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1. Finite groups and their actions

Endre Szabó, in a joint work with N. Gill and L. Pyber, proved the following. Let G be a finite simple group of Lie type of bounded rank. If we have a series of subsets such that the product of their sizes is at least $|G|^c$, then we can choose conjugates of these subsets such that the product of these conjugates contains each element of G. This is a common generalization of two earlier results. Namely, Roger and Saxl proved the special case when $G = SL_n(q)$ and the subsets are conjugacy classes, and Gill, Pyber, Short and Szabó proved the special case when the subsets are all equal.

Endre Szabó, in a joint work with B. Csikós, I. Mundet, L. Pyber, gave an upper bound on the number of stabilizer subgroups of a finite p-group acting on a topological manifold - this result is a crucial step in the proof of the conjecture of Ghys. The result is obtained via studying the Borel-Serre spectral sequence calculating the equivariant cohomology groups with compact support, and using some interesting finite group theory.

2. Groups in algebraic geometry and number theory

Attila Guld has studied finite groups acting on a flag variety over a function field. He proved that these groups have an abelian subgroup of bounded index, i.e. the automorphism group of the flag variety has the Jordan property. This result is an interesting special case when one wants to characterize which birational automorphism groups have the Jordan property.

Attila Guld, in a series of two papers, has proved that finite subgroups of the birational automorphism group of a complex variety have a class two nilpotent subgroup of bounded index. This is a serious improvement on the earlier work of Prokhorov and Shramov, where they obtained a soluble subgroup of bounded index. Work of D. Szabó shows, that Guld's result is "best possible", each finite nilpotent group of class 2 apears in certain birational automorphism groups.

Endre Szabó, in a joint work with A. Pál, studied the fibration method over real functionfields. They proved the following. Let $\mathbb{R}(C)$ be the function field of a smooth, irreducible projective curve C over the real numbers. Let X be a smooth, projective, geometrically irreducible variety equipped with a dominant morphism f onto a smooth projective rational variety with a smooth generic fibre over $\mathbb{R}(C)$. Assume that the cohomological obstruction introduced by Colliot-Thélène is the only one to the local-global principle for rational points for the smooth fibres of f over $\mathbb{R}(C)$-valued points. Then the same holds for X, too. The proof adopts the fibration method similarly to Harpaz–Wittenberg.

Endre Szabó, in a joint work with A. Pál, studied the vanishing conjecture for n-fold Massey products. They proved that a stronger version of the vanishing conjecture holds for fields of virtual cohomological dimension at most 1. In the proof they use a theorem of Haran. They also prove the same for PpC fields, using results of Haran–Jarden.

3. Bordism groups an cobordism groups

The study of smooth maps with a prescribed set τ of allowed singularities, the so called τ-maps. is an important direction in differential topology. A natural equivalence relation among such maps is the τ-cobordism, i.e. cobordism with the same set of allowed singularities. The most important tool in the subject is the classifying space of τ-maps. Szűcs gave a construction of the lassifying space via block-gluing, and defined a key-fibration which connects the classifying space of τ-maps with the classifying space of τ'-maps, where τ' is a smaller set of allowed singularities. These are the main tools, but usually, for calculations, more geometric understanding is neccessary.

Tamás Terpai, jointly with A. Szűcs and Cs. Nagy, has developed a connection between the stable homotopy groups of spheres and the cobordism groups of codimension 1 Morin (corank 1) maps. The results are based on a more detailed description of both the block-glueing and the key fibration constructions of the classifying space of such maps as well as on relating this space to the classifying space of immersions.

4. Moduli space of Higgs bundles

The study of Hodge moduli spaces on curves is a topic of intense current interest in Complex Geometry. One intriguing direction in this field is the Algebraic Topology of the mentioned spaces. By the foundational results of N. Hitchin (in the case of compact curves) and C. Simpson (in the case of noncompact curves, endowed with parabolic structures) the moduli spaces carry two very different algebraic structures: one of them (called Dolbeault structure) arises when we view them as spaces of Higgs bundles up to gauge transformation, the other one (called Betti structure) occurs when we view them as representation varieties of the fundamental group of the underlying curve up to global conjugacy.

In the first part of the period of the grant, Szilárd Szabó and Péter Ivanics, with co-author A. Stipsicz, carried out a detailed analysis of the complex 2-dimensional Dolbeault moduli spaces where the underlying curve is the projective line and the rank of the Higgs bundles (equivalently, of the representations) is 2, allowing arbitrary irregular singularities. In the literature, these cases are referred to as the Painlevé cases. They gave a complete list of the singular fibers of the Hitchin fibration in all Painlevé cases except for the 5th one. In addition, they described the related wall-crossing phenomena, which explain the change in the spaces and singular fibers when the parameters of the spaces (parabolic weights) cross certain hyperplanes.

Both algebraic structures naturally endow the cohomology space of the moduli space with a different filtration by integers, namely the Dolbeault structure induces the so-called perverse Leray filtration and the Betti structure induces the weight filtration. The P=W conjecture states that these two filtrations agree. It is a hard conjecture because the relationship between these two algebraic structures is highly transcendental, making use of existence results for certain systems of nonlinear partial differential equation called the Hermite–Einstein equations (in the direction Dolbeault to Betti) and for harmonic metric equations (in the opposite direction).

In the second half of the period, based on the geometric understanding of the spaces obtained in the first part, Szilárd Szabó made progress on this conjecture. Namely, he proved it in all the Painlevé cases using a novel approach. The strategy of the proof was to employ recent powerful analytical results on the asymptotic decoupling of the Hermite–Einstein equations and the ensuing abelianization of its solutions pioneered by T. Mochizuki over the complement of the ramification locus and further generalized by R. Mazzeo and his co-authors near the ramification locus.