Integrability and holographic duality

The main aim of our project was to elaborate a program in order to determine exactly the three point (3pt) couplings of the simplest nontrivial 4 dimensional (4D) gauge theory. This theory, which is the maximally supersymmetric theory in 4 dimension, is conformal and considered to be the hydrogen atom of all gauge theories, including the theories of the strong and electroweak interactions. Together with the already determined scaling dimensions the 3pt couplings provide the missing fundamental data of this conformal field theory, from which it can be completely reconstructed. The feasibility of our project derived from the fact that the gauge theory is holographically dual to string theory on the product of the 5D anti de Sitter space and the 5D sphere. As string theory on this background is described by an integrable two-dimensional quantum field theory, we could exploit our expertise, to investigate a non-perturbative 4D gauge theory in terms of its spectrum and correlation functions.

We organized the work around four topics as follows

1. Diagonal 3pt functions
2. Non-local operator insertion, the string field theory vertex
3. Boundary generalization of the program
4. Other simplified models of string theory

The project with small detours and differences in the timing and objectives followed the original plan. Additionally, we also solved relevant unforseen problems and opened new research directions. During the project we achieved the following results

1 Diagonal 3-point functions

There is a special class of 3-point functions in the maximally supersymmetric 4D gauge theory, where the generic methods do not work and we need to take a different approach. These are the 3-point functions, in which one of the operators does not have any R-charge and the other two operators have large and opposite charges. These are called heavy-heavy-light (HHL) 3-point functions and are special also in the sense that at strong coupling they have a geometrical meaning: they are related to classical string solutions with operator insertions. We had already managed to map these 3-point functions to diagonal form factors, i.e. to diagonal matrix elements of local operators in [1].

Since we had already performed a weak coupling check of the relation between HHL 3pt functions and diagonal form factors [2] we focused on the strong coupling side. As a first step we proposed a novel expression for the classical limit of diagonal form factors in which we integrated the corresponding observable over the moduli space of classical solutions. In infinite volume the integral had to be regularized by proper subtractions and we presented the one, which corresponds to connected form factors. In finite volume the integral is finite and we expressed it in terms of the infinite volume diagonal form factors and subvolumes of the moduli space. By applying the results to the HHL 3pt functions in the AdS/CFT setting we could express their strong coupling limit in terms of the classical limit of the sine-Gordon diagonal matrix elements of the exponential operators [R21][1]. We then started to work for generic couplings in order to calculate the finite size effects of diagonal form factors.

The leading corrections are polynomial in the inverse of the volume, for which only a conjecture existed in the literature, until we managed to prove it in [R20]. We did it by carefully evaluating the diagonal limit of a non-diagonal form factor and our results are valid for any diagonally scattering theories. We also elaborated on how these results can be extended for non-diagonally scattering theories such as for the sine-Gordon theory in [R37]. In order to understand all the exponentially small finite size corrections of diagonal form factors in the sine-Gordon theory we analyzed the continuum limit of its integrable lattice regularization. This approach enabled us to determine the continuum limit

[1] [Rn] refers to the nth paper in list of publication of the project (printview).

of the norm of the Bethe states and provided the finite volume expectation values of various local operators [R23,R30,R37]. Our results summed up the contributions of virtual particles and provided compact exact expressions for the diagonal finite volume form factors. We tested these results in the pure multi-soliton sector by comparing - their ultraviolet limit to Liouville 3-point functions, while - their infrared limit to the previously conjectured LeClair-Mussardo type formula. We also performed a similar analysis for the diagonal form factors of all operators in the sinh-Gordon theory [R35]. In order to apply these formulae for the HHL 3pt function of the gauge theory we need the infinite volume form factors. These could be only obtained from completing our second project on the non-local operator insertion.

2 Non-local operator insertion, the string field theory vertex

We had previously formulated very restrictive functional relations for the asymptotic 3pt functions, namely for the string field theory vertex in [3]. Due to the very complicated structure of the scattering matrix, however, none could find any solution even for the AdS form factor equations yet. We then decided to factorize the problem into a kinematical and a dynamical part. The first is responsible for taking into account the AdS kinematics, while the second should be the solution of the ordinary form factor equations. We found the solution in the first case, which correctly reproduced the plane-wave limit [R5]. In the meantime another approach based on cutting the string field theory vertex into two hexagons proved to be successful for the asymptotic 3-pt function [4]. We demonstrated in [R22] the relation between our program and the one based on hexagons. We showed that gluing two hexagons leads to the our octagon, while gluing the octagons leads to our non-local operator form factor. Finally, gluing the remaining side sums up all the finite volume effects of a nondiagonal form factor. In order to understand the finite size effects of the non-diagonal form factors we developed a systematic method to calculate both the finite volume energy levels and the form factors from the momentum space finite volume two-point function [R27]. In order to demonstrate how the method works we extracted the leading exponential volume correction both to the energy of a moving particle state and to the simplest non-diagonal form factor. We tested these results against second order Lagrangian and Hamiltonian perturbation theory in the sinh-Gordon theory and we obtained perfect agreement. We managed to extend these results for the leading exponential volume correction for the generic non-diagonal form factor. We then developed numerical techniques (Hamiltonian truncation) in the Lee-Yang and sinh-Gordon models and checked our formulas [R36]. The generalization for the non-diagonally scattering sine-Gordon models was recently achieved by Hegedüs. Presently we are extending this analysis to derive all the exponentially small finite size corrections for non-diagonal form factors and planning to apply them to the 3pt functions.

3 Boundary generalization of the program

In the boundary generalization of our program we had to analyze the finite volume form factors of boundary changing operators. To gain intuition and experiences with these operators we developed axioms for their form factors in relativistic theories [R4]. We found and classified all solutions satisfying the axioms in toys models such as in the scaling Lee-Yang and free boson theories. We also analyzed the effect of virtual particles on the spectrum in the presence of boundaries in the AdS/CFT setting [R7]. In order to develop a spectral curve formulation of open boundaries we investigated the Bethe Ansatz equations of the O(N) sigma models [R10] first. We then analyzed boundary conditions in a family of toy models relevant for string theory, namely we made an attempt to map the integrable boundary conditions for 2 dimensional non-linear O(N) σ-models [R15]. We did it at various levels: classically, by demanding the existence of infinitely many conserved local charges and also by constructing the double row transfer matrix from the Lax connection, which led to the spectral curve formulation of the problem, at the quantum level, we described the solutions of the boundary Yang–Baxter equation and derived the Bethe–Yang equations. We then showed how to connect the thermodynamic limit of the boundary Bethe–Yang equations to the spectral curve. Integrable boundary conditions are characterized by the solutions of the boundary Yang-Baxter equation. We managed to classify all solutions for rational scattering matrices [R43]. We then developed a new way of solving the corresponding boundary asymptotic Bethe Ansatz equations based on Q-system equations [R45].

We then analyzed boundary 3-point functions in planar N = 4 super Yang-Mills theory realized by operators on the 1/2 BPS Wilson loop [R17,R18]. The operators we considered were 'defect changing operators', which change the scalar coupled to the Wilson loop. We first performed the computation at two loops in general set-up, and then studied a special scaling limit called the ladders limit, in which the spectrum is known to be described by a quantum mechanics with the SL(2,R) symmetry. In this limit, we resumed the Feynman diagrams using the Schwinger-Dyson equation and determined the structure constants at all order in the rescaled coupling constant. Besides providing an interesting solvable example of defect conformal field theories, our result gives invaluable data for the integrability-based approach to the structure constants.

4 Other simplified models of string theory

Another direction of research was to analyze simplified models of string theory, which share interesting physics with the AdS backgrounds, but might admit explicit solutions. Such a theory is the topological string theory, which is a mathematical model of strings focusing on the fundamental properties of spacetime geometry. We derived the analog of the large N Gross-Taylor holomorphic string expansion for the refinement of q-deformed U(N) Yang-Mills theory on a compact oriented Riemann surface [R11]. The derivation combined Schur-Weyl duality for quantum groups with the Etingof-Kirillov theory of generalized quantum characters which are related to Macdonald polynomials. In the unrefined limit we reproduced the chiral expansion of q-deformed Yang-Mills theory.

M-theory is thought to be a unified framwork of string theory, but it's microscopic formulation is yet unknown. The fundamental degrees of freedom of the theory are thought to be membranes. In our work we studied a topological version of M-theory, based on membranes on manifolds of G_2 holonomy. We gave a new construction of topological membranes based on the so-called AKSZ construction. The main motivation to develop the AKSZ formulation was the study of fluxes, as related to generalized geometry. In addition, we studied a certain Courant sigma-model with constant R-flux, which we found related to quantum mechanincs with a non-associate star product [R28].

We studied AKSZ-type BV constructions for the topological A- and B-models within a double field theory formulation that incorporates backgrounds with geometric and non-geometric fluxes. We related them to a Courant sigma-model, on an open membrane, corresponding to a generalized complex structure, which reduces to the A- or B-models on the boundary. We introduced S-duality at the level of the membrane sigma-model based on the generalized complex structure, which exchanges the related AKSZ field theories, and interpret it as topological S-duality of the A- and B-models. Our approach leads to new classes of Courant algebroids associated to (generalized) complex geometry [R29].

Within string theoretic holographic dualities we studied Renyi entropies for geometries with Lifshitz scaling and hyperscaling violation and interpreted the results in terms of the thermodynamic stabilities of the dual black holes [R48].

5 Some new relevant directions

We initiated research also into new subjects and achieved relevant progress.

5.1 Gluon scattering amplitudes in AdS/CFT

Gluon scattering amplitudes in the maximally supersymmetric gauge theory are dual to the vacuum expectation values of light-like Wilson loops. Due to the holographic duality these VEVs can be calculated at strong coupling by evaluating the area of the minimal surface, which is spanned over the Wilson loops. These minimal surfaces are described by thermodynamic Bethe ansatz type integral equations. In order to expand analytically these equations one needs to know the relation between the masses of the particles and the parameters of the Lagrangian. In collaboration with a Japanese group we made relevant progress into this direction. We managed to calculate explicitly this mass-coupling relation in the simplest model, which has two arbitrarily tunable mass scales [R2,R3].

5.2 Integrable quantum mechanical systems

The Ruijsenaars-Schneider quantum mechanical models realize many interesting aspects of the sine-Gordon quantum field theory. As a first step in this direction we constructed a Lax pair for the classical hyperbolic van Diejen system with two independent coupling parameters. Built upon this construction, we showed that the dynamics can be solved by a projection method, which in turn allows us to initiate the study of the scattering properties [R12]. We also constructed global action-angle variables for their certain two-parameter family of deformations. Following Ruijsenaars' ideas on the translation invariant models, the proposed action-angle variables come from a thorough analysis of the commutation relation obeyed by the Lax matrix, whereas the proof of their canonicity is based on the study of the scattering theory. Also, in an appendix by S. Ruijsenaars, a novel proof of the spectral asymptotics of certain exponential type matrix flows was presented [R24].

5.3 One point functions in AdS/CFT

We investigated a problem, in which the AdS5xS5 background was deformed to accommodate some additional D-branes. On the gauge theory side this situation is equivalent to adding a codimension 1 defect in the Yang-Mills theory, which breaks part of the gauge symmetry and also the supersymmetry of the original model. In this model local gauge invariant operators acquire nontrivial vacuum expectation values, which can be calculated as overlaps between an integrable boundary state and finite volume multiparticle states. We managed to classify these overlaps at leading order in perturbation theory [R39]. We carried on our boundary investigations by determining the 1-loop 1-point functions in the defect version of the maximally supersymmetric 3+1 dimensional gauge theory. We then extended this result to include the all-loop wrapping free 1-point functions for various defect settings in [R44,R50] . We also investigated the general form of this type of overlaps and developed a new way, how such overlaps can be calculated in the truncated conformal space approach in [R51].

6 Summary and outcome

In summarizing, we elaborated a program to calculate the 3pt functions of the simplest nontrivial 4D gauge theory. One unforeseen difficulty was the solution of the form factor equations. We overcome this obstacle by constructing the form factor solution from gluing together hexagons formfactors. These gluing procedure is more complicated, however, than we expected before, so we had to understand better the finite size effects of non-diagonal form factors. We already advanced considerable into this direction and these extended research have many potential application besides the AdS/CFT correspondence including statistical physics and condensed matter systems. We have already won an NKFIH K_20 support K134946 to continue our successful research on this promising topic. During our present project we also successfully analyzed the boundary generalization of the 3pt functions, we investigated simplified string theories, integrable quantum mechanical systems and various other observables in the AdS/CFT correspondence including 1-point functions and gluing scattering amplitudes. We have also achieved many relevant, but not strongly connected results using integrability.

All in all, we published 51 journal articles, most of them in the leading, high impact journals of our field (see the list of publication on the portal [R1-R51]. During the project László Holló, Zoltán Kökényesi, Tamás Gombor and Márton Lájer defended their theses. István Vona got his Master degree and has been proceeding as a PhD student. We organized three workshops (PallaFest, Celebrating the 70th birthday of László Palla, Budapest, Workshop on 3-point functions in Gauge/String theories, Sao Paulo, Integrable Approaches to 3pt functions in AdS5/CFT4, Budapest).

References

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