OTKA 116892 PD final report

Highly eccentric signals in gravitational wave physics

(1 Sep 2015 – 31 Dec 2018)

The aim of the proposal was the theoretical investigation of the compact binary systems and the gravitational waves formed by the binaries which focused on the impact of the physical parameters of the source, i.e. *spin* and *eccentricity* with the help of the *post-Newtonian* formalism the part of the approximation theory of the general relativity. I have given the impacts of these physical properties for description of binary evolution in a single-author paper which have already been cited by famous experts in its publication year. Continuing this research with Dr. Zoltán Keresztes, associate professor at University of Szeged (US), we have extended the description of the motion and placed it in a more general context. In this way we studied the exact equations for the motion of the spinning particle in curved spacetime under analytical and numerical investigations as opposed to the planned use of the post-Newtonian methods in proposal. In this latter study, unexpected numerical difficulties were encountered in the examination of equations which increased the time of the research to a greater extent than expected, this made it necessary to extend the application by 4 months.

In connection with the topic for the parameter estimation of the gravitational waveforms included in the proposal, Dr. László Árpád Gergely (US), in cooperation with the gravitational group led by the associate professor, then Krisztina Kövér (a 2nd year BSc Student) prepared a TDK (Student Association's Conference) dissertation in which she gave a statistical estimation of the parameters found in spin-dominated gravitational waveforms.

During the application, 2 international and 4 other publications were published which are 1 referenced article, 1 preparing for submission article, 1 BSc TDK thesis and 3 posters. The results of the application were presented in 5 talks on national and international conferences.

Summary of my results

The orbital evolution of highly eccentric spinning binaries

I have investigated the motion of the eccentric compact binaries with leading-order spin-orbit interaction and its dissipative contributions due to gravitational radiation. The Lagrangian of this interaction is not unique, because in general relativity the *spin supplementary conditions* (SSCs) are necessary when introducing the spin. There are three SSCs for the spin-orbit interaction: the covariant (the *Frenkel-Mathisson-Pirani* or the *Tulczyjew-Dixon* SSC which are equivalent to each other for the spin-orbit interaction), the *Corinaldesi-Papapetrou* and the *Newton-Wigner-Pryce* SSCs. I have written the Lagrangians for each SSC and concluded that the conserved quantities, i. e. the energy and the magnitude of the orbital angular momentum depend on SSC. The Lagrangian depends on the acceleration in two cases, therefore we should generalize the standard formalism to the Ostrogradsky's mechanics. Furthermore, I constructed the generalized Hamiltonian with one higher-order canonical moment, and added the generalized Hamilton's equations. I showed some examples for the elimination of the acceleration terms of the Lagrangians with the help of constrained dynamics and the *double zero method* proposed by Barker and O'Connell. I calculated the equation of motion using both the Lagrangian and the Hamiltonian mechanics, and I compared the different formulas in some of the literature. The transformation between SSCs has been developed. After the integration of the equation of motion I computed the orbital parameters for radial and angular motion in all SSC. I also calculated the gravitational multipole momentums for the classic motion of the spinning binaries, and the energy and the orbital angular momentum losses under the transferring of the gravitational waves depending on SSC. I proved that the SSC dependence of these disappears if we use the averaging method over one orbital period. I wrote about the dependence of the gravitational waveform on multipoles where it has been proved that the leading-order spin-orbit term of the measurement signal does not depend on SSC, but the next-to-leading-(half)-order terms do [1].

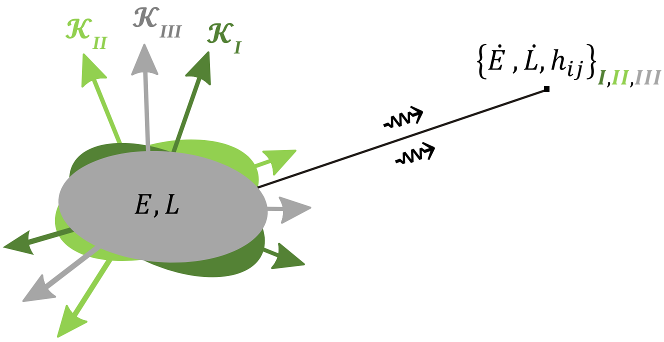

Fig. 1. Dissipative quantities due to gravitational waves far from the spinning binary in different SSC frames (Mikóczi 2017).

Parameter estimation on spinning gravitational waveforms

The gravitational waves carry information about the physical parameters of their sources, one of which the most important are the mass and the spin. The theoretical gravitational waveforms containing these properties allow the increase of the measurements' accuracy of the gravitational wave detectors.

Krisztina Kövér, the student of the US has performed the mathematical statistical method-based parameter estimation for spin-dominated waveforms developed by US which completed my previous results on eccentric non-spinning waveforms (Mikóczi et al. 2015).

Gravitational waves transport energy and angular momentum away from the system described by Einstein's quadrupole formula, under which the orbital eccentricity of the binary containing the black holes and/or the neutron stars gradually decreases, therefore they circularize on near circular orbit before their merge. Considering the aforementioned theorem, we first examined the circular orbit's limit, taking into account the leading-order spin-orbit interaction of the binaries. In this context, we have calculated the time dependence of the angular quantities applied in the dynamics and using the this calculated time dependence we determined the time-dependent spin-dominated waveforms, i.e., the leading-order spin-orbit contributions. Then using the stationary phase approximation, we transformed the former time-dependent waveforms into the explicit frequency domain. In addition, we derived the polarization of the gravitational waves and the antenna function characteristics describing the relative position of the detector in order for the complete waveform to be defined.

Subsequently, we have accomplished the parameter estimation of the full signal in frequency domain using a Bayesian statistical method, the Fisher-matrix analysis. With this method the root-mean square of each parameter and the correlations between the individual parameters in the gravitational waveform can be estimated. We have given these theoretical estimations with the noise function of the American LIGO detector for various mass and angle configurations, such as stellar black holes and neutron stars. Based on our results, we can say that in most cases the mass parameters are well measurable and by increasing the dimensionless spin parameter the root-mean squares of the other parameters decrease [III].

Extended spinning bodies in curved spacetime

The development of the *eccentric zoom-whirl orbits* of my research plan has been extended with the numerical investigation of exact equations from the general relativity, which placed the direction of my research plan in a more general context. Due to the nonlinearity of the equations, the numerical elaboration and examination of the orbits proved to be more complicated than expected, which was complicated by the fact that the authors used several spin definitions, therefore the comparison of the results was not easy either. For this reason, the reproduction of numerical results in the literature and the elimination of numerical instabilities have unexpectedly increased the research time.

Under my research project we have examined the motion of the massive extended spinning bodies in the background of the Kerr black hole. The non-geodesic motion is governed by the Mathisson-Papapetrou-Dixon (MPD) equations which are not closed. In order to close the system of MPD equations, it is necessary to choose spin supplementary conditions (SSC), of which two covariant SSCs, i.e., Tulczyjew-Dixon (TD) and Frenkel-Mathisson-Pirani (FMP) have been chosen, which will be applied for the treatment of the numerical problems. To investigate the non-linearity of the equations, we developed a numerical program with which we demonstrated the three-dimensional spin movement of the body during circularization. We have shown that the zoom-whirl-like orbits exist in case of spinning bodies, which are non-plane motions in general. During the numerical investigations we used several different frames to characterize the spin, such as the *zero angular momentum observer* (ZAMO), *static* (SO) and *comoving* observers. We have found some interesting orbits for spinning particles when the body gets in and out of the ergosphere of the Kerr black hole, meanwhile the spin vector rotates significantly. We also defined an angular momentum-like quantity which in fact describes the motion of the orbital plane.

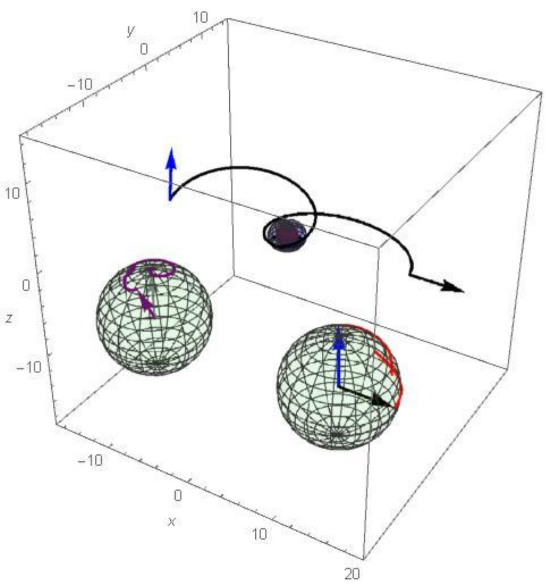

Fig. 2. A zoom-whirl orbit for an extended spinning particle in the Kerr spacetime

In 2017, Bini et al. examined systems where the contribution from spin of the body was considered as perturbation in the geodesic equations. Namely, the angular velocity of a particle with small spin, moving on the geodesic orbit and determined by the static observer, was given in the comoving system fixed to the body. Consequently, the equations of the spinning particle could be considered geodesic, but are only valid for small spin particle cases. We extended this work to the non-geodesic MPD equations allowing the study of arbitrary spin particles. Moreover, we defined the three-dimensional angular velocities of the body in the comoving coordinate system using ZAMO and SO observers. Based on this, we obtained additional contributions to the angular velocities, presented in the work of Bini et al., resulting from the movement of the spinning particle.

We used the above mentioned covariant SSCs (FMP and TD) to determine the orbits. Usually, there is no emphasized center of mass in FMP SSC, while the center of mass of a spinning body is clearly fixed in TD SSC. So called, helical orbits, causing further, undesired spin axis movement effects, can be formed in FMP SSC as opposed to TD SSC. In order to avoid these helical orbits, after examining several initial conditions, we were able to find orbits that are close to those analyzed in TD SSC. Our studies showed that the orbit of a body with spin is significantly modified as it approaches the central black hole.

Subsequently, we also determined the spin dynamics of a spinning body by defining three separate spin vectors: one in FMP SSC and two in TD SSC. In this latter SSC case, the definition of the two spins is formally the same, however in one case we used the *comoving frame with the centroid*, while in the other one the *zero 3-momentum frame* was used.

In both SSC cases, we determined the direction of the three spin vectors for various initial spin settings using the comoving coordinate system established using both SO and ZAMO observers, which was necessary because only the ZAMO observer could be used to characterize the orbit within the ergosphere. As a consequence, we also demonstrated such zoom-whirl orbits by which the spinning particle enters the ergosphere and after a few circulations exits and moves away from the black hole (Fig. 2). For the investigated orbits, we presented the angular velocities of the three spin vectors in polar coordinates for both observers (SO, ZAMO). Finally, we examined periodic zoom-whirl orbits, previously analyzed for non-spinning cases (Levin and Perez-Giz 2008), and demonstrated how the spin of the particle modifies those.

Our scientific paper about this work is being finalized, which will then be sent to the journal Phys. Rev. D for which I will send a notification to NKFIH [2].

Spinning neutron stars with magnetic dipoles

The fast rotating neutron stars or pulsars have a magnetic field and due to this their motion can be given by *magnetohydrodynamic* (MHD) equations. The magnetic field can be approached with a magnetic dipole momentum, which is generally not parallel to the rotation axis of the neutron star. Thus the spinning dipole causes the pulsations to the remote observer, which can be accurately measured by radio astronomy.

The gravitational wave detectors measured a signal generated by a merged neutron star binary in August 2017 for the first time (GW170817), so it was important to study the dynamics of double neutron star systems.

The spin precession equations containing magnetic dipole moments can be written, similar to the spin evolution equations of the bodies, which can be discussed with the post-Newtonian approximation. With this approach I studied the spin dynamics of spinning binaries with magnetic dipole moments. Since the evolution of spin precession is on a smaller scale than the orbital period time scale, I derived the averaged spin precession equations for one radial Newtonian orbital period taking into account the precession of the dipoles. Based on these, I have shown that due to dipole contributions, the magnitudes of the individual spins are not conserved quantities in contrast with dipole-free cases. In 2008, Racine found a conserved quantity, which could be used to precisely solve the averaged spin precession equations for equal-mass cases, whereas it is necessary to use a perturbation method for unequal cases. I have shown that this scalar quantity will not be a constant in equal-mass cases because quadratic dipole terms remain in the time evolution, although they can be removed by adding another scalar which is proportional to the square of the total spin vector. My study has shown that with this new scalar quantity we cannot write the solution or find any scalar quantity that would remain at least in dipole and spin in quadratic terms. In the remainder of the work I would like to examine the equations using perturbation techniques or perturbative solutions. Also, the relevance of dipole moments in the dynamics of neutron stars can be examined by the numerical solution of equations [3].

Publications

[1] **B. Mikóczi**, *Spin supplementary conditions for spinning compact binaries* Phys. Rev. D **95**, 064023 (2017).

[2] Z. Keresztes and **B. Mikóczi**, *Motion of spinning bodies in Kerr spacetime* (under finalization).

[3] **B. Mikóczi**, *Motion of the spinning magnetar binaries* (in preparation).

Talks and poster presentations at national and international conferences

I. **B. Mikóczi,** *Spin supplementary conditions for spinning compact binary systems* #, 21st International Conf. on Gen. Rel. and Gravitation (GR21), NewYork, USA, 10-15 July 2016

II. **B. Mikóczi,** *Forgó testek mozgása az általános relativitáselméletben* *, University of Szeged, Elméleti Fizikai szeminárium, Szeged, 29 Nov 2015

III. K. Kövér, **B. Mikóczi**, M. Tápai és L. Á. Gergely, *Spin-dominált gravitációs hullámformák Fisher-analízise* #, Fizikai Vándorgyűlés, Szeged, 24-27 Aug 2016

IV. **B. Mikóczi**, *Forgó testek mozgása az általános relativitáselméletben**, 100 éves az általános relativitáselmélet workshop, Zrínyi Miklós Nemzetvédelmi Egyetem, Budapest, 9 Nov 2016

V. K. Kövér, *Fisher analysis of the spin-dominated gravitational waveform*, local TDK (first award), Szeged, 24 Nov 2016, supervisors: László Á. Gergely and **B. Mikóczi**.

VI. K. Kövér, OTDK, Debrecen, 11 Apr 2017 Supervisors: László Á. Gergely and **B. Mikóczi**.

VII. **B. Mikóczi**, *Does the gravitational waveform depend on the spin supplementary conditions?* * (20+5)', 33RD Institut d'Astrophysique de Paris Colloquium: *The Era of Gravitational Wave Astronomy* (TEGrAW), France, Paris, 26-30 June 2017

VIII. **B. Mikóczi**, *Zoom-whirl orbits of spinning bodies in rotating spacetimes* *, Gravity@Malta, Gravitational Waves, Black Holes and Fundamental Physics, COST Action, Valletta, Malta, 22-25 Jan 2018

IX. **B. Mikóczi**, *What kind of spin can be measured by gravitational wave detectors?* Joint Space-Science Institute (JSI) Workshop: *Gravitational Wave Physics and Astronomy Workshop*, College Park, University of Maryland, USA, 1-4 Dec 2018 # *(sparkler talk)

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