Final report
NKFIH project 115926
Network processes, partial differential equations and their applications

The research has been carried out in the period 2016-01-01 - 2021-12-31 according to the research plan in the following main research fields

- Network processes
- Metastable periodic orbits in cellular neural networks and qualitative behavior of dynamical systems in biology
- Nonlinear functional partial differential equations
- Nonsmooth elliptic eigenvalue problems

The research results are presented below according to these main fields.

Network processes

Our research concerning complex networks is motivated, on one hand, by understanding epidemic propagation on networks and, on the other hand, by studying the dynamical behaviour of recurrent neural networks.

In [SP1], a new model, called super compact pairwise (PW) model is derived with only four equations in the context of describing susceptible–infected–susceptible (SIS) epidemic dynamics on heterogenous networks. This model is based on a new closure relation that involves not only the average degree but also the second and third moments of the degree distribution. Its derivation uses an a priori approximation of the degree distribution of susceptible nodes in terms of the degree distribution of the network. The new closure gives excellent agreement with heterogeneous PW models that contain significantly more differential equations.

Explicit bounds on the expected value of one-step processes are derived in [SP2], bracketing it between the mean-field model and another simple ODE. While the first one is a well-known approximation, this lower bound is new, and unlike an asymptotic result for infinite networks, these bounds can be used for networks with finite size. The new bounds require that the Markov transition rates are density dependent polynomials that satisfy a sign condition.

In [SP3] the SIR homogeneous mean-field and pairwise models and the heterogeneous mean-field model are studied. It is proved that the implicit equation for the final epidemic size has a unique solution, and that through writing the implicit equation as a fixed point equation in a suitable form, the iteration of the fixed point equation converges to the unique solution. The Markovian SIR epidemic model on finite networks is also studied by using the generation-based approach. Explicit analytic formulas are derived for the final size distribution for line and star graphs of arbitrary size.

An adaptive network model using SIS epidemic propagation with link-type dependent link activation and deletion is considered in [SP4]. Bifurcation analysis of the pairwise ODE approximation and the network-based stochastic simulation is carried out, showing that three typical behaviours may occur; namely, oscillations can be observed besides disease-free or endemic steady states. The oscillatory behaviour in the stochastic simulations is studied using Fourier analysis, as well as through analysing the exact master equations of the stochastic model.

Deterministic limit of a class of continuous time Markov chains is considered in [SP5], based purely on differential equation techniques. Starting from the linear system of master equations, ordinary differential equations for the moments and a partial differential equation, called Fokker–Planck equation, for the distribution is derived. Introducing closures at the level of the second and third moments, mean-field approximations are introduced. The accuracy of the mean-field approximations and the Fokker–Planck equation is investigated by using two differential equation-based and an operator semigroup-based approach.

Mathematical modelling of epidemic propagation on networks is extended to hypergraphs in [SP6]. This is motivated by the need to account for both the community structure and the nonlinear dependence of the infection pressure on the number of infected neighbours. The exact master equations of the propagation process are derived for an arbitrary hypergraph given by its incidence matrix. Based on these, moment closure approximation and mean-field models are introduced and compared to individual-based stochastic simulations. The simulation algorithm, developed for networks, is extended to hypergraphs. The effects of hypergraph structure and the model parameters are investigated via individual-based simulation results.

In [SP7], the SIS epidemic on a dynamic network model is considered with addition and deletion of links depending on node status. The resulting pairwise model is analyzed using bifurcation theory to map out the spectrum of all possible epidemic behaviours. It is found that depending on parameter values the network can become disconnected and show bistable-like behaviour whereas the endemic steady state sees the emergence of networks with qualitatively different degree distributions.

An adaptive network model using SIS epidemic propagation with link-number preserving link activation and deletion is considered in [SP8]. It is proved that transcritical, saddle-node and Andronov–Hopf bifurcations may occur. These bifurcation curves are determined analytically in the plane of two parameters by using the parametric representation method, together with the two co-dimensional Takens–Bogdanov bifurcation point. It is shown that this parameter plane is divided into four regions by the above bifurcation curves. The possible behaviours are: (a) globally stable disease-free steady state, (b) stable disease-free steady state with two unstable endemic equilibria and a stable periodic orbit, (c) stable disease-free steady state with a stable and an unstable endemic equilibrium and (d) a globally stable endemic equilibrium. Numerical evidence is shown that homoclinic bifurcation, giving rise to an unstable periodic orbit, and cycle-fold bifurcation also occur.

The monograph [SP9] gives a review of mathematical models of epidemic propagation on networks and summarizes the know results about them. Chapter 1 introduces the reader to the fundamentals of disease transmission models and the underlying networks. Chapter 2 frames disease transmission on a network as a continuous-time Markov chain. Chapter 3 builds a hierarchy of models starting at the node level which depend on the node–neighbour pairs. Chapters 4 and 5 focus on mean-field and pairwise models and their analysis. In Chapter 6, percolation theory methods are used to derive the low-dimensional edge-based compartmental model of SIR epidemic. Chapter 7 brings the different SIR models together. Chapters 8 and 9 extend the earlier models to adaptive networks and to non-Markovian epidemics, leading to integro-differential and delay differential equations. Chapter 10 starts from a Markov chain to derive the Fokker–Planck equation, and uses the resulting partial differential equation to investigate epidemic processes. Finally, Chapter 11 shows that our models can perform well in empirically observed networks.

In [SP10], comparison theorems from classical ODE theory are used in order to rigorously show that the N-Intertwined Mean-Field Approximation (NIMFA) model provides an upper estimate on the exact stochastic process describing SIS epidemic propagation on networks. The results are presented in a more general framework where alternative closures, other than that assuming the independence of nodes connected by an edge, are possible. Moreover, a succinct summary of the stability analysis of the resulting more general mean-field models is also provided.

Epidemic propagation is controlled conventionally by vaccination or by quarantine. These methods have been widely applied for different compartmental ODE models of epidemic propagation. When epidemic spread is considered on a network, then it is natural to control the propagation process by changing the network structure. Namely, SI links, connecting a susceptible individual to an infected one, can be deleted. This would lead to a disconnected network, which is not realistic, hence new SS links can be created in order to keep the network well connected. Thus it seems to be promising to drive the process to a target with no infection and a prescribed average degree by deleting SI links and creating SS links in an appropriate way. It was shown previously that this can be done for the pairwise ODE approximation of SIS epidemic propagation. In [SP11], this is extended to the original stochastic process by using the control signals computed from the ODE approximation.

Focusing on SIR epidemics on regular but clustered networks, it is shown in [SP12] and [SP13] that for the simplest and then even for the most complex closure we can determine the epidemic threshold as an asymptotic expansion in terms of the clustering coefficient.

The 20th European Conference on Mathematics for Industry, ECMI 2018 was held in Budapest from 18^{th} to 22^{nd} June 2018, the leader of this project was the chair of the organizing committee. Following the traditions of ECMI, the conference focused on various fields of industrial and applied mathematics, such as Applied Physics, Biology and Medicine, Cybersecurity, Data Science, Economy, Finance and Insurance, Energy, Production Systems, Social Challenges, Vehicles and Transportation. These themes nicely fit to current distinguished national research programs supported by the EU and the National Research, Development and Innovation Office. In addition to the nine plenary talks, given by world class researchers, we had 50 minisymposia, and 45 contributed talks and poster presentations, running in 7 parallel sessions. Altogether there were more than 350 participants, from around 40 countries. The conference proceedings [SP14] appeared in 2019 and was edited jointly by Ferenc Izsák, István Faragó and the leader of this project.

The global behaviour of the compact pairwise approximation of SIS epidemic propagation on networks is studied in [SP15] based on the reduction of the system to two equations. It is shown that transcritical bifurcation occurs in the system at a certain value of the infection rate, depending on the first and the second moments of the degree distribution. For subcritical values of the infection rate the disease-free steady state is stable, while for supercritical values a unique stable endemic equilibrium appears. It is also proven that for subcritical values the disease-free steady state is globally stable under certain assumptions on the graph that cover a wide class of networks.

A class of dynamical systems is introduced in [SP16], for which the local stability of the trivial steady state determines the global behaviour of the system. The dynamical behaviour is captured by a simple property of the right hand side of the corresponding system of ODEs. It is shown that this condition is satisfied by three network models, namely the individual-based and degree-based ODE approximations of SIS epidemic propagation on networks and the Hopfield model with non-negative weights. The general result enables us to describe the global behaviour of these systems that was not available for the first and third model and was proven in a significantly more complicated way for the second.

The problem of deriving triple closures for pairwise epidemic models is investigated in detail in [SP17] together with the assumptions behind some of the well-known closures as well as their validity. A top-down approach is used starting at the level of the entire graph and working down to the level of triples. Using this approach many of the existing closures are derived and new ones are proposed and theoretically connected the two well-studied models of multinomial link and Poisson link selection.

The aim of [SP18] and [SP19] is to understand the effect of inhibitory neurons on neural networks' dynamics. This is examined for a special excitatory-inhibitory neural network where the network is complete. In this special case the dynamics has an order preserving property if the activation function is a positive bounded monotone increasing function. With a special choice of activation functions, namely step functions, the dynamics can be completely characterized. This is carried out in the case of two and three valued step functions. The three valued case can exhibit stable limit cycles.

Metastable periodic orbits in cellular neural networks and qualitative behavior of dynamical systems in biology

A ring of 2M identical neuron cells with piecewise linear and saturated bidirectional nearest--neighbor coupling nonlinearities is considered in [GB1]. For certain values of the two coupling parameters, existence of a hyperbolic periodic solution with cyclic symmetry is established. The dominant Floquet multiplier converges to 1 as M tends to infinity and the remaining 2M-2 nontrivial Floquet multipliers converge to 0. In both cases sharp, exponentially small estimates are given. Waveform asymptotics as well as the asymptotics of the dominant eigenvector are also presented. The entire work was motivated by electrical circuit experiments.

In [GB2], Leon Chua's local activity concept is reconsidered and its connections to the theory of control are described. In [GB3] the authors point out how an eight-dimensional circulant system's highly degenerate bifurcation can be decomposed into three consecutive nondegenerate bifurcations by smoothing the nonlinearities. In addition, it is shown that the nonsmooth and the smooth dynamics - when restricted to the respective topological and smooth two-dimensional invariant spheres - are topologically equivalent.

A circle of eight identical Chua-Yang oscillators with two-parameter nearest-neighbor coupling is investigated in [GB4]. To the depth of five consecutive bifurcations, the dynamics governed by the standard piecewise linear saturated activation function as well as the dynamics governed by tangent hyperbolic as a smooth activation function are considered and compared. In both cases, essential dynamics is restricted to an invariant 2D topological sphere with $\mathbf{Z}^8 x \mathbf{Z}^2$ symmetry containing all equilibria except the origin.

On various networks, the standard linear averaging dynamics on the neighboring vertices is composed by the same nonlinear, strictly increasing real function in each coordinate. Based on the elements of algebraic graph theory, asymptotically stable periodic orbits - also of long periods - are constructed in [GB5].

The successful invasion of a multi-species resident system by mutants has received a great deal of attention in theoretical ecology but less is known about what happens after the successful invasion. In the framework of Lotka-Volterra systems, various possibilities like substitution, replacement, historically independent replacement are investigated, examples and counterexamples are given in [GB6].

For the understanding of human nature, the evolutionary roots of human moral behaviour are a key precondition. [GB7] contributes to the question: Can the altruistic moral rule “Risk your life to save your family members, if you want them to save your life” be evolutionary stable? Following the Darwinian tenet and adapting the Maynard Smith & Price definition of evolutionary stability to genotypes, the authors introduce a new population genetics model and demonstrate that the self-sacrificing behaviour in monogamous and exogamous families can confer a selective advantage under the right conditions for altruistic, grateful and provider mutants. They find that their conditions for the evolutionary stability of the self-sacrificing phenotype in the classical, additive model boil down to Hamilton’s rule for evolutionary stability of the self-sacrificing life history strategy.

Heteroclinic equilibrium connections in Chua-Yang ring networks with two-parameter nearest neighbor couplings are investigated in [GB8]. The dynamics is governed by the standard piecewise linear saturated activation function. Heteroclinic cycles created by terminating families of Hopf periodic orbits are studied. The approach is based on numerical experimentation supported by rigorous a posteriori mathematical arguments. Previous results are generalized to 7D, 9D, and 10D. Odd and even dimensional cases are considerably different, and the odd dimensional case is harder.

Nonlinear functional partial differential equations

A system of a semilinear hyperbolic functional differential equations (where the lower order terms contain nonlocal dependence on the unknown function) and a quasilinear parabolic functional differential equation with initial-boundary conditions are considered in [SL1]. Existence of weak solutions is shown for finite and infinite time intervals and some qualitative properties of the solutions are proved.

In [SL2], a system of a semilinear hyperbolic functional differential equation with initial-boundary conditions and a quasilinear elliptic functional differential equation (containing time as a parameter) with boundary conditions are considered. Existence and qualitative properties of weak solutions are proved for an unbounded time interval.

Weak solutions of boundary value problems for quasilinear elliptic equations, containing non-local terms (e.g. integrals of the unknown function) are considered in [SL3]. In the paper, several types of equations are shown, for which non-uniqueness occurs, namely, for arbitrary N, equations are constructed which have N solutions (with zero boundary condition).

Then, in [SL4], similar nonlinear parabolic equations (with zero initial and boundary condition) and systems of nonlinear elliptic equations (with zero boundary condition) are constructed. The proof of these results are based on arguments for fixed points of certain real functions and operators, respectively.

As a continuation of studies on partial functional equations, general first and second order evolution equations in the infinite time interval with functional (nonlocal) terms is studied in [SL5], based on the theory of monotone operators. In the case when the nonlocal terms are operators of Volterra type, existence and uniqueness of the solution was proved; with terms without Volterra property, existence of multiple solutions was shown. In both cases qualitative properties of the solutions were proved as time tends to infinity.

Nonsmooth elliptic eigenvalue problems

Using Ekeland's variational principle, a critical point theorem of Schechter type for extrema of a functional in an annular conical domain of a Banach space is proved in [VCs1]. The result can be considered as a variational analogue of Krasnoselskii's fixed point theorem in cones and can be applied for the existence, localization and multiplicity of the positive solutions of variational problems. The result is then applied to p-Laplace equations, where the geometric condition on the boundary of the annular conical domain is established via a weak Harnack type inequality given in terms of the energetic norm. This method can be applied also to other homogeneous operators in order to obtain existence, multiplicity or infinitely many solutions for certain classes of quasilinear equations.

In [VCs2], the critical point theory developed by Schechter for C^1-functionals in Hilbert spaces is generalized to locally Lipschitz functionals defined on a closed ball of a reflexive Banach space with strictly convex dual. Using a constrained deformation theorem which involves the duality mapping, various critical point alternatives are obtained. Applications to partial differential inclusions governed by the *p*-Laplacian are also provided in the last section of the paper.

Using the bounded mountain pass lemma and the Ekeland variational principle in [VCs3], a bounded version of the Pucci-Serrin three critical points result is proved in the intersection of a ball with a wedge in a Banach space. The localization constraints are overcome by boundary and invariance conditions. The result is applied to obtain multiple positive solutions for some semilinear problems.

Motivated by relevant physical applications, Schrodinger equations with state-dependent potentials are studied in [VCs4]. Existence, localization and multiplicity results are established for positive standing wave solutions in the case of oscillating potentials. To this aim, a localized Pucci-Serrin type critical point theorem is first obtained. Two examples are then given to illustrate the new theory.

In [VCs5], the authors prove different versions of the general minimax theorem of Willem and of the Mountain Pass Theorem of Ambrosetti and Rabinowitz on a wedge intersected with a ball in a reflexive locally uniformly convex smooth Banach space. These results are applied to localize two nontrivial solutions for Dirichlet problems involving nonhomogeneous operators in the context of Orlicz–Sobolev spaces. As a special case, the existence of two nontrivial positive solutions located on a certain ball is also obtained for p - Laplacian boundary value problems.

The number of solutions of a semilinear elliptic problem is studied in [VCs6], for a bounded domain with smooth boundary. A sufficient condition in terms of the parameters is shown guaranteeing that the equation subject to homogeneous Dirichlet boundary condition possesses a nonnegative solution. Moreover, the asymptotic behavior of this solution is analyzed, and it is shown that it converges uniformly to the distance function to the boundary of the domain.

Statistical methods for mathematical modelling of time dependent processes

In [ZA1] the authors investigated the 0.5-grid daily temperatures of Europe from the last 65 years. An increased speed of the global warming was observed, and the Gaussian model-based clustering was applied to determine the areas, which are most affected by the process. It turned out that large parts of Central Europe are among the ones with the quickest temperature increase, which has a wide-range of effects from agriculture to a possible spread of tropical epidemics. The stability of the results was checked by changing the starting point of the linear regression – this approach might be valuable for climatologists in finding the "best" starting point for assessing the global warming in Europe.

The main approach of paper [ZA2] is the dependent weighted bootstrap. It is a simulation method, where both the fitted regression and the dependency among the data is taken into account. The authors present a simulation study showing that for serially dependent data it is the most accurate in case of estimating the coefficient in a linear regression. The paper shows an analysis of the gridded European temperature data.

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