we have shown that the cross-dependent components modify neither the potential nor the macroscopic behavior whereas these terms give contribution to the average payoff that can easily be determined. In the light of this feature the social dilemma can occur if the optimum of cross-dependent terms is achieved for a pure strategy deviating from those preferred by the potential.

Recognizing the inherent relationship between the anti-symmetric part of the payoff matrix and the adjacency matrix of directed graphs we could derive some new quantities (e.g., the portion of hierarchical any cyclic components and the measure of anti-symmetry of the hierarchical components) that characterize the macroscopic feature of large directed networks.

These briefly described results were published in 42 research articles including the above mentioned review and another one published in Physics Reports. These papers are downloadable from the homepages of senior researchers (https://www.energia.mta.hu/~szabo/ and https://www.energia.mta.hu/~szolnoki/). Our works were also presented and discussed in several conferences. Naturally, we are still working on some results obtained during this project and writing further papers is in progress.

Finally we must note that the straightforward application of the important results of (evolutionary) game theory can be hardly detected in our social life and governance. At first glance this seems to be surprising because the mentioned applications would help us reach a higher level of well-being. Probably the lack of these efforts is partially prevented by the poor general knowledge of the fundamental concepts and mathematical background of game theory. To overcome this difficulty we believe that to disseminate recent developments for broader audience has paramount importance. For this goal we permanently collaborate with not just university students, but also work with secondary school students and their teachers. These efforts made us possible not just to publish mutual scientific papers with them but also a PhD thesis was also defended successfully based on these efforts. Our collaborators, who are worth to name, were Blahota, Bodó, Bunth, Danku, Hódsági, Király, Leitner, Sulyok, Szombati and Varga.