Dynamics of Strongly Correlated Quantum Systems

Exact results and integrability breaking in the dynamics of spin chains

Quenches in the XXX and XXZ spin chains

A quantum transfer matrix approach was used to derive TBA equations for the Loschmidt echo after global quenches in the XXZ spin chain[1], establishing a clear correspondence between the quantum transfer matrix formalism and the quench action approach. These results are also an important step towards developing techniques which can provide analytical formulas for the time evolution of physical quantities, and indeed we obtained such results for the Loschmidt echo, where we found interesting non-analytic behaviour[2].

We derived overlaps for a large class of initial states on the XXZ spin chain[3], which is necessary for deriving exact results for time evolutions after a quantum quench. Furthermore, we also determined the exact propagator of finite size XXZ spin chain, which can be applied to study time evolution in the finite-size system, as opposed to the existing results which are mainly for the thermodynamic limit.[4]

Concept of, and exact results for, integrable quenches

Regarding the feasibility of exact results, it is important to find out what is the class of quenches for which this can be potentially obtained. Integrability of the Hamiltonian is not sufficient: the initial state must also satisfy appropriate conditions.

We proposed a definition of integrable initial states for quantum quenches in lattice models and showed that this class includes the states related to integrable boundary conditions, and all global quenches for which closed-form analytical results could be obtained in the previous literature[5]. We also constructed novel classes of integrable initial states[6] which are important as for quenches in this class not only the stationary state, but also the entire time evolution can be constructed exactly by Bethe Ansatz methods, in very non-trivially interacting and strongly correlated systems.

We derived exact results for more complicated systems with a nested Bethe Ansatz[7,8], which extends exact methods to a much wider class of systems than those with a simple Bethe Ansatz.

New results for the generalized Gibbs ensemble

Recently it was shown that for the GGE to describe the stationary state after a quantum quench, it must generally involve an infinite number of novel quasi-local conserved charges. We have provided an alternative construction by considering truncated GGE's (tGGE's) that only include a finite number of well localized conserved operators. We have shown that the tGGE's can approximate the steady states with arbitrary precision[9].

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We have also constructed the GGE for integrable systems with a nested Bethe ansatz[10], extending the construction of GGE involving alllocal and quasi-local conserved charges to integrable lattice models solvable with group symmetry SU(N), N≥3. We showed that a complete set of charges for the GGE can be obtained from the known fusion hierarchy of transfer matrices, and they completely fix all the quasi-particle rapidity distributions.

Correlation functions for systems with nested Bethe Ansatz

Using the algebraic Bethe Ansatz we developed an expansion theorem for correlation functions in nested Bethe Ansatz, applicable both to one-point and two-point functions, and used it to computed the ground state density-density correlator in the Gaudin-Yang model of spin-1/2 Fermi particles[11].

Boundary free energy of the open XXZ chain

We have determined the exact boundary free energy, i.e. the contribution of the open end to the free energy for the open XXZ spin chain[12].

Deformations and overlaps in integrable spin chain

We established a deep connection between two classes of deformations of integrable models[13]. One class includes the TTbar-deformation of 1+1 dimensional integrable quantum field theory and related solvable irrelevant deformations proposed recently. The other class is a specific type of long-range integrable deformation of quantum spin chains introduced in the context of N=4 super-Yang-Mills theory. We also proved a factorisation formula for the expectation value of the operators which trigger the deformation on the lattice; similar results in quantum field theory play an essential role in the solvability of such deformations.

We derived an exact expression for the overlaps between the Bethe eigenstates of the SO(6) spin chain and matrix product states built from matrices whose commutators generate an irreducible representation of so(5)[14], which play the role of boundary states in a domain wall version of N=4 SYM theory which is known to be dual to a D3-D7 probe brane system. We also showed that the same methodology makes it possible to prove an overlap formula obtained earlier and relevant for the similar D3-D5 probe brane system.

We developed a new method to compute the exact overlaps between integrable boundary states and on-shell Bethe states for integrable spin chains, based on the coordinate Bethe Ansatz[15]. It leads to a rigorous proof of the factorized overlap formulas in a number of cases, some of which were inaccessible to earlier methods.

Integrability breaking in spin chains

Recently it was suggested that certain perturbations of integrable spin chains lead to a weak breaking of integrability in the sense that integrability is preserved at the first order of the coupling. Here we examine this claim using level spacing distribution[16]. We found that the volume dependent cross-over between integrable and chaotic statistics is markedly different between weak and strong breaking of integrability, supporting the claim that perturbations by generalised currents only break integrability at higher order. In addition, for the massless case we find that the critical coupling as a function of the volume L scales with a $1/L^2$ law for weak breaking as opposed to the previously found $1/L^3$ law for the strong case.

Inhomogeneous quenches and generalised hydrodynamics

Inhomogeneous quantum quenches

We investigated inhomogeneous quenches obtained by locally joining two semi-infinite systems separately at equilibrium.

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We studied scaling and front dynamics in the transverse field Ising chain, obtaining integral expressions for all two-point correlation functions of the Jordan-Wigner Majorana fermions at any time and for any value of the transverse field[17] and computing analytically the profiles of various physical observables in the space-time scaling limit, showing they can be obtained from a hydrodynamic picture based on ballistically propagating quasiparticles. Going beyond the hydrodynamic limit, we analysed the approach to the non-equilibrium steady state and found that the leading late time corrections display a lattice effect. We also observed the phenomenon of energy back-flow where the energy locally flows from the colder to the hotter region.
We also studied entanglement in the emerging non-equilibrium steady state[18], showing that the Rényi mutual information between two adjacent segments of the chain, scales logarithmically in the subsystem size. Surprisingly, for Rényi indices $\alpha>2$ we found cases where the coefficient of the logarithmic dependence is negative. The fact that the naively defined Rényi mutual information might be negative has been pointed out before, however, we provided the first example for this scenario in a realistic many-body setup.

Currents in Generalised Hydrodynamics

Generalized Hydrodynamics is a recent theory that describes large scale transport properties of one-dimensional integrable models. One of its cornerstones is a conjectured expression for the currents of the conserved charges in local equilibrium. We computed an exact result for the mean values of current operators in Bethe Ansatz solvable systems, valid in arbitrary finite volume[19].
This result was taken further using a certain type of long-range deformation of the local spin chains, which is immediately applicable also to higher-rank models. As a concrete example we derived the current mean values in the SU(3)-symmetric fundamental model, solvable by the nested Bethe Ansatz[20].
We gave an explicit construction for the currents of the integrable spin chains into the canonical framework of Yang-Baxter integrability[21], which can be applied in a large variety of models including the XXZ spin chains, the Hubbard model, and even in models lacking particle conservation such as the XYZ chain. It also provides a simplified proof of the recent exact results for the current mean values and explains how their quasi-classical nature emerges from the exact computations.

Dynamics in integrable quantum field theories

Correlation functions of integrable QFT at finite temperature/density

We have made a crucial breakthrough on a previous result of our group, finding a more compact formula for the correlation functions that reduces a double series expansion to a single one, thereby achieving a partial resummation, and also extending its validity to more general thermodynamic states[22].

Time evolution using spectral expansion with singular overlaps

We have examined the extension of the method of spectral expansion to quantum quenches in the attractive regime of sine-Gordon theory[23]. We have shown that previous attempts were mistaken as they neglected the singular behaviour of the overlaps and computed the leading effect of singularities on the time evolution.

Overlaps in integrable quenches of quantum field theories

Recently our group developed a formalism to obtain quench overlaps in quantum field theory and obtained a candidate solution for certain quenches from free boson to sinh-Gordon field theory. Using the truncated conformal space approach, we have shown that analytic continuation of these results can be used to describe quenches in sine-Gordon field theory, thereby confirming the validity of the original result and significantly expanding its usability[24].

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Generalised hydrodynamics in massless flows

We also applied the recently developed generalised hydrodynamics to describe out-of-equilibrium behaviour in quantum field theories describing massless flows (i.e. crossover behaviour) between different universality classes in one-dimensional quantum critical systems. The results of this investigation lead to a new version of Zamolodchikov's famous c-theorem[25].

Generalised hydrodynamics vs. semiclassical approach

We carried out a systematic comparison between the Generalised Hydrodynamics and semiclassical descriptions in the prototypical example of the sine-Gordon field theory, studying the "bipartitioning protocol"[26]. The semiclassical predictions are analytically recovered from GHD in a particular non-relativistic limit and also for low temperatures, with sub-ballistic the transport in both cases. Away from these limits, the semiclassical predictions are only approximate and the transport is always ballistic.

Novel experimental realisation of the E_8 integrable QFT

We also provided the theoretical background for a novel experimental realisation of a famous integrable model[27,28]. We contributed with theoretical modelling to an unambiguous experimental realization of the massive E8 phase in the quasi-1D antiferromagnetic material BaCo2V2O8, via nuclear magnetic resonance and inelastic neutron scattering measurements, identifying the full 8 single-particle E8 excitations for the first time and opening new experimental route for exploring the dynamics of quantum integrable systems and physics beyond integrability.

Non-equilibrium dynamics in quantum field theories

Semiclassical theory of front propagation

We obtained a semiclassical description for the propagation of fronts and approach to equilibrium in inhomogeneous quenches in the O(3) sigma model[29].

Application of sine-Gordon field theory to cold atom experiments

We have studied the out-of-equilibrium evolution in the quantum sine-Gordon field theory[30], which is relevant for experiments conducted on coupled quasi-one-dimensional bosonic condensates and demonstrated that the phase-locking observed in the experiment cannot be explained by sine-Gordon dynamics itself, reaching the conclusion that the out-of-equilibrium behaviour observed in the experiment can only be modelled by going beyond the sine-Gordon model, more likely including additional degrees of freedom.

Kibble-Zurek scaling in interacting QFT

Using the Truncated Conformal Space Approach, we investigated the microscopic details of the Kibble-Zurek mechanism in terms of instantaneous eigenstates in a genuinely interacting field theory[31]. We demonstrated dynamical scaling in the non-adiabatic time window and provided analytic and numerical evidence for specific scaling properties of various quantities. We also found that the higher cumulants of the excess heat exhibit universal scaling in generic interacting models for a slow enough ramp.

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Horizon violation after a quantum quench in sine-Gordon model

We studied time evolution non-equilibrium quantum field theory and found the surprising effect that in the presence of topological excitations correlations can develop outside of horizon and indeed even between infinitely distant points[32]. We demonstrated this effect for a wide class of global quantum quenches to the sine-Gordon model. We found a very surprising and entirely novel phenomenon: out-of-equilibrium dynamics in sine-Gordon model generically violates the so-called horizon effect, due to long-range EPR-like correlations induced between the solitonic quasi-particles of the system, already present - albeit hidden - in the initial state.

Dynamics of quenches in the scaling Ising QFT

We developed a theoretical description for dynamics of quenches starting from the E_8 Ising QFT, both for integrable and non-integrable post-quench dynamics[33]. Using numerical results obtained with the Truncated Conformal Space Approach, we have clarified the domain of validity of two analytic approaches developed in the literature.
We also developed a perturbative formalism to construct so-called quench overlaps, which characterise the initial state of the system in terms of the quasi-particle excitation and showed that it successfully accounts for properties of overlaps, using the scaling Ising field theory as a test case[34]. We have shown that the initial state is generally not of the squeezed state form usually assumed for theoretical computations, even when the initial and the post-quench systems are both integrable.

Entanglement dynamics and spreading of correlations out of equilibrium

Confinement in spin chains

Building upon our discovery of dynamical confinement in non-integrable quenches in the Ising spin chain[35], we demonstrated that the same effect is realised in the Heisenberg-Ising spin ladder without the need of an external field, which makes it suitable for an experimental realization with ultracold atoms. We found that the resulting spectrum contains two kinds of particles, intrachain and interchain mesons[36]. The intrachain mesons can also be qualitatively assessed through an effective mean-field description, the interchain ones are new and they represent general features of spin ladders with confinement.

Entanglement growth and quasi-particle spectrum

In the paramagnetic phase of the Ising spin chain, we identified the non-equilibrium effect of Gibbs mxing entropy, the "dynamical Gibbs effect", which leads to a characteristic dependence of entropy growth rate on the quasi-particle spectrum[37]. Later we have shown that this effect extends to the 3-state Potts case, which has a richer dynamics, and so provides a more detailed understanding of the phenomenon[38]. Furthermore, in the absence of explicit symmetry breaking, we found universal ratios involving Rényi entropy growth rates and magnetisation relaxation for small quenches[39]. We also demonstrated that the magnetisation relaxation rate provides an observable signature for the "dynamical Gibbs effect".

Chiral entanglement in massive quantum field theories

We also considered chiral entanglement in quantum field theories which are obtained as flows generated by a relevant operator from a conformal fixed point[40]. Breaking scale invariance induces entanglement between left- and right-moving degrees of freedom, the so-called chiral entanglement. We found that the contribution O(1) in system size is universal; it was previously suggested as a possible benchmark for topological states in two spatial dimensions. Our results also provided an in-depth verification of an Ansatz for the ground state of massive quantum field theories recently proposed by J. Cardy.

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Supervised students

The following students participated in the project work:

Name	Level	Supervisor	Graduation
Dávid Horváth	PhD (BME)	Gábor Takács	2019
Olivér Rákos	MSc	Balázs Pozsgay	2018
Octavio Pomponio	MSc (Bologna Univ.)	Gábor Takács	2018
Octavio Pomponio	PhD (Bologna Univ.)	Gábor Takács	Expected in 2021
Kristóf Hódsági	MSc	Márton Kormos	2018
Kristóf Hódsági	PhD	Márton Kormos	Expected in 2022
Levente Pristyák	BSc	Gábor Takács	2017
Levente Pristyák	MSc	Balázs Pozsgay	2019
Levente Pristyák	PhD	Balázs Pozsgay	Expected in 2023
Ivan Kukuljan	PhD (Ljubljana univ.)	Gábor Takács	2019
Márton Borsi	MSc	Balázs Pozsgay	2020
Dávid Szász-Schagrin	MSc	Gábor Takács	2020
Dávid Szász-Schagrin	PhD	Gábor Takács	Expected in 2024

The research resulted in the completion of 1 BSc, 6 MSc and 2 PhD theses, with four more PhD theses expected in the foreseeable future.