If we have to summarize it in one sentence then we can say that the project was very successful. It can be measured in many different ways. E.g. the participants of the project received several excellent prizes:

Imre Bárány– Széchenyi prize in 2016,

Zoltán Füredi – Széchenyi prize in 2018,

Ervin Győri – Szele prize in 2018,

János Pach – Szele prize in 2019

Gábor Tardos – Gödel prize 2020 ,

Endre Szemerédi- Szent István Order of Merit 2020

to mention just the most important ones.

The participants of the project wrote 261 research papers about the results they achieved, and a fundamental book about extremal set family. At different conferences, we gave about 300 lectures about these results despite of the Covid-19 pandemic. The number of plenary talks is more than 100. The most important was Gábor Tardos was invited speaker on ICM2018. Beyond that, many of us were invited speakers on satellite conferences of ICM (Bárány, Füredi, Győri, OH Katona and Tardos) and even more on other conferences.

One of the important tasks was to help young researchers to start their research works. We employed Gábor Mészáros on the budget of the project who wrote his PhD thesis under the supervision of the project leader. His nice work might be measured by the fact that he won the Ralph Faudree Assistant Professorship at the University of Memphis. After he left, we employed Tamás Mezei (PhD student of the project leader at CEU at that time) in the project. He defended his PhD thesis and he got a young researcher position in Renyi Institute. Then we employed Lucas Colucci and helped him to defend his PhD thesis written in the subject of the project and under the supervision of the leader of this project. In the last months, we employed one more PhD student of the project leader. Debarun Ghosh is expected to defend his thesis in 2021. We included other PhD students and young researchers (Beka Ergemlidze, Abhishek Methuku, Nika Salia, Casey Tompkins) too in the work as un-paid participants. During the project, Mezei, Methuku, Ergemlidze, Colucci received their PhD degree, Salia and Ghosh are expected to defend their thesis in 2021.

Before the research papers, I should mention that Gerbner and Patkos wrote a 336 page book on extremal problems on subsets of a finite set. The book was published by Chapman and Hall/CRC and is intended for graduate students and researchers.

Now, we try to summarize the most important results of the team in the subject of the project.

Extremal graph theory

We proved the quarter of a century old conjecture of Erdős that every K4-free graph with n vertices and $t_2(n) + m$ edges contains m pairwise edge disjoint triangles.

We proved the list version of the Bollobás Eldridge packing conjecture for graphs with few edges. It yields a new technique which is already used in another paper to prove a good approximation of Zak's conjecture, but several other applications are expected too.

We proved several theorems for terminal-pairability problems in several papers. Usually there is given a base graph and a non-simple demand graph. The problem is to find conditions, when there exist a bunch of pairwise edge disjoint paths in the base graph, that for any vertex pair the number of the adjacent paths to the pair is the same as the value given by the demand graph. The question is also known in theoretical computer science under the name of edge-disjoint paths problem. This problem can be studied in algorithmic mean (as in TCS). In general it is known to be NP-complete, but the problem solved for several special cases. From an extremal point of view, it is easy to see that in the bipartite case n/3 is an upper bound for the vertex degrees in the demand graph. Considering this we came up with a tidy solution that if the maximal degree is below roughly n/4 then the problem is solvable. We affirmatively answered and generalized the question of Kubicka, Kubicki and Lehel (1999) concerning the path-pairability of high-dimensional complete grid graphs. As an intriguing by-product of our result we significantly improve the estimate of the necessary maximum degree in path-pairable graphs, a question originally raised and studied by Faudree, Gyárfás, and Lehel (1999).

We also investigated the terminal-pairability problem in the case when the base graph is a complete bipartite graph, and the demand graph is also bipartite with the same color classes. We improved the lower bound on maximum value of $\Delta(D)$ which still guarantees that the demand graph D is terminal-pairable in this setting.

Furthermore, we investigated the terminal-pairability problem in the case when the base graph is a complete bipartite graph, and the demand graph is a (not necessarily bipartite) multigraph on the same vertex set. We improved the lower bound on the maximum value of $\Delta(D)$ which still guarantees that the demand graph D has a realization in $K_{n,n}$.

Vera T. Sos asked in 1991 what is the minimum number of 3-edge-colorings of the complete graph on n vertices that are needed to ensure that every triangle gets 3-colored in at least one of them.
This question is still open. We gave the exact answer to a related question: we give the minimum number of orientations of the edges of the complete graph on n vertices needed to ensure that each triangle gets cyclically oriented in at least one
of them..

We proved surprising stability of the Erdős-Gallai Theorem on cycles and paths and it is expected to be widely used in extremal graph theory.

We investigate the number of 4-edge paths in graphs with a given number of vertices and edges, proving an asymptotically sharp upper bound on this number. The extremal construction is the quasi-star or the quasi-clique graph, depending on the edge density.

We investigated processes for constructing non-negative topological orderings of weighted directed acyclic graphs. We answered a question of Erickson by showing that every non-negative topological ordering that can be realized by a mark-unmark sequence can also be realized by a mark sequence.

We determined the 2-color Ramsey number of a connected triangle matching which is any connected graph containing n vertex disjoint triangles.

We modeled learning using a graph-theoretical model, which captures salient characteristics of the learning process. We investigated bootstrap percolation with excitatory and inhibitory vertices.

We investigated the number of 4-edge paths in graphs with a given number of vertices and edges, proving an asymptotically sharp upper bound on this number. The extremal construction is the quasi-star or the quasi-clique graph, depending on the edge density. An easy lower bound is also proved. This answer resembles the classic theorem of Ahlswede and Katona about the maximal number of 2-edge paths, and a recent theorem of Kenyon, Radin, Ren and Sadun about k-edge stars.

We have finished publishing the approximate version, general case of the proof of the Loebl-Komlos-Sos conjecture, on embedding trees into graphs under the condition that the median degree of the graph is sufficiently large. It was an enormous work, the papers appeared in four papers of about 170 pages in total.

We gave an asymptotic formula for the minimum number of edges contained in triangles among graphs with n vertices and e edges.

We determined the order of magnitude of the Kneser rank of random graphs (with high probability). We applied this for other graph representations defined by Boros, Gurvich and Meshulam.

Suppose we are given a set L of positive integers, and a graph where every cycle has length in L. What is the maximum number of cycles in that graph? We determine the order of magnitude, and for directed graphs we determine the asymptotics.

Improving our earlier result we showed that for every integer $k\geq 1$ there exists a c(k) such that in every 2-colored complete graph apart from at most c(k) vertices the vertex set can be covered by 200k2logk vertex disjoint monochromatic k-th powers of cycles.

We proved that 2k-regular 4-cycle-free graphs on 4k+1 vertices contain a clique of size k+1. This is best possible as shown by the kth power of the cycle.

It is well-known that in every k-coloring of the edges of the complete graph of n vertices, there is a monochromatic connected component of order at least n/(k−1). We studied an extension of this problem by replacing complete graphs by graphs of large minimum degree.

A new graph model has been introduced in 2015 which is a combination of fixed grid edges and additional random ones. These graphs model the connections between the neurons in the brain. The critical values for non monotone bootstrap percolation (i.e., vertex activation process) were determined only in the mean field approximation before. In 2017, the critical values of percolation have been determined in the real graph model. Numerical results show that the critical values in the mean field do not approximate the ones in the real graph model but rather those values, where the behavior of the relative density of the active points changes from monotone decreasing to monotone increasing.

We studied a possible generalization of restricted degree sequences introduced at the Joint Degree Matrix problem. Here the vertex set is partitioned and we give structural conditions on the number of edges among certain classes. Our results can be considered as possible initial steps into the topic.

We considered the oriented vertex Turán problem in the hypercube: for a fixed oriented graph F, determined the maximum size of a subset U of the vertices of the oriented hypercube Q_n such that the induced subgraph does not contain any copy of F.

We showed that the switch Markov chain is rapidly mixing on stable degree sequences of simple, bipartite, and directed graphs.

Generalizing Turán's classical extremal problem, Alon and Shikhelman investigated the problem of maximizing the number of T copies in an H-free graph, for a pair of graphs T and H. We found asymptotic and in some cases exact results in the cases when T and H are paths.

Bollobas and Gyori initiated studying of maximum number of triangles in C_5 graph, and gave an upper bound, it was improved by Alon and Shikelman. Now, we further improved the upper bound.

We estimated the rainbow Turan number of a path with k+1 edges, improving an earlier estimate of Johnston, Palmer and Sarkar.

We investigated properties of minimally t-tough graphs. We proved that the maximum degree in such graphs is not more than n/3+1. It is also shown, that for every positive rational number t any graph can be embedded as an induced subgraph into a minimally t-tough graph.

We proved that for any positive rationalnumber $t \leq 1$ and for any $k \geq 2$ and $r \geq 6$ integers recognizing t-tough bipartite graphs is coNP-complete, and this problem remains coNP-complete for k-connected bipartite graphs, and so does the problem of recognizing 1-tough r-regular bipartite graphs.

We proved a necessary and sufficient condition for the decomposability of a multigraph into two odd subgraphs (i.e all degrees are odd). We also presented a polynomial time algorithm for finding such a decomposition or showing its non-existence

We found sufficient conditions for the isomorphic orientability of isomorphic undirected graphs that partition the edge set of the complete graph, in such a way that the union of the so obtained oriented graphs gives a transitive tournament.

The minimum number of colors is determined for infinitely many n that suffice for a coloring of the edges of the n-vertex complete graph so that no 4-vertex path subgraph is left monochromatic while at the same time at least one copy of the 4-vertex path on every 4 vertices is totally multicolored.

We studied the maximum number of copies of a graph H in graphs with fixed number of edges and vertices. We showed that for any H, if the number of edges is large enough compared to the number of vertices, then the so-called quasi-clique has asymptotically the largest number of copies of H.

We considered coverings of edges of a graph with cliques. Our goal was to minimize the sum of the orders of the cliques in the covering. We determine this number for balanced complete multipartite graphs asymptotically.

We determined asymptotically the number of cycles and paths of given length if $K_{2,t}$ is forbidden.

We counted cycles in a graph when another cycle is forbidden. We determined the order of magnitude if both cycles are even. We obtained asymptotic result in some cases. We also prove several theorems in case multiple cycles are forbidden.

The 3-color Ramsey number of paths is determined (and the extremal 3-colorings follow the pattern of the 2-color case). However, for more than three colors a new pattern emerges, showing that further advances need new methods.

It is well-known that in every r -coloring of the edges of the complete bipartite graph $K_{m,n}$ there is a monochromatic connected component with at least (m+n)/r vertices. We studied an extension of this problem by replacing complete bipartite graphs by bipartite graphs of large minimum degree.

We showed that a 3-connected claw-free graph G always has a cycle passing through any given five vertices but avoiding any other one specified vertex. We also show that this result is sharp by exhibiting an infinite family of 3-connected claw-free graphs in which there is no cycle containing a certain set of six vertices but avoiding a seventh specified vertex.

We examined the so called generalized Turán number for even cycles, that is we count the number of 2k-cycles if 2l-cycles are forbidden. We determined the order of this quantity for every k and l and achieved asymptotics in some cases.

Generalizing Turán's classical extremal problem, Alon and Shikhelman investigated the problem of maximizing the number of T copies in an H-free graph, for a pair of graphs T and H. We focused on the case when T and H are paths, where we found asymptotic and in some cases exact results.

In the classical extremal graph theory, we determined the Turán number of the square of a short path. The case of longer paths seems to be difficult, though we found a plausible conjecture.

The rainbow Turan number of F is defined as the maximum number of edges in a properly edge-colored graph on n vertices with no rainbow copy of F. We determined the rainbow Turan number of paths and cycles.

An elementary proof is given for the non-3-colorability of 4-chromatic Schrijver graphs.
Another coloring type result: We showed that every balanced distribution of k colors can be realized as a Gallai coloring provided that k is at most n/2.

We studied the edge disjoint caterpillar realizations of tree degree sequences. We gave the necessary and sufficient conditions when two tree degree sequences have edge disjoint caterpillar realizations.

It is well known that in every r-coloring of the edges of the complete bipartite graph K(n,n), there is a monochromatic component wit at least 2n/r vertices. We proved that there is such a component that is balanced, i.e. meets both sides in at
least n/r vertices for r=2,3 but it fails if r is large. In a related paper we do not require balancedness and completeness, but large minimum degree.

We determined Ramsey numbers of path-matchings with a small error. This is a very interesting generalization of the theorem about the Ramsey numbers of matchings.

We gave upper bounds in terms of graphs Ramsey numbers and sharper bounds and exact results when the target graph is a Berge triangle or a Berge K_4.

Anti-Ramsey number is another well-studied releted notion. We determined the exact value of the anti-Ramsey number for star forests and the approximate value of the anti-Ramsey number for linear forests. In a related paper we studied the maximum number of colors in an edge-coloring of K_n with no properly colored copy of G. We determined it for long paths and for a few other small graphs

We started to work on extremal problems in planar graphs. We determined the maximum Wiener index in triangulated plane graphs. . We also determined the maximum Wiener index in quadrangulation graphs. These results are confirming conjectures of Czabarka et al.

We also determined the maximum number of pentagons in a planar graph. We also started to work on the maximum number of edges in a planar graph not containing a given cycle as a subgraph.

It was proved earlier by Sali and Simonyi that self-complementary graphs can always be oriented in such a way that the union of the oriented version and its isomorphically oriented complement gives a transitive tournament. We investigated the possibilities of generalizing this theorem to decompositions of the complete graph into three or more isomorphic graphs. We found that a complete characterization of when an orientation with similar properties is possible seems elusive. Nevertheless, we gave sufficient conditions that generalize the earlier theorem and also imply that decompositions of odd vertex complete graphs to Hamiltonian cycles admit such an orientation. These conditions are further generalized and some necessary conditions are given as well.

We gave several sufficient conditions and one necessary condition to guarantee a Gallai coloring .

Real-world networks evolve over time via the addition or removal of vertices and edges. In current network evolution models, vertex degree varies or can even grow arbitrarily, yet there are many networks in which it saturates, such as the number of active contacts of a person, or it is fixed, such as the valence of an atom in a chemical complex, thus requiring an entirely different description.
We introduced a novel class of models that encapsulates degree preserving dynamics in the simplest form, resulting in structures significantly different from previous ones. We discuss their properties as function of the evolution of the network's matching number and present several generative models based on this framework, from growing uniform degree distribution graphs to growing random regular graphs, which, to our best knowledge, is the first model of this kind. Moreover, this process can generate scale-free networks with arbitrary exponents, but without involving any degree-based preferential attachment. Within this approach, we also introduced configuration-like models that realize given degree sequences, with tunable degree-mixing properties.

Extremal hypergraphs

Various hypergraph versions of the Erdős-Gallai theorem on paths were proved. The well-known theorem of Erdos and Gallai asserts that a graph with no path of length k contains at most (k−1)n/2 edges. Recently, we gave an extension of this result to hypergraphs by determining the maximum number of hyperedges in an r-uniform hypergraph containing no Berge path of length k for all values of r and k except for k = r + 1. Now, we settled the remaining case by proving that an r-uniform hypergraph with more than n edges must contain a Berge path of length r + 1.

The maximum size of an r-uniform hypergraph without a Berge cycle of length at least k has been determined for all k≥r+3 by Füredi, Kostochka and Luo and for k<r (and k=r, asymptotically) by Kostochka and Luo. We settled the remaining cases: k=r+1 and k=r+2, proving a conjecture of Füredi, Kostochka and Luo.

We study the structure of r-uniform hypergraphs containing no Berge cycles of length at least k for k-1< r, and determined that such hypergraphs have some special substructure. In particular we determined the extremal number in such hypergraphs, gave an afirmative answer to the conjectured value when k=r.

We estimated the Ramsey number of uniform hypergraphs defined by taking Berge copies of a graph H.

We investigated the maximum possible number of k-chains in the set systems avoiding certain subposets P. We showed that the answer's order of magnitude depends of whether the height of P is larger then k or not.

We studied the so-called tilted Sperner families with patterns. We improved the best known bound on the maximum cardinality of such a family.

The notion of a tree is well known and investigated for graphs. We extended the definition for hypergraphs using tight-paths to replace the ordinary graphs and gave different bounds for the number of edges for hypergraphs defined in this way.

We introduced a new model for epidemic propagation. The traditional models used only graphs, the new model uses hypergraphs. This makes it possible to include more complex scenarios.
We obtained several results about forbidden partially ordered set problems in the Boolean lattice. The most important one is the improvement of the bound in the famous case when the forbidden configuration is the diamond poset.

We determined the exact Turan number for a number of hypergraphs. These investigations might lead to the proof of the Erdos-Sos-Kalai conjecture.

We proved a very strong generalization of the celebrated Erdos-Ko-Rado theorem.

Upper bounds to the size of a family of subsets of an n-element set that avoids certain configurations are proved. These forbidden configurations can be described by inclusion patterns and some sets having the same size. Our results are closely related to the forbidden subposet problems, where the avoided configurations are described solely by inclusions.

We generalized the established definitions of "Berge path" and "Berge cycle" to general graphs in a straightforward way. A hypergraph H is a Berge copy of a graph G if there is a bijection between the hyperedges of H and the edges of G such that the image of a hyperedge is a subset of it. For given G, we study extremal properties of hypergraphs that do not contain any Berge copy of G.

We considered the problem of 2-coloring geometric hypergraphs. Specifically, we showed that there is a constant m such that any finite set S of points in the plane can be 2-colored such that every axis-parallel square that contains at least m points from S contains points of both colors.

In 3-uniform hypergraphs, we determined the linear Turan number of the five cycle in the Berge sense asymptotically.

We studied a variant of forbidden subposet problems, where we only forbid those copies of a ranked poset, where elements of the same rank have the same rank in the Boolean poset as well. We proved asmptotically sharp results for some posets.

We determined the asymptotics for the Turan number of the 3-uniform Berge copies of the complete bipartite graph with parts of size 2 and t>6.

We proved two general lemmas concerning the Turán number of Berge hypergraphs. Both are generalizations of some known results.

We showed that the saturation number is linear for any Berge hypergraph of uniformity three, four or five.

We extended results on a Ramsey type problem raised in 1977 and extended to hypergraphs in 2017. The results show the connection of the problem to a classical area of design theory, the existence of large sets of t-designs.

The exact value of the Turan number of a Berge triangle in triple systems is known for fifteen years (Gyori, Frankl, Furedi, Simonyi). For larger complete graphs asymptotic and later exact results for large n were discovered. The exact value for Berge-K_4 is determined in this paper for every n.

We proved two general lemmas concerning the maximum size of a Berge-F-free hypergraph and used them to establish new results and improve several old results.

We studied the structure of r-uniform hypergraphs containing no Berge cycles of length at least k for $k \leq r$. In particular, we determine the extremal number of such hypergraphs, giving an affirmative answer to the conjectured value when $k=r$ and giving a simple solution to a recent result of Kostochka-Luo when $k<r$.

The maximum size of an r-uniform hypergraph without a Berge cycle of length at least k has been determined for all $k \geq r+3$ by Füredi, Kostochka and Luo and for $k<r$ (and $k=r$, asymptotically) by Kostochka and Luo. We settled the remaining cases: $k=r+1$ and $k=r+2$, proving a conjecture of Füredi, Kostochka and Luo.

We estimated the maximum number of hyperedges in a 3-uniform hypergraph on n vertices without a (Berge) cycle of length five, improving the estimate of Bollobás and Győri.

We asymptotically determined the Turán number of t-heavy and t-wise Berge copies of long paths and cycles and exactly determined the Turán number of t-heavy and t-wise Berge copies of cliques. (A hypergraph H is a t-heavy copy of a graph F if there is a copy of F on its vertex set such that each edge of F is contained in at least t hyperedges of H. H is a t-wise Berge copy of F if additionally for distinct edges of F those t hyperedges are distinct.)

We obtain upper bounds on the number of hyperedges in 3-uniform hypergraphs not containing a Berge cycle of given odd length. We improve the bound given by Füredi and Özkahya.

We gave a short, concise proof that there exists a k-uniform hypergraph H (for $k \geq 5$) without exponent, i.e., when the Turán function is not polynomial in n. More precisely, we have $ex(n,H)=o(n^{k-1})$ but it exceeds n^{k-1-c} for any positive c for $n>n_0(k,c)$.

We disproved a conjecture of Kuzmin and Warmuth claiming that every family whose VC-dimension is at most d admits an unlabeled compression scheme to a sample of size at most d. We also studied the unlabeled compression schemes of the joins of some families and conjecture that these give a larger gap between the VC-dimension and the size of the smallest
unlabeled compression scheme for them.

We established basic results on the maximum number of k-chains in a P-free family formed by subsets of an n-element set. We show that the order of magnitude of this quantity depends only on the height of the forbidden poset P.

We proved asymptotically optimal upper bounds on $La_{rp}(n,P)$ for tree posets of height 2 and monotone tree posets of height 3, strengthening a result of Bukh in these cases.

We showed that the 2-color Ramsey number of Berge cliques is linear. In particular, we show that R3(BK_s,BK_t)=s+t−3 for s,t≥4 and max(s,t)≥5 where BK_n is a Berge-K_n hypergraph.

We determined the Turan number of the Berge K_4 exactly: for n>5 the maximum number of triples in a Berge-K_4 free triple system is obtained by the balanced complete 3-partite triple system.

We investigated the variant of a classical theorem of Bollobas, for intersecting set-pair systems under the assumption that each intersection is of size one. Somewhat surprisingly these systems can be still exponential but we get several particular properties of these systems and some applications and connections to other areas, as perfect graphs and clique partition problems.

An ordered hypergraph is a hypergraph whose vertex set is linearly ordered, and a convex geometric hypergraph is a hypergraph whose vertex set is cyclically ordered. We consider analogous extremal problems for uniform hypergraphs, and determine the order of magnitude of the extremal function for various ordered and convex geometric paths and matchings. Our results generalize earlier works of Brass-Karolyi-Valtr, Capoyleas-Pach and Aronov-Dujmovic-Morin-Ooms-da Silveira. We also proved a new generalization of the Erdos-Ko-Rado theorem in the ordered setting. In another paper, we proved a nice splitting theorem for ordered hypergraphs.

For a fixed linear triple system $, the linear Turan number ex_L(n,F) is the maximum number of triples in a linear triple system with n points that does not contain F as a subsystem. We obtained tight bounds for several important acyclic linear triple systems.

Geometry related problems

We studied the realizations of n angles by placing m points in the d dimensional space. The main result is that there are sets of 2m-3 angles that cannot be realised by m points.

We studied the problem of guarding orthogonal art galleries with horizontal mobile guards (alternatively, vertical) and point guards, using "rectangular vision". We prove a sharp bound on the minimum number of point guards required to cover the gallery in terms of the minimum number of vertical mobile guards and the minimum number of horizontal mobile guards required to cover the gallery. Furthermore, we show that the latter two numbers can be calculated in linear time. We proved a sophisticated partitioning theorem what implies the 20 page proof theorem of Aggarwal, but actually it is much stronger, e.g. it follows the answer to other open questions.

We prove that if an n-vertex graph G can be drawn in the plane such that each pair of crossing edges is independent and there is a crossing-free edge that connects their endpoints, then G has O(n) edges.

We showed that there is a constant m such that any finite set S of points in the plane can be 2-colored such that every axis-parallel square that contains at least m points from S contains points of both colors.

We showed that if a curve in the d dimensional Euclidean space intersects every hyperplane at most d+1 times, then the curve can be split into C(d) convex curves. (A curve is convex by definition if every hyperplane intersects it at most d times. The question is connected to geometric Ramsey type theorem, and the result implies the right order of magnitude for the corresponding Ramsey number.

A theorem of Steinitz from 1914 says the following. Given finitely many (at most) unit vectors in the d dimensional Euclidean space whose sum is zero, there is an ordering of these vectors so that along this ordering all partial sums are bounded, namely, all of them have norm at most 2d. We proved several extensions and generalizations of this important result.

Given a tree T on n vertices where every degree is at most 3 or 4, one want to embed to in a planar grid of small size so that the edges go to rectilinear zig-zags with no crossing. The question is to determine the smallest size grid where this is possible, and to give an algorithm to find the corresponding embedding. We gave such algorithms that improve on the previous upper bounds.

Verifying a conjecture of Hadwiger and Debrunner Alon and Kleitman proved that for any p>q>d there exist a number N such that for any family compact convex sets in d-space that satisfy the (p,q)-property one can find N points such that each set in the family contains one of the points. Here the (p,q)-property means that among any p of the sets one can find q with a point in common. We greatly improved the estimate for N as a function of p, q and d.

We studied whether for a given planar family F of points there is an m such that any finite set of points can be 3-colored such that any member of F that contains at least m points contains two points with different colors. We proved that when F is the family of all homothetic copies of a given convex polygon, then such an m exists.

We proved that if an n-vertex graph G can be drawn in the plane such that each pair of crossing edges is independent and there is a crossing-free edge that connects their endpoints, then G has O(n) edges.

The intrinsic volumes of Gaussian polytopes were considered. A lower variance bound for these quantities is proved, showing that, under suitable normalization, the variances converge to strictly positive limits. The implications of this missing piece of the jigsaw in the theory of Gaussian polytopes are discussed.

We proved that if an n-vertex graph G can be drawn in the plane such that each pair of crossing edges is independent and there is a crossing-free edge that connects their endpoints, then G has O(n) edges.

We proved that the intersection hypergraph of a family of n pseudo-disks with respect to another family of pseudo-disks admits a proper coloring with 4 colors and a conflict-free coloring with O(log n) colors.

We also proved that the intersection hypergraph of a family of n regions with linear union complexity with respect to a family of pseudo-disks admits a proper coloring with constantly many colors and a conflict-free coloring with $O(\log n)$ colors. Our results serve as a common generalization and strengthening of many earlier results, including ones about proper and conflict-free coloring points with respect to pseudo-disks, coloring regions of linear union complexity with respect to points and coloring disks with respect to disks.

We studied whether for a given planar family F there is an m such that any finite set of points can be 3-colored such that any member of F that contains at least m points contains two points with different colors. We conjecture that if F is a family of pseudo-disks, then m=3 is sufficient. We proved that when F is the family of all homothetic copies of a given convex polygon, then such an m exists.

Dimension free versions of the Carathedory, Helly, and Tverberg theorems were proved with several consequences.

A new proof of the "first selection lemma" is given, based on some special Radon partitions. (in Applications of the universal theorem for Tverberg partitions)

A geometry type extremal result is as follows. We showed that any set of n points in general position in the plane determines n^(1−o(1)) pairwise crossing segments. The best previously known lower bound was of order n^1/2, proved more than 25 years ago.

Another hypergraph/geometry related coloring problem is to determine the minimum number of colors that always suffice to color every planar set of points such that any disk that contains enough points contains two points of different colors?
It is known that the answer to this question is either three or four. We showed that three colors always suffice if the condition must be satisfied only by disks that contain a fixed point. Our result also holds, and is even tight, when instead of disks we consider their topological generalization, namely pseudo-disks, with a non-empty intersection.

We proved that the intersection hypergraph of a family of n pseudo-disks with respect to another family of pseudo-disks admits a proper coloring with 4 colors and a conflict-free coloring with O(log n) colors. Along the way we proved that the respective Delaunay-graph is planar.

It was known that unlike intersection graphs of segments in the plane, their complement, the disjointness graphs of segments in the plane is chi-bounded. We extend this to disjointness graphs of segments in higher dimensional spaces and establish that their bounding function is at most slightly higher than the bounding function for disjointness graphs of planar segments.

The Erdős–Szekeres problem is to find the largest number f(n) such that among n points in general position in the plane one can always find f(n) in convex position. Andrew Suk's breakthrough result established that Erdős and Szekeres's original upper bound for f(n) is asymptoticly tight. We improved on this result in two ways: we decreased the error term in Suk's bound and extended the result from the ordinary plane to any pseudoline-arrangement.

Other related problems

We improved the conjectures optimal pebbling number construction in square grids. Evenmore, the best upper bound is improved too. We have written further papers on various topics about optimal pebbling too. We also determined the optimal pebbling numbers of staircase graphs up to width 7. In a forthcoming paper we gave lower bounds on both the optimal pebbling and rubbling numbers by the distance k domination number. With this bound it is proved that for each k there is a graph G with diameter k and optimal pebbling and rubbling number equal to 2^k.

We determined the optimal pebbling number for several classes of induced subgraphs of the square grid, which we call staircase graphs.

We studied the following problem: Given a weighted directed acyclic graph, we investigated the orderings where every prefix is non-negative and outdirected. One can get such an ordering by marking the vertices in that order. A generalization is when one is allowed to 'unmark' the vertices. It is proven here that this is not useful in some cases.

We proved several extremal results on 01 and r-matrices not containing special configurations. A (0,1)-matrix is simple if it has no repeated columns. Let F be a set of (0,1)-matrices. Let forb(m,F) denote the maximum number of columns possible in a simple (0,1)-matrix A that has no submatrix which is a row and column permutation of any B from F. We investigated the case when F consists of pairs of minimal quadratic and simple cubic configurations continuing a research started by Anstee and Koch. A stability theorem is also proven for matrices avoiding a certain basic configuration.

We studied a new group testing model, where the elements are considered participants, and receive the results of tests they are involved in.

Let P and Q be families of finite relational structures. We considered structures of a fixed type, for example directed graphs. We say P and Q form a duality pair if for any structure X of this type either there is a homomorphism from an element of P to X or a homomorphism from X to an element of Q but not both. We characterized the antichains P forming a duality pair with a suitable finite set Q.

We studied combinatorial parameters of a recently introduced bootstrap percolation problem in finite projective planes. We obtained sharp results on the size of the minimum percolating sets and the maximal non-percolating sets.

We continued our research on matrices not containing given configurations, particularly multi-symbol forbidden configurations.

We also proved new bounds on Armstrong codes.

We introduced a new concept of handling NULL values in database tables. It is based on interpreting the Null as information unkonwn, so strongly possible worlds are defined by replacing the NULL values by data already occuring in the given column of table. Earlier possible worlds were defined by replacing NULL values by any possible value from the attribute's domain. Then possible keys, functional dependencies are those that are satisfied by some possible world and certain keys and functional dependencies are those that are satisfied by every possible world. Our strongly possible keys and functional dependencies are intermediate between possible and certain constraints. Our papers contain implication properties, mathematical and algorithmic problems of implications, verifications of the constraints.