

Project closing report (final report)

ERC_HU_15 118286

Results obtained during the first year

The Principal Investigator (Laszlo Pyber) has submitted an ERC ADG proposal (again) in 2016, entitled "Growth in groups and Graph Isomorphism now". This proposal was evaluated positively by the panel and got to the second round of evaluation.

Papers and preprints

1) *R.M.Guralnick, A.Maróti and L. Pyber, Normalisers of primitive permutation groups, to appear in Advances of Mathematics, arXiv:1603.00187 , 44 pages*

Let G be a transitive normal subgroup of a permutation group A of finite degree n . The factor group A/G can be considered as a certain Galois group and one would like to bound its size. The main result of the paper is that $|A/G| < n$ if G is primitive with 5 exceptions. Many other results of this type are also established.

2) *G.Kun, On sofic approximations of property (T) groups, arXiv:1606.04471, 14 pages*

We prove Bowen's conjecture that every sequence of finite graphs that locally converges to the Cayley graph of a countably infinite group with Kazhdan Property (T) is essentially a vertex-disjoint union of expander graphs.

3) *A.Sheffer, E.Szabó and J.Zahl, Point-curve incidences in the complex plane, to appear in Combinatorica, arXiv:1502.07003 , 10 pages*

We prove an incidence theorem for points in the complex plane. Earlier similar results were obtained by Pach and Sharir in the real plane. Our key technical lemma controls the number of complex curves that can be contained inside a real hypersurface.

This lemma may be of independent interest.

4) *K.Cziszter, On the Noether number of p -groups , arXiv:1604.01938 , 14 pages*

A group of order p^n (p prime) has an abstract indecomposable polynomial if and only if the group has a cyclic subgroup of index at most p or it is isomorphic to one of two particular groups of small order.

5) *K.Cziszter, M.Domokos and I.Szöllősi*, *The Noether numbers and the Davenport constants of the groups of order less than 32*, arXiv:1702.02997, 27 pages

The computation of the Noether numbers of all groups of order less than thirty-two is completed. For each of these groups the Noether number exceeds the small Davenport constant, whereas the first example of a group whose Noether number exceeds the large Davenport constant is found, answering partially a question posed by Geroldinger and Gryniewicz.

Results obtained during the final half year

The Principal Investigator (Laszlo Pyber) has submitted an ERC ADG proposal (again) in 2016 entitled "Growth in Groups and Graph Isomorphism now. This proposal was evaluated positively by the panel in the second round of evaluation and the Principal Investigator will receive actual funding from the ERC.

Papers and preprints

1) *L.Pyber* *A CFSG-free analysis of Babai's quasipolynomial GI algorithm*
arXiv:1605.08266v2, 8 pages

Babai's GI algorithm was hailed by the theoretical computer science community as the greatest breakthrough of the last 10 years.

The analysis of Babai's algorithm relies on the 10000 page Classification of the Finite Simple Groups (CFSG).

Very surprisingly we manage to give a slightly weaker but still quasipolynomial version which avoids reference to the this monumental theorem.

The key idea was obtained by the PI while trying to improve an important result of Helfgott-Seress in the area of growth in groups. It is expected that in the end this will lead to a CFSG-free proof of the Helfgott-Seress result as well.

2) *P.Hegedűs, A.Maróti and L.Pyber*, *Finite groups with large Noether number are almost cyclic*, arXiv:1706.08290v2 14 pages, submitted

Let $\beta(G)$ be the Noether number of a finite group. We show that $|G|/\beta(G)$ is bounded if and only if G has a characteristic cyclic subgroup of bounded index.

This is similar to the Jordan-type theorems which play an important role in the area of growth.

In the course of the proof we obtain surprisingly strong estimates for the Noether numbers of finite simple groups of Lie type.

3) *B.Csikós, M.Horváth*, *Harmonic manifolds and tubes*, arXiv:1705.00311v2

Earlier the authors showed, that in a connected locally harmonic manifold, the volume of a tube of small radius about a regularly parametrized simple arc depends only on the length of the arc and the radius. In this paper it is shown that this property

characterizes harmonic manifolds even if it is assumed only for tubes of geodesic segments.

4) *G.Horváth, C.L.Nehaniv and K.Podoski, The maximal subgroups and the complexity of the flow semigroup of finite (di)graphs, arXiv:1705.09577v1, submitted*

The authors refine and prove Rhodes's conjecture on the structure of the maximal groups in the flow semigroup for finite, antisymmetric, strongly connected digraphs. Building on this result they fully describe the structure and actions of the maximal subgroups of the flow semigroup acting on all but k points for all finite digraphs and graphs for all k greater than 1. A linear algorithm is presented to determine these so-called "defect k groups" for any finite (di)graph

Finally they prove that the complexity of the flow semigroup of a 2-vertex connected (and strongly connected di)graph with n vertices is $n-2$, completely confirming Rhodes's conjecture for such (di)graphs

5) *D.Levy and A.Maróti, Set-direct factorizations of groups, arXiv:1707.04643v1, 19 pages*

The authors consider factorizations $G=XY$ where G is a general group, and X and Y are normal subsets of G and any g has a unique representation $g=xy$ with x in X and y in Y . This definition coincides with the customary and extensively studied definition of a direct product decomposition by subsets of a finite abelian group. The main result states that a group G has such a factorization if and only if G is a central product of the subgroups A and B generated by X resp. Y and the intersection of A and B satisfies certain abelian factorization conditions. In particular, simple groups have no non-trivial set-direct factorizations

24. August, 2017

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