OTKA FINAL REPORT **Principal investigator: János Lógó**

Period: 1.10.2016-31.03.2022 **NKFI ID: 119440**

The title of the project: Selected problems in structural topology optimization: from basic theories to engineering applications

(Detailed report)

At first the statistical data is reported. During the research period (1 Oct 2016-31 March 2022) 39 journal papers and 34 conference presentations were performed. Several journal papers were published in high quality (D1, Q1) journals, such as Thin-Walled Structures, Advance Engineering Software, Computers and Structures, Meccanica, Computers & Structures, Engineering Structures, Structural and Multidisciplinary Optimization, Symmetry, Structures. The aggregate impact factor is **82,03**. The international recognition of this research can be mirrored by the number of the **independent citations**: **220** (MTMT2) or cc. **300** (Google). One doctoral dissertation of the Hungarian Academy of Science (Ádány S., 2019), one habilitation (Ádány, S. 2019), two PhD dissertations (Pintér, E, 2019, Haffar M. Z., 2022) were defended and additional two PhD dissertations are ready for the defense (home defenses performed successfully).
We elaborated 8 topics. The build-up of the research is rather horizontal than vertical, that is, not necessarily organized along a line but the tasks can be investigated more or less parallel. During the research new theories, analytical solutions and numerical confirmations, numerical methods, algorithms, and computer programs with test problems (benchmark) were developed. The results of deterministic and the stochastic models were compared and benchmark problems were elaborated. The exact list of the publications can be found after the short version of the final report. Here only some relevant publications are mentioned at the specific topic.

In brief the following selected results were achieved:

In topic (1) - **analytical solutions:**

(a) Superposition for Plastic Design

Analytical solution was derived for plastic stress-based optimization of truss-like structures with two, alternative loads of different magnitude and direction satisfying the least-weight optimality conditions.

It is found that for certain load cases the optimal plastic design became statically determinate, therefore it is also the optimal elastic design and the obtained result is in accordance with the Nagtegaal-Prager solutions, the conclusion is in agreement with Sved. (The detailed results can be found: Topology Optimization: Deterministic and Probabilistic Problems: Budapest University of Technology and Economics, Faculty of Civil Engineering, Department of Structural Mechanics, Vásárhelyi Pál Doktori Iskola 2019)

Two alternative point loads with magnitude and direction at the same point of application and a line support at a given distance from the point are given. In this case, if either the lines of action of the component loads both enclose angles less than $\pi/4$ with the line of support or both enclose angles less than $\pi/4$ with the line perpendicular to the line of support, then the optimal plastic truss design is statically determinate and therefore it is also the optimal elastic truss design.

Let $\mathbf{P}1$ and $\mathbf{P}2$ denote the two-point loads acting at the origin of the Cartesian coordinate system (x, y). Let the line support be parallel to axis y given at x = h, where h is a positive constant. The component loads $\mathbf{Q}1$ and $\mathbf{Q}2$ are calculated by the superposition principle as

$$\mathbf{Q}_1 = \frac{1}{\sqrt{2}}(\mathbf{P}_1 + \mathbf{P}_2),\ \mathbf{Q}_2 = \frac{1}{\sqrt{2}}(\mathbf{P}_1 - \mathbf{P}_2)$$

The regions are the same as those shown in Figure 1(a). The conditions are satisfied if the $\mathbf{Q}i$ component load vectors both lie either in the blue regions or both lie in the red regions. According to the single load case above, if both component loads enclose angles less than $\pi/4$ with axis x, then the individual optimal solutions comprise of a single bar each and hence the superposition is a 2-bar truss. On the other hand, if they both enclose angles less than $\pi/4$ with axis y, then the individual optimal solutions are 2-bar trusses with angles $\pm\pi/4$ and hence the superposition is also a 2-bar truss with angles $\pm\pi/4$. In both cases the optimal design is a 2-bar statically determinate truss. The obtained result is in accordance with the article of Nagtegaal and Prager.

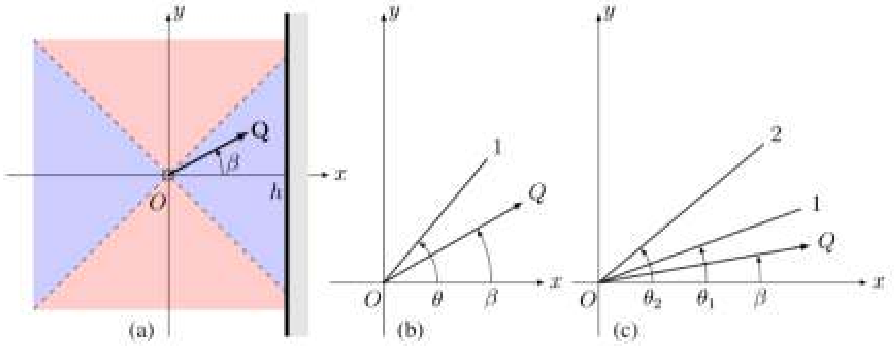

Figure 1: Truss design for a single point load. (a) Regions of the direction of the load. (b) Line of forces, single-bar truss. (c) Line of forces, 2-bar truss

(b) lateral-torsional buckling

In the field of lateral-torsional buckling of thin-walled beams we have derived analytical solutions for the geometrically nonlinear analysis with considering geometric imperfections. The new analytical results justify and explain some earlier observations from numerical (e.g. shell finite element) studies, e.g. the asymmetric nature of the bifurcation if the member cross-section is mono-symmetric.

Haffar M.Z., Ádány S.,: ***Analytical solutions for the GNI analysis for lateral-torsional buckling of thin-walled beams with doubly-symmetric and mono-symmetric cross-sections,***, Proceedings of the Coupled Instabilities in Metal Structures (CIMS 2021), Lódz, Poland, July 12-14, 2021. p. 8,, 2021

In topic (2) - **plated structures:**

A new way (Figures 4. and 5) was investigated for avoiding the development of checkerboard patterns in structural topology optimization, using an additional in-plane rotational freedom. The efficiency of a few, from the many existing formulations, with differing complexity are put into comparison, such as the standard 4-noded bilinear element, the Allman-type solution, the shell element from SAP2000 and finally an element constructed on the basis of micropolar theory. Since the emergence of checkerboarded regions is a general phenomenon, the optimization problem is as simple as possible, being a weight minimization with a compliance constraint, solved with the optimality criteria method and a FEM discretization of the design domain.

Algorithm 1

Initialization
$t_{\min} \leftarrow 1e-5$
$t_i \leftarrow t_0 \quad \forall i$
$C_0 \leftarrow 1.5\ \mathbf{u}^T\mathbf{K}\mathbf{u}$
$p \leftarrow 1.0$
$p_{\max} \leftarrow 3.0$
$N \leftarrow 0$ ▷ N : number of steps
$N_{\max} \leftarrow 150$ ▷ max. number of iterations

while (KKT conditions in (4) are not met) and ($N \leq N_{\max}$) **do**
  if N mod $5 = 0$ **then** $p = Min(p_{\max}, p + 0.2)$
  end if
  $t_i \leftarrow$ updated by (6) and (2d) $\quad \forall i$
  $\lambda \leftarrow$ updated by (5)
  update the state variables by equation (2c)
end while

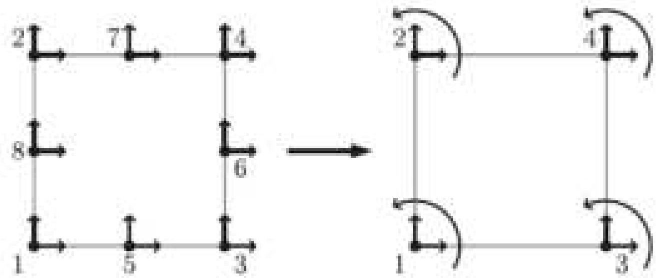

Fig. 4. Standard biquadratic 9-node element.

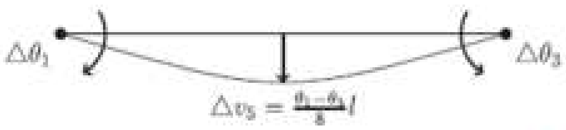

Fig. 5. The bottom side of a quadrilateral element and the midedge normal displacement from corner rotations.

B. Balogh, J. Lógó, The application of drilling degree of freedom to checkerboards in structural topology optimization, Advances in Engineering Software, Volume 107, 2017, Pages 7-12, ISSN 0965-9978, https://doi.org/10.1016/j.advengsoft.2017.02.005.

Optimization of cold-formed steel (CFS) members requires automatic design procedures, which cover a wide range of cases (e.g. geometries). The currently used procedures (e.g. by using the effective width approach) cover the design of typical cases, but have various limitations, and sometimes not easy to make them automatic. An important step toward automation of design is to use numerical method with general constraining ability, such as the constrained finite strip method (cFSM) or the recently introduced constrained finite element method (cFEM). The practical advantage of these methods is that the various deformation modes (such as global, distortional, local-plate, in-plane shear, etc.) can automatically be separated, which separation is necessary in the design procedures. The concept of such automated design procedure was presented in a paper.

Hoang T, Ádány S: ***On the use of constrained finite element method in the design of cold-formed steel Z purlins***, Proceedings of the Eurosteel 2017 conference, Sept 13–15, 2017, Copenhagen, Denmark. Ernst & Sohn Verlag für Architektur und technische Wissenschaften GmbH & Co. KG, Berl, 2017

At the time of starting this actual OTKA project, the cFSM and cFEM methods were already known, but with various limitations. During this research project the various limitations were addressed and solutions were proposed. The achieved results are briefly summarized as follows.

a) cFEM extension and application for members with holes

Thin-walled members are frequently produced with holes. For example, cold-formed steel studs and joists are provided with holes to accommodate electrical conduits and other services. In bridge construction, steel box girders commonly have access holes for inspection drilled into the flange or web. In airplanes, holes can also be found in numerous structural details, e.g. in the ribs attached to the main spar of an airplane wing, in order to reduce self-weight. Hence, a general design procedure must be able to handle members with holes. Since cFEM is essentially a shell finite element calculation, holes can easily be introduced, the real challenge was separation of the various behaviour modes. Two basic approaches for handling the holes were finally proposed. Moreover, it was also shown how general deformation modes (i.e. when regular shell finite element method is employed for the analysis of thin-walled members, without any specific constraints, and therefore the deformation components are not separated) can be identified, i.e. how the participations of the deformation components can be quantified. The case of so-called slotted web girders was separately discussed, and for this particular case an approximate analytical calculation was also proposed to calculate the flexural buckling.

Adany S: ***Non-uniform modal decomposition of thin-walled members by the constrained finite element method***, In: Proceedings of the Annual Stability Conference. Konferencia helye, ideje: San Antonio, United States, 2017.03.21-2017.03.24. San Antonio: Structural Stabi, 2017

Ádány S.: ***Constrained shell Finite Element Method for thin-walled members with holes***, Thin-Walled Structures, Vol 121, pp. 41-56., 2017

Ádány S: ***Modal identification of thin-walled members with and without holes by using CFEM***, Proceedings of the Eighth International Conference on Thin-Walled Structures, July 24-27, 2018. Lisbon, Portugal, 2018

Visy D., Ádány S., Geleji B., Szedlák M.: ***Flexural buckling of thin-walled lipped channel columns with slotted webs: Numerical and analytical studies***, Engineering Structures, Vol 197, 2019. pp. 1-12, 2019

Ádány S.: ***Modal identification of thin-walled members by using the constrained finite element method***, Thin-Walled Structures, , Vol 140, 2019, pp. 31-42, 2019

b) Rounded corners

An important practical appearance of thin-walled members is cold-formed steel (CFS) members. A particularity of cold-formed steel is that the members always have rounded corners due to the production technology. The effect of rounded corners cannot be neglected in practical situations. We have addressed the problem from two particular aspects: when the curved corner has a small radius, and when the curved part has large radius. In two publications we have discussed how cFSM can be extended to members with rounded corners (with small radius). The problem is that though the original cFSM/cFEM can technically be applied to members with rounded corners, if done so, the separation of local and distortional modes does not meet engineering expectations. We proposed to introduce a „corner element", which makes it possible to achieve practically acceptable constraint results, and we have completed extensive parametric studies to show how the cFSM could be applied in the practice for CFS members with rounded corners. The other aspect is the case of large radius. We have demonstrated that if the cross-section contains curved parts (with large radius), shell-like buckling is a potential failure mode, which can be critical in certain cases. Current design recommendations, however, disregard this failure mode. Since the currently available practical CFS members are produced with small radii, this is not a problem. However, it is known from the literature that formal optimization of CFS members frequently leads to cross-section shapes with large-radius curved parts, thus, this question needs to be further investigated in the future.

Jobbágy D, Ádány S: ***Local buckling behaviour of thin-walled members with curved cross-section parts***, Thin-Walled Structures 115: pp. 264-276., 2017

Beregszászi Z., Ádány S.: ***Modal buckling analysis of thin-walled members with rounded corners by using the constrained finite strip method with elastic corner elements***, Thin-Walled Structures, Vol 142, pp. 414-425, 2019

Beregszászi Z., Ádány S.: ***Method with Rigid Corner Element for the Buckling Analysis of Thin-Walled Members with Rounded Corners***, Periodica Polytechnica Civil Engineering, 63(1), pp. 192–205, 2019

c) Stiffened plates

Originally, cFSM and cFEM were developed and validated to thin-walled members, such as beams and columns. Within the reported OTKA project, we have shown that cFEM can meaningfully be applied to plate-like structures. As far as we know, this was the first time when a formal modal decomposition method was applied to any plate-like structure, hence, first we introduced the deformation modes of trapezoidal sheeting and longitudinally stiffened plates, then we have discussed in detail the design of longitudinally stiffened plates with closed, trapezoid-shape stiffeners. By utilizing the modal decomposition ability of cFEM, and by performing an extensive parametric study (advanced nonlinear shell finite element calculations) we have pointed out that the current European design recommendations for stiffened plates involve some inaccuracies. We have proposed several possible modifications. Particularly, we have proposed two new mathematical models for the capacity predictions.

Ádány S., Aldalaien Q.T.: ***Modal buckling analysis of trapezoidal sheeting***, Proceedings of the Annual Stability Conference, Structural Stability Research Council, St. Louis, Missouri, April 2-5, 2019, p12., 2019

Ádány S., Li Y.: ***Buckling of stiffened plates with modal decomposition***, 9th International Conference on Steel and Aluminium Structures (ICSAS19), Bradford, UK, 3-5, July, 2019. pp 445-456., 2019

Haffar M.Z, Kövesdi B., Ádány S.: ***Buckling of compressed plates with closed-section longitudinal stiffeners: Two new mathematical models for resistance prediction***, Structures, Vol 33, pp 3526-3539, 2021

Haffar M.Z., Ádány S., Kövesdi B.: ***A new approach for the design of plates with trapezoidal longitudinal stiffeners***, Proceedings of the 14th International Conference on Metal Structures, Poznan, Poland, June 16-18, 2021, pp. 500-506, doi: 10.1201/9781003132134-6, in Modern Trends in Res, 2021

Kövesdi B., Haffar M.Z, Ádány S.: ***Buckling resistance of longitudinally stiffened plates: Eurocode-based design for column-like and interactive behavior of plates with closed-section stiffeners***, Thin-Walled Structures, Volume 159, February, 107266, 2021

d) cFEM development – transverse plates

Further to longitudinal stiffeners, transverse stiffeners are also frequently applied in the practice for various thin-walled members. Moreover, other transverse plates might be present, such as end-plates or gusset plates. Therefore we investigated the effect of transverse plates on the buckling behaviour of thin-walled members, with a special focus on modal decomposition. Accordingly, we have extended the constrained finite element method (cFEM) to handle members with transverse plate elements, and we have shown – via several numerical examples – how the presence of end-plates and/or transverse stiffeners modify the modal behaviour of the members. As part of this investigation, in order to be able to validate the extended cFEM, we have derived analytical solutions for some basic cases of global buckling of members with transverse stiffeners. Both the analytical solutions and the numerical results show that transverse plates have important effect, sometimes even in cases (such as global buckling) when such effect is neglected by the current engineering practice.

Hoang T., Ádány S.: ***Torsional Buckling of Thin-Walled Columns with Transverse Stiffeners: Analytical Studies***, Periodica Polytechnica Civil Engineering, 64(2), pp. 370-386, 2020

Hoang T., Ádány S.: ***The effect of transverse stiffeners on the torsional buckling of thin-walled columns***, Proceedings of the Annual Stability Conference, Structural Stability Research Council, Atlanta, Georgia, April 21-24, 2020. p 16., 2020

Hoang T., Ádány S.: ***Constrained finite element method for thin-walled members with transverse stiffeners and end-plates***, Thin-Walled Structures, Volume 159, February, 107273, 2021

Hoang T., Ádány S.: ***The effect of transverse stiffeners and end-plates on the lateral-torsional buckling of beams***, Proceedings of the Annual Stability Conference, Structural Stability Research Council, Louisville, Kentucky,USA, April 13-16, 2021. p 16., 2021

e) Other topics

During the reported period of the OTKA project, we started to investigate some further related topics. From these topics we have already published some preliminary results, and further results are planned to be presented in the following years.

Another sub-topic is the investigation of the transverse membrane deformations of thin-walled members. These deformations are small, and sometimes thought to be negligible, but formal modal decomposition highlighted their importance. We have proposed new "transverse extension modes" to be used in cFSM and cFEM. We have shown that the proposed new modes are practically beneficial, since they make it possible to easily reproduce beam-model-based classic buckling solutions, or they make it possible to easily handle the barrel-type buckling mode of hollow section members.

Finally, we have started a major extension of cFEM, by introducing the "cFEM with mapping" idea. Even though the existing cFEM is more flexible modal decomposition method than many of its alternatives, it has certain limitations due to mainly two specialties: cFEM requires a highly regular rectangular finite element mesh, and cFEM uses one specific shell finite element based on Kirchhoff-Love plate theory. In order to release the limitation associated with the above particularities, we have proposed "cFEMmap" method, which is based on the mapping of the displacements. It makes possible to perform modal decomposition with various shell finite elements (e.g. non-rectangular ones, potentially with considering trough-thickness shear deformations) and on thin-walled members with irregular geometries (e.g. members with holes, tapered members, curved members).

Haffar M.Z., Ádány S.: ***Analytical solutions for the GNI analysis for lateral-torsional buckling of thin-walled beams with doubly-symmetric and mono-symmetric cross-sections***, Proceedings of the Coupled Instabilities in Metal Structures (CIMS 2021), Lódz, Poland, July 12-14, 2021. p. 8, 2021

Hoang T., Ádány S.: ***New transverse extension modes for the constrained finite strip analysis of thin-walled members***, Proceedings of the Coupled Instabilities in Metal Structures (CIMS 2021), Lódz, Poland, July 12-14, 2021. p. 8, 2021

In topic (3) - **inflated membranes:**

A Dynamic Relaxation based numerical procedure was developed to analyze air-inflated structures. Three air-inflated structures, a beam, a single arch, and a structure composed of four arches, were analysed at different membrane material properties (different moduli of elasticity) and different internal pressure values. The load-displacement curves, the load-bearing capacity, the wrinkling load (the smallest load that results in zero membrane force at least in one finite element in one direction), and the distribution of the membrane forces were determined for several cases. The results show that below the wrinkling load, the load-displacement curves are close to linear and almost independent of the internal pressure. Above the wrinkling load, the gradient of the load-displacement curves decreases radically. The relationship between the internal pressure and the load-bearing capacity is close to linear. The effect of the modulus of elasticity on the load-bearing capacity is negligible; however, the modulus of elasticity has a significant impact on the displacements. The relationship between the modulus of elasticity and the displacements is nonlinear.

Since the building codes do not provide the pressure coefficients of membrane structures, normally, they are determined by the help of wind tunnel tests or CFD (Computational Fluid Dynamics) calculations. In the current project, the pressure coefficients of a structure composed of six air-inflated arches and a mast-supported tensile structure were determined through wind tunnel test and CFD analysis. The pressure coefficient fields were presented in detail.

The climate change results in higher flood levels, which means that in many cases, the permanent embankments are no longer sufficient. There is an increasing claim for temporary, removable structures to increase the height of the embankment systems. A mobile flood protection device, which can be used as an independent flood protection system or as an extension of a permanent embankment, was analysed with the procedure developed for inflated structures. The two main structural parts of the analyzed flood protection device are the water-filled membrane tube or bag made of PVC coated polyester and the hollow steel section supporting frame. The main advantages of the structure are light weight, good mobility, easy and quick installation. The equilibrium shape of the filled membrane tube, the membrane forces, the contact forces between the membrane and the steel frame, and the reaction forces are calculated and compared for different internal and external water levels. The overall stability of the whole structure was also analysed.

In topic (4) - **woven coverings of polyhedral surfaces:**

A convex polyhedron P covered by a two-way two-fold orthogonal weaving of uniform strands has to meet some metric and topological conditions: any vertices in a development of P on a flat surface must be located at gridpoints of a suitably chosen uniform rectangular grid and must have at most eight vertices. As a consequence, not all 257 combinatorial cases of the edge structure can be realized as woven polyhedra, which naturally gives rise to the following question: is there any weavable polyedron in a given combinatorial class? A partial answer is obtained by a numeric simulation of embedding 2D developments into the 3D space: edge graphs of eight-vertex convex polyhedra can always be obtained as duals of octahedrite graphs which can systematically be generated for any number of unit square tiles in the weaving; once the octahedrite dual is given, an iterative method is applied for finding the only convex realization of any given developments. Numeric results for octahedrite duals with a surface not larger than 30 units (1003 cases) show that the number of triangulated polyhedra (which is generically obtained from an octahedrite dual) in the corresponding 14 combinatorial classes are of highly uneven distribution. On the other hand, polyhedra with one or more non-triangular faces appear typically because of symmetry; the smallest non-generic and nonsymmetric weavable polyhedron has a surface of 28 units.

In topic (5) - **topological optimization of pattern formation of the receptacle of lotus:**

The research program aiming the mechanical model for the optimal configuration of lotus receptacles had limited results during the project period, however it initiated successful program in a related field. According to the initial plan, based on the few hundred available configuration an algorithm was derived, which created the input data for the Surface Evolver program, which allows the calculation of shape-finding problems dry foams. To incorporate the variable width of the walls a material model was prepared, but initial tests were showing that the applied numerical method had convergence issues with some configurations. Deeper analysis did not reveal the numerical reason behind the issue, so we could not overcome that problem during the project period. The formation of the receptacle structure is a slow dynamical process. While our method aims to handle the result as a static state, it is worth to analyse the dynamics of curved linear segments. A real life example for this type of problems is the mechanical vibration of masonry arches. Load bearing capacity of historic buildings with damaged masonry arches is an important question, which must be answered before their restoration process. One possible way of the analysis requires the measurement of the free vibration of the arch, so finding the normal modes is a crucial step in the decision of the positioning of the instruments. We presented on two mechanical models (a quasi-continuous and a discrete one) the typical behaviour of the arches and the key parameters affecting the vibration in the symmetric modes. Beside the stuck topic, we included in our analysis the piecewise linear elastic civil engineering structures, where the nonlinear nature of the vibration could be damped under certain circumstances, when we search for periodic paths only. Even the periodic vibrations of these structures shows energy loss during the numerical calculation, especially, if the modal analysis uses only a reduced number of modes. Characterization of this energy loss as a damping was performed in the project. For the characterization of the limits of this loss a set of eigenvalue problems were formulated and numerical tests were performed to obtain free vibration periods. Furthermore, a theoretical extension of the cable-stayed beam was introduced with continuous support. The parameter-dependent vibration and buckling modes were derived and semi-analytically calculated for this piecewise linear elastic structure-family.

In topic (6) - **Optimization in morphology of polyhedra:**

We have published results of optimization in morphology of polyhedra. In papers by Tarnai's and co-authors a new optimization method was developed for the isoperimetric problem in the space and optimality of various polyhedra were proven or conjectured for solution of the problem. In addition to it a new way of visualizing compatibility path in four-dimensional parameter space was shown and effectively applied for an antiprismatic mechanism as an example.

Morphology and combinatorics of spherical polyhedra was also studied with focus on historical aspects. The truncated icosahedron and networks based on its tessellation form the bases of structural design from geodesic domes through pneumatic structures to radomes or soccer balls. The authors have identified the occurrence of a ball design based on the truncated icosahedron in archive video footages shot in Germany in the late 1930s preceding the first ever known such design by several decades. The colouring of the ball indicates mathematical knowledge of the designer. The authors identified flaws in the scheme. The icosahedral symmetry of the polyhedron allows a fivefold rotation around all pentagonal faces suggesting a possible colouring with five different colours. The authors investigated the coloring schemes and proposed ways for combinatorically uniform colouring. One possible arrangement is based on the fitting of a cube into a dodecahedron in order to attribute colours to the hexagonal faces of the truncated icosahedron. This scheme complies with the fivefold rotational symmetry though the distance between identical colours is not unique. Another possible colouring is proposed by the concept of the compound of five tetrahedra, which features five tetrahedra of different colour arranged in a rotationally symmetric way fitted into the dodecahedron, and thus perfectly complying with the set rules of uniformity. It has been shown that four versions of this scheme exist with respect to the permutation of colours and handedness.

Further to the topology of ball design, we have studied truncated icosahedra as the basis of the 32-panel ball design from optimization point of view. Flat panel balls are modelled as thin membranes subject to internal pressure. Unlike most tensile structures such as tents, balls form closed surfaces, which are expected to comply strict conditions, most importantly regarding geometry. In this research we performed a parametric study on a series of ball configurations, including a few commercially available ball types. The roundness and the stress distribution are numerically evaluated and compared in terms of the model parameters and the pressure.

As a natural extension of Archimedean polyhedra, we also investigated semi-regular Archimedean tilings as possible models of equiauxetic (i.e., with Poisson's ratio -1) tissues. The considered bar-and-body frameworks are periodic assemblies with single or double links between polygonal units. Finite mechanisms for both models were identified for all tilings, as well as the inexistence of fully symmetric state of self-stress has been proven for single-link models.

In topic (7) - **probabilistic topology optimization with very large problem size, correlated parameters and reliability constraints:**

(a) general concept

A novel approach was elaborated to development of optimality criteria based finite element code for topology optimization of elasto-plastic structures. The novelty of this work is related to the concept of function object called functor and its application to efficient FE code development. First, the general problem of topology optimization under stress constraints is briefly formulated (Figure 6).

Then, the programming aspects of topology optimization using traditional object-oriented and functor based programming are discussed. The advantages of the functor based approach (Figure 7) are related to simplicity of designing the FE code architecture and reusability of this code. In particular, the metric known as 'Lack of cohesion of methods' is useful in comparing these two different paradigms. Finally, the paper is also illustrated with numerical examples of topology optimization using the proposed methodology.

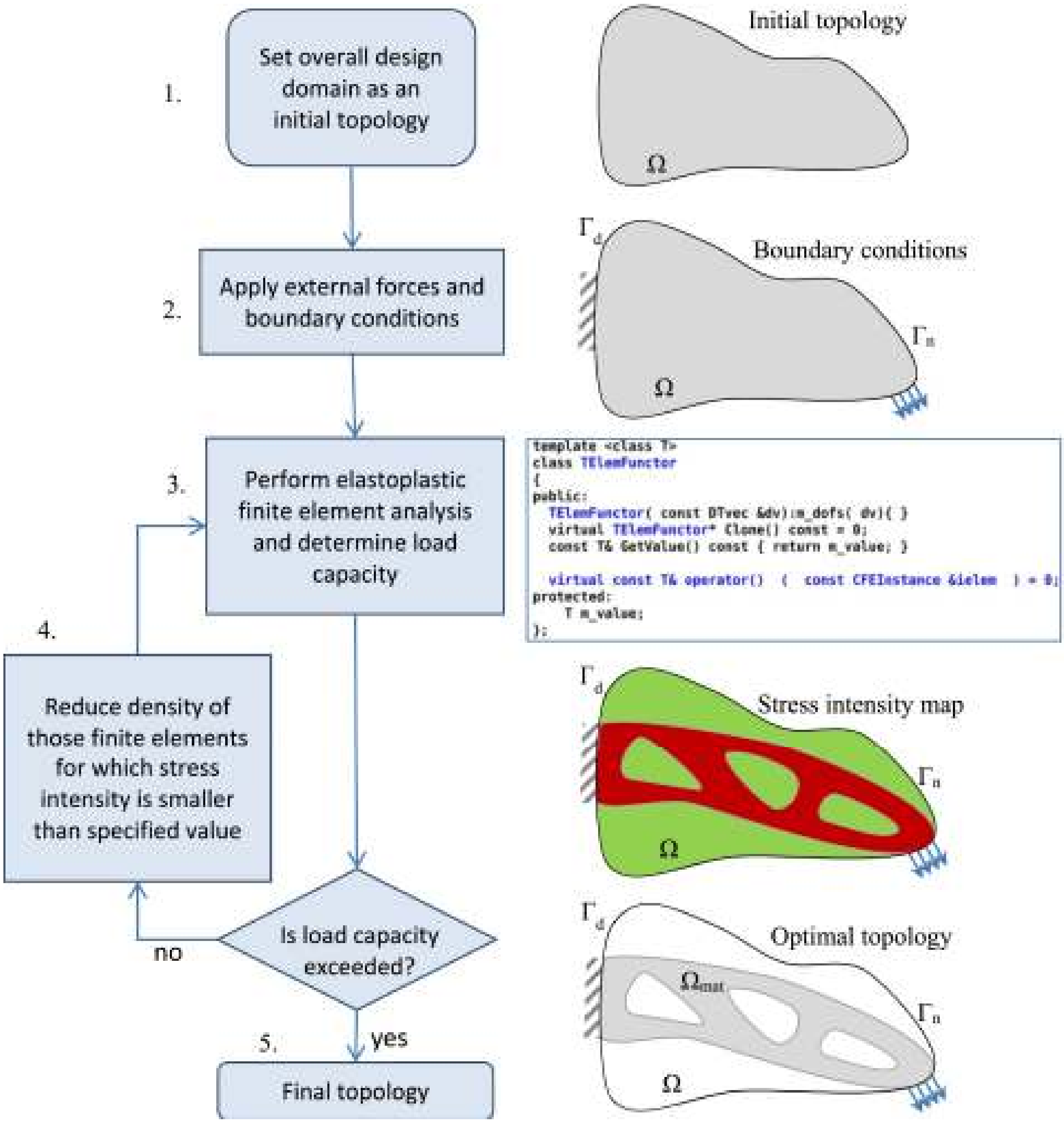

Figure 6. Computational steps in structural topology optimization.

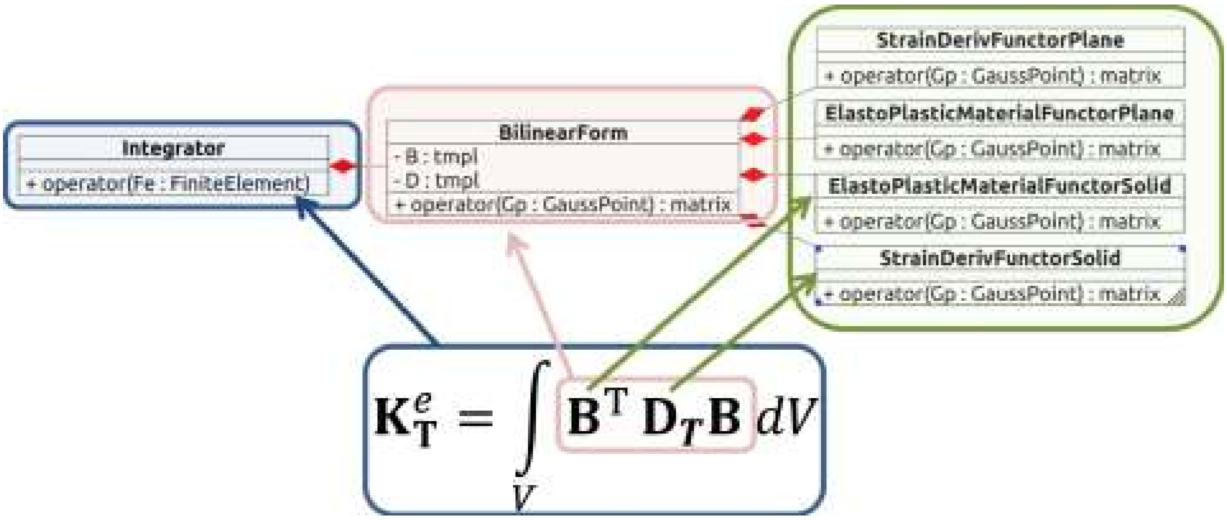

Figure 7. UML scheme of the functor oriented FE code architecture for element matrix computations

P. Tauzowski, B. Blachowski, J. Lógó, ***Functor-oriented topology optimization of elasto-plastic structures***, Advances in Engineering Software, Volume 135, 2019, 102690, ISSN 0965-9978, https://doi.org/10.1016/j.advengsoft.2019.102690. (https://www.sciencedirect.com/science/article/pii/S0965997818311426)

(b) plastic solution with stress constraints

A study is devoted to a novel method for topology optimization of elastoplastic structures subjected to stress constraints (Figure 8). It should be noted that in spite of the classical solutions of the different type of elastoplastic topology problems are more than 70 years old, the integration of the Prandtl-Reuss constitutive equations into the topology optimization process is not very often investigated in the last three decades. In the presented methodology where the classical variational principles of plasticity and the functor-oriented programming technique are applied in topology design, the aim is to find a minimum weight structure which is able to carry a given load, fulfills the allowable stress limit, and is made of a linearly elastic, perfectly plastic material. The optimal structure is found in an iterative way using only a stress intensity distribution and a return mapping algorithm. The method determines representative stresses at every Gaussian point, averages them inside every finite element using the von Mises yield criterion, and removes material proportionally to the stress intensities in individual finite elements.

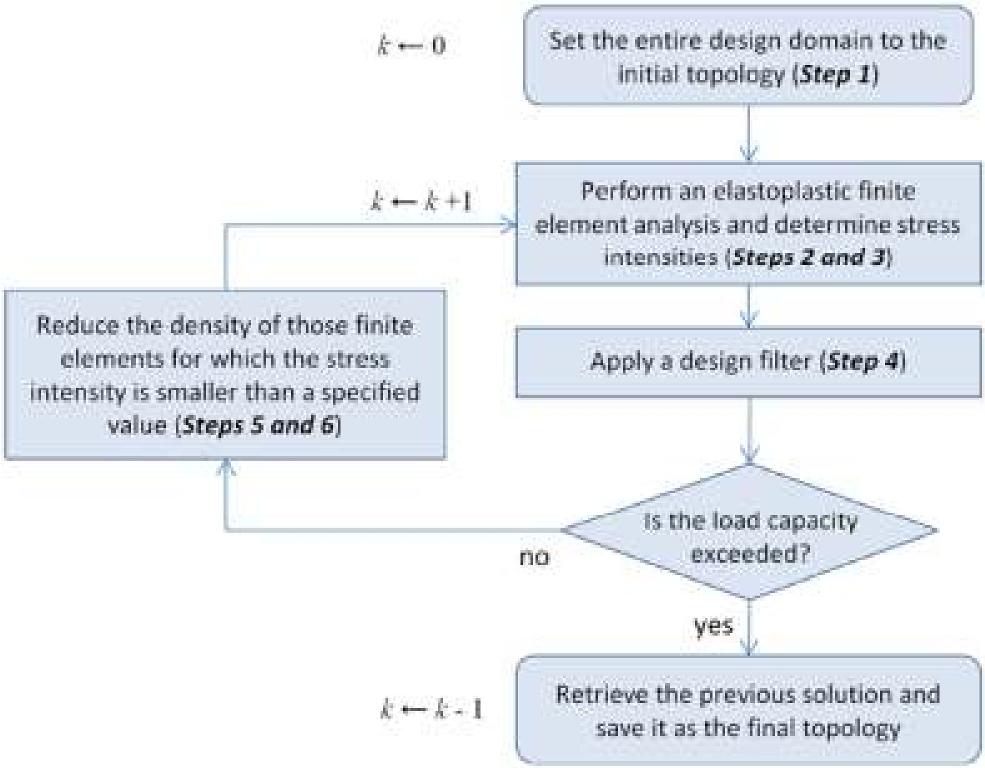

Figure 8. Flowchart of the proposed stress intensity driven topology optimization

The procedure is repeated until the limit load capacity is exceeded under a given loading. The effectiveness of the methodology is illustrated with three numerical examples. Additionally, different topologies are presented for a purely elastic and an elastoplastic material, respectively. It is also demonstrated that the proposed method is able to find the optimal elastoplastic topology for a problem with a computational mesh of the order of tens of thousands of finite elements. (Figure 9)

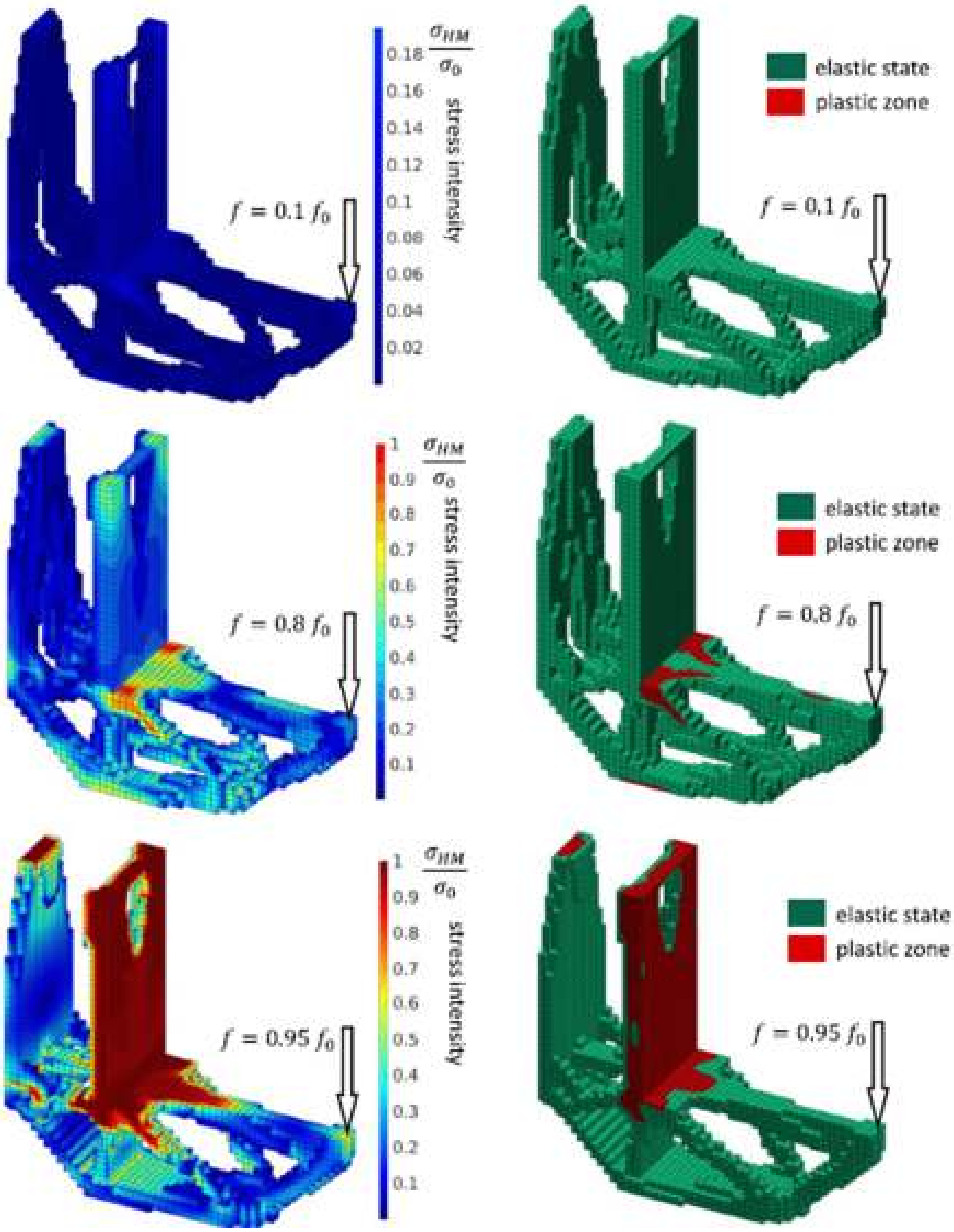

Figure 9. Optimal topologies

(c) reliability based plastic design

The objective of this study is to propose a relatively simple and efficient method for reliability based topology optimization for structures made of elasto-plastic material (Figure 10). The process of determining the optimal topology of elastic-perfectly plastic structures is associated with the removal of material from the structure. Such a process leads to weakening of structural strength and stiffness causing at the same time the increase the likelihood of structural failure. An important aspect of engineering design is to track this probability during the optimization process and not allow the structure safety to exceed a certain level specified by the designer. The purpose of this work is to combine the previously developed yield-limited topology optimization method with reliability analysis using first order approach. Effectiveness of the proposed methodology is demonstrated on benchmark problems proposed by Rozvany and Maute, and the elasto-plastic topology design of L-shape structure which is frequently used in different approaches for stress constrained topology optimization.

Piotr Tauzowski, Bartlomiej Blachowski, János Lógó, ***Topology optimization of elasto-plastic structures under reliability constraints: A first order approach***, Computers & Structures, Volume 243, 2021, 106406, ISSN 0045-7949,

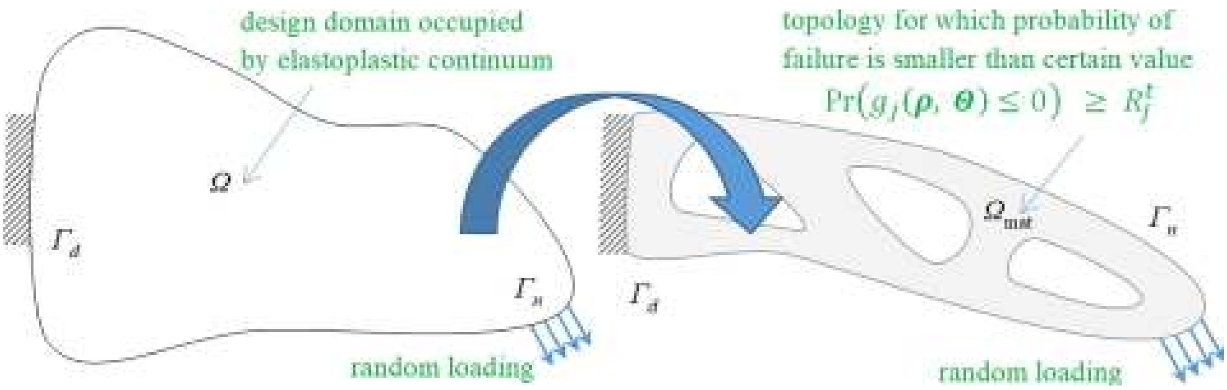

Figure 10. Reliability-based topology optimization of elastoplastic structure.

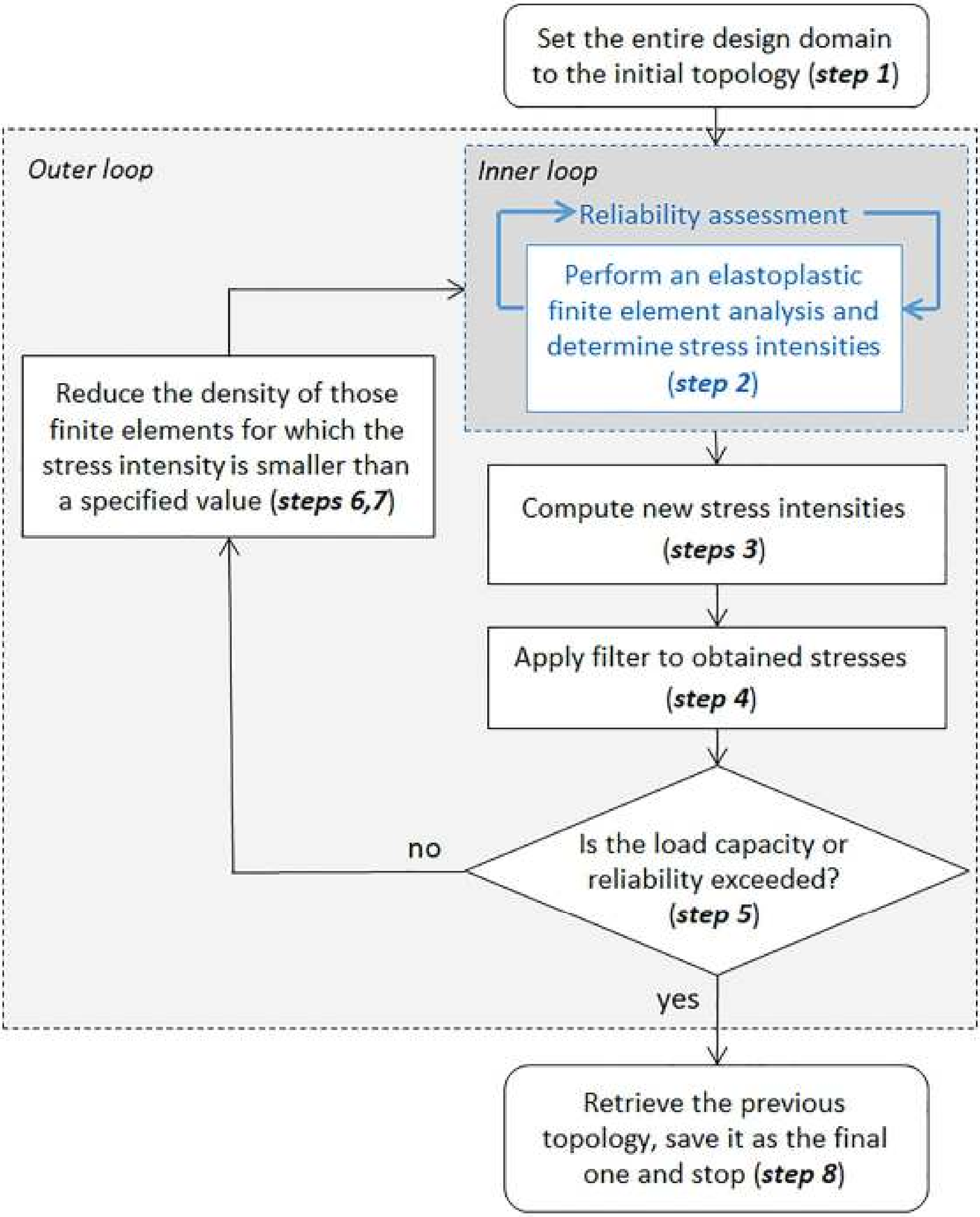

Figure 11. Algorithm

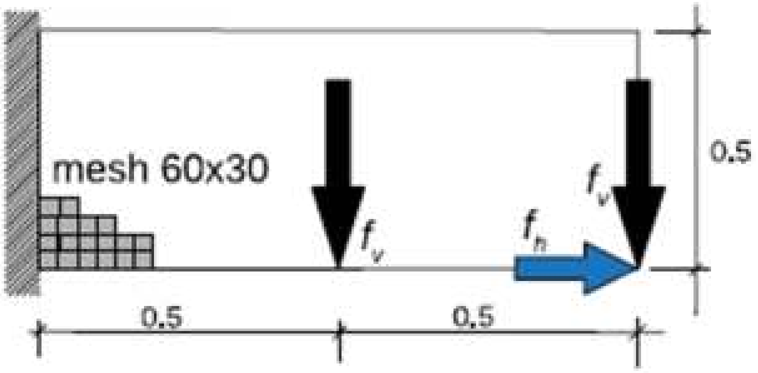

Figure 12. example

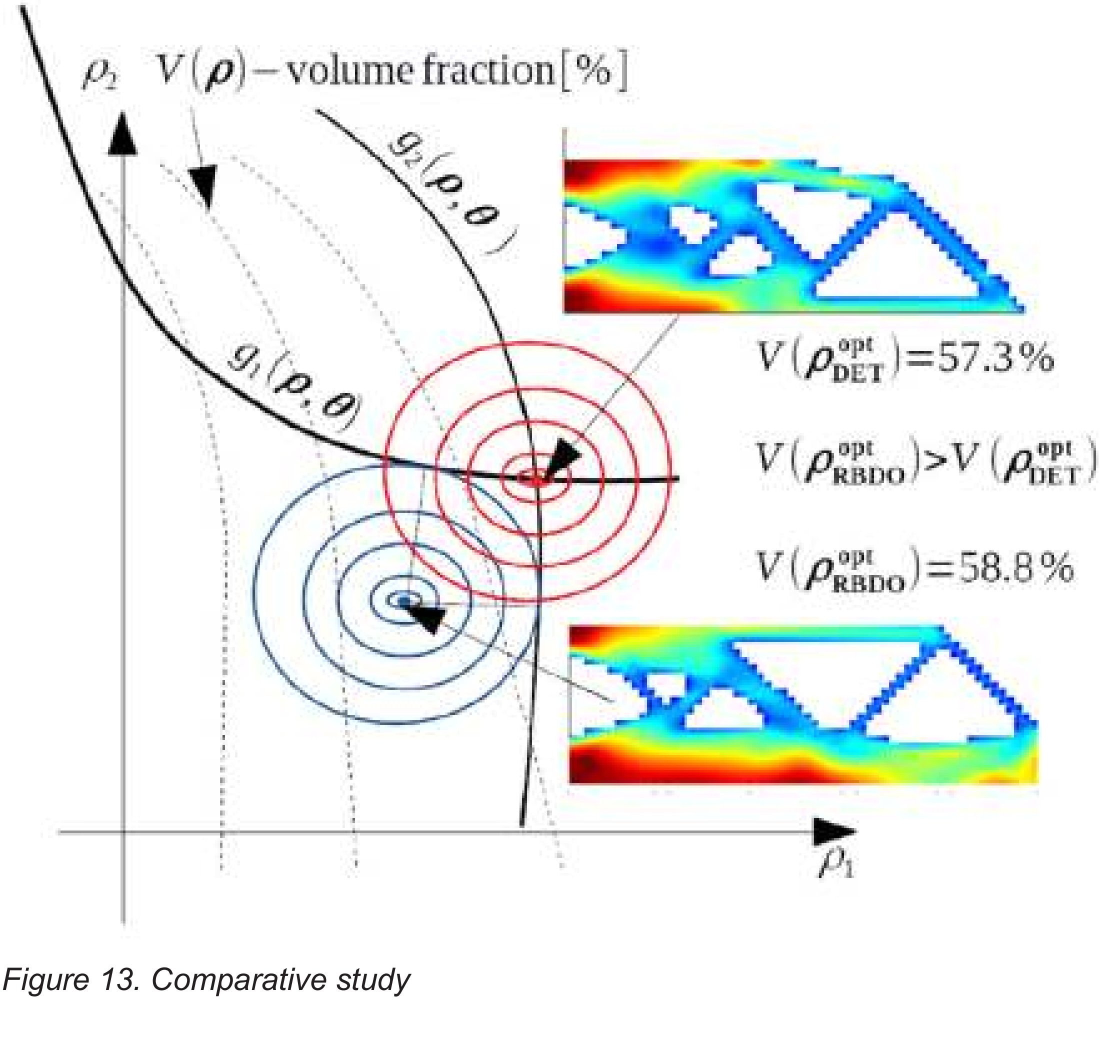

Figure 13. Comparative study

(d) other achievements

We have published results on application of optimization for the material modelling of structural elements as optimization procedures can also be applied for computational problems that are characterized as inverse calculations. Physical quantities not accessible or measurable directly may be determined inversely via measurements of global behaviour of test pieces. A straightforward application is the determination of material properties of structural elements through ordinary bending loading tests. For natural materials such as wood or other materials exhibiting a large scattering in mechanical properties not only between species but within the same stock as well, measurements aiming to obtain material properties of a sample are only loosely correlating with data from measurements conducted on different specimens aiming to study the global structural behaviour of structures. This research aimed to both measure the global structural behaviour and use them to extract important material properties for later modelling. An optimization procedure can be carried out to iteratively determine the material properties such that the measured load-deflection curves are best approximated by the numerically simulated ones.

The flexural behavior of timber beams externally reinforced using carbon fiber-reinforced polymers (CFRP) is investigated. Measurement data from four-point bending tests are available from previous tests. Numerical modelling of the mechanical behaviour of the structure included standard beam theory as well as finite element modelling. Linear and non-linear finite element analysis were performed and validated by experimental four-point bending tests from previous research carried out on timber beams. The general commercial software ANSYS was used, and three- and two-dimensional numerical models were evaluated using the linear elastic orthotropic material model for the timber beams in the linear range and the 3D nonlinear rate-independent generalized anisotropic Hill potential model for the nonlinear behavior. CFRP was modelled as a composite of linear elastic orthotropic material model for the fibers and a linear elastic isotropic model for the epoxy resin. Simulated and measured load-deflection responses were compared and the material properties for timber-CFRP were numerically determined using inverse calculation. The matching of simulated and measured data demonstrated the applicability of the procedure. Through inverse calculation not only complex behaviour of the composite reinforced structure is analysed but the details of its constituents as well.

Unlike most designed and manufactured materials, behaviour of natural materials are strongly affected by natural imperfections such as knots or fibre deviations in wood, etc. For the accurate modelling of structural elements made of timber, these factors should be taken into consideration. Structural modelling of timber beams incorporating natural features and imperfections is investigated. It includes the detailed description of geometry of the beam with respect to non-linear fibres, rings, and knots in particular opposed to simplified pure geometry typical in most research. Capturing of geometry is conducted using photographic techniques and testing of real specimens. The geometrical and mechanical characteristics of knots and related local disturbed fibre patterns were numerically modelled. A series of measurement are conducted to supplement the model with actual material properties for the finite element modelling. Moreover, the model allows defining different fibre patterns in the knot vicinity. Results proved that openings can represent knots when found in the tension zone with careful adjustment of the related three-dimensional fibre deviations. Moreover, the results emphasize the need for accurate modelling for the fibre deviations rather than the knot itself.

In topic (8) - **A unified – neither probabilistic nor fuzzy – approach for optimal design**

A new algorithm was elaborated for volume-constrained expected compliance minimization of continuum structures with probabilistic loading directions using analytically determined exact objective and gradient functions. The algorithm is based upon the finding that for a particular set of statistical parameters the integration in the expected compliance function can be done symbolically and automatically using symbolic manipulation software.

Mathematica was used to integrate and simplify the highly nonlinear expected compliance function. It was demonstrated by examples that the result of the symbolic pre-processing step is a simple linear function defined on a particular subset of the inverse stiffness entries which is needed in the compliance computation. The coefficient set of this linear function forms the base of the exact analytical gradient computation used in the optimal solution searching optimality criteria (OC) method to define the steepest descent direction.

Naturally, the applied OC method can be replaced by any other appropriate nonlinear solver. Matlab codes of the algorithm for 2D and 3D structures with exact analytical sensitivities have been developed using the topology optimization codes presented by Andreassen et al. for 2D and Liu and Tovar for 3D structures as starting points.

Illustrative examples with Mathematica and Matlab codes are presented to demonstrate the essence and viability of the proposed approach and highlight the potential of the automatic symbolic computation in structure optimization.

Csébfalvi, A. ***Robust Topology Optimization: A New Algorithm for Volume-constrained Expected Compliance Minimization with Probabilistic Loading Directions using Exact Analytical Objective and Gradient***, Periodica Polytechnica Civil Engineering, 61(1), pp. 154–163, 2017. https://doi.org/10.3311/PPci.10214

Using a recently developed unified approach, benchmark results are presented for structural optimization when the only source of uncertainty is the variability of the applied load directions. The worst-load-direction oriented framework can be applied to a broad class of engineering optimization problems. In each case, the central element of the solution searching algorithm is a standard multi-load structure optimization problem, which using an appropriate method, can be solved within reasonable time. The varying load directions are handled by additional linear or nonlinear relations, which describe the allowable perturbations of the nominal load directions. The result of the optimization is a performance measure minimal design which is invariant to the investigated uncertainty type and satisfies the response constraints. In order to illustrate the viability and efficiency of the approach, problem-specific models, algorithms and detailed benchmark results are presented for volume minimization of 2D continuum structures with compliance constraints and weight minimization of 2D truss structures with displacement and stress constraints. In each case, the computational cost of the proposed approach is comparable with its fixed load direction oriented equivalent because the worst-load-direction identification process is searching on the space of allowable direction perturbations, which generally means an easier and smaller computational problem than the standard multi-load structure optimization.

A. Csébfalvi, ***Structural optimization under uncertainty in loading directions***: Benchmark results, Advances in Engineering Software, Volume 120,2018,Pages 68-78, ISSN 0965-9978, https://doi.org/10.1016/j.advengsoft.2016.02.006.

A new compliance-function-shape-oriented robust approach was elaborated for the volume-constrained continuous topology optimization with uncertain loading directions. The pure set-based algorithm try to rearrange (take away) some amount of the material volume, originally used to minimize the nominal-compliance, to make a more balanced compliance function-shape on the set of feasible directions which is less sensitive to the directional fluctuation. The objective is the area of the compliance function shape defined on the set of feasible directions. The area-minimal shape searching process is controlled by the maximum allowable increase of the nominal-compliance. The result will be a more robust compliance function shape which can be characterized by a higher nominal-compliance but a smaller curvature about it in any direction. Using the terminology of the classical variational problems, the proposed approach can be classified as a curve length or surface-area minimizing inner-value problem where the inner condition, namely the maximum allowable increase of the nominal-compliance, expressed as a percentage of the original nominal compliance, the searching domain is defined implicitly as integration limits in the objective formulation and a usual equality relation is used to prescribe the allowable material volume expressed as a percentage of the total material volume. Two examples are presented to demonstrate the viability and efficiency of the proposed robust approach.

Csébfalvi, A. ***A New Compliance-function-shape oriented Robust Approach for Volume-constrained Continuous Topology Optimization with Uncertain Loading***

Directions, Periodica Polytechnica Civil Engineering, 62(1), pp. 219–225, 2018. https://doi.org/10.3311/PPci.11398

Uncertainty is an important consideration in topology optimization to produce robust and reliable solutions. There are several possibilities to take into account the uncertainty in the topology optimization of continuum structures. In this paper, we assume that the only source of uncertainty is the variability of the applied load directions. Most models in this area apply parametric statistical tools to describe the directional uncertainty of the applied loads to produce robust structures which are insensitive to the directional uncertainty as much as possible. In the most popular parametric statistical approach the expected-compliance, based on the directional normality assumption, is used as the preferred measure of robustness. We prove indirectly that this approach is far from the engineering practice and may give hardly interpretable or totally misleading results, which is demonstrated by two carefully selected counter-examples. The counter-examples validate the fact that the expected-compliance, as a statistical abstraction based on more or less theoretical assumption about normality, not a general applicable measure of robustness. It will be shown, that the non-parametric and really robust volume constrained worst-load-direction-oriented minimax-compliance model, used in this study only as a proofing tool in a very simple form, is a viable alternative of the parametric expected-compliance model and its results and its problem solving process as a whole are very close to the engineering thinking. The worst-load-direction-oriented minimax-compliance-model provides expressive, rigorous, generally applicable, and objective information about the robustness. The parametric expected-compliance in itself as the preferred measure of robustness is unable to characterize the compliance variability, in contrast of the minimax approach which can be describe the compliance variability by a robust range-like measure computed very easily as the difference of the maximal- and minimal-compliance on the set of the feasible loading directions.

A. Csébfalvi, J. Lógó, ***A critical analysis of expected-compliance model in volume-constrained robust topology optimization with normally distributed loading directions,***

using a minimax-compliance approach alternatively, Advances in Engineering Software, Volume 120, 2018, Pages 107-115, ISSN 0965-9978, https://doi.org/10.1016/j.advengsoft.2018.02.003.

In this study, the possible numerical treatments of a new compliance-function-shape-oriented robust truss sizing model with directionally uncertain loads have been analyzed. The applied robustness measure is the total-compliance-variation; the corresponding model is a total-compliance-variation minimization model. The model, according to the number of uncertain loads, can be classified as a curve-length, surface-area, volume, and generalized volume minimization problem where the objective is integral and the weight increase is constrained. The numerical treatment of the model is a challenging problem. In this study, for the numerical treatment a one-step asymptotically optimal algorithm and a two-steps heuristic algorithm are presented. The central element of the one-step algorithm is a mesh with mesh-point-specific displacements, stress constraints, and compliance values. The algorithm can be formulated as a sample-based iterative process using one of the two alternative objectives. The one-step algorithm is computationally expensive but able to find an optimal solution with appropriate maximum running time and mesh-size settings. The two-steps algorithm firstly defines a lower dimensional searching space without displacements and stress constraints and uses it as the searching space in the second step. The hybrid metaheuristic search method is a modified ANGEL algorithm, which is a hybridization of the ant-colony (AN), the genetic (GE) and a local search (L) algorithms. The two-steps algorithm is faster but its result is not necessary optimal. To illustrate the efficiency of the solution possibilities the popular ten-bar truss sizing problem will be used with one, two, and four directionally uncertain point loads as the benchmark problem.

Anikó Csébfalvi, János Lógó, ***Investigation of the possible numerical treatments of a compliance-function-shape-oriented robust truss sizing model with uncertain loading directions***, Advances in Engineering Software, Volume 149, 2020, 102899, ISSN 0965-9978, https://doi.org/10.1016/j.advengsoft.2020.102899.

We critically investigated the role of the combined compliance average and spreading measures in the volume-constrained continuous robust topology optimization with uncertain loading magnitude and direction. In the robust topology optimization the generally expected and most popular robustness measure is the expected compliance. In the expectancy oriented approach, the compliance increment which is needed to get the robust design is an implicitly defined response variable. In order to open the possibility of the creative contribution of the designer to the best robust design searching process, this measure is sometimes combined with a spreading-oriented measure, which may be the variance or standard deviation. The best weighting schema can be done by a try-and-error-like algorithm in which the weights are design variables and the compliance-increment remains an implicitly defined response variable. In this study, it is shown that all of the compliance oriented approaches which are based on a single or combined statistical measure can be replaced by a new compliance-function-shape-oriented robust approach in which the allowed compliance-increment will be an explicitly defined design variable and for a given increment value the robust solution will be the theoretically best one. A popular volume-constrained symmetric bridge problem with uncertain loading magnitude and direction is presented to demonstrate the viability and efficiency of the proposed robust approach.

Csébfalvi, A. ***Critical Investigation of the Combined Compliance Average and Spreading Measures in the Robust Topology Optimization with Uncertain Loading Magnitude and Direction***, Periodica Polytechnica Civil Engineering, 64(4), pp. 1275–1283, 2020. https://doi.org/10.3311/PPci.16681