Zárójelentés a 115804 számú pályázathoz, 2015.09.01.-2020.08.31.

We continued our research in summability theory of Fourier transforms and Fourier series. Especially, we investigate a general summability method of multi-dimensional Fourier transforms, the so called θ-summability. We generalize the well known result due to Lebesgue about the convergence of Fejér means at Lebesgue points for higher dimensional functions. Under some conditions on θ, we show that the Marcinkiewicz-θ-means of a function $f\in W(L_1,\ell_\infty)(\R^d)$ converge to f almost everywhere. In addition, we characterize the set of convergence. We introduce a new version of Lebesgue points and show that the convergence holds at each modified strong Lebesgue point. The same holds for a weaker version of Lebesgue points, for the so called modified Lebesgue points of $f\in W(L_p,\ell_\infty)(\R^d)$, whenever $1<p<\infty$. As an application, we generalize the classical one-dimensional strong summability results of Hardy and Littlewood, Marcinkiewicz, Zygmund and Gabisoniya for $f\in W(L_1,\ell_\infty)(\R)$ and for strong θ-summability. We characterized the set of one-dimensional functions for which strong summability holds at each Lebesgue point. More exactly, if f is in the Wiener amalgam space $W(L_1,\ell_q)(\R)$ and f is almost everywhere locally bounded, or $f\in W(L_p,\ell_q)(\R)$ $(1<p<\infty,1\leq q<\infty)$, then strong θ-summability holds at each Lebesgue point of f. Similar theorems are proved for the triangular summability and for the restricted summability over a cone as well as for Fourier and Walsh-Fourier series.

In this topic, we published a book at Springer (F. Weisz: Convergence and Summability of Fourier Transforms and Hardy Spaces. Applied and Numerical Harmonic Analysis. Springer, Birkhäuser, Basel. 2017, 446 pp.). The main purpose of the book is to investigate the convergence and summability both of one-dimensional and multi-dimensional Fourier transforms as well as the theory of Hardy spaces. The theta-summation is studied which contains all well known summability methods, such as the Fejér, Riesz, Weierstrass, Abel, Picard, Bessel and Rogosinski summations. After the classical books of Bary (1964) and Zygmund (1968), this is the first book which considers strong summability introduced by current methodology. A further novelty of this book is that the Lebesgue points are studied also in the theory of multi-dimensional summability. Besides the classical results, recent results of the last 20-30 years are studied. The book will be useful for researchers as well as for graduate or postgraduate students. Especially the first two chapters can be used well by graduate students and the other ones rather by PhD students and researchers.

In a survey paper, we present some results on convergence and summability of one- and multi-dimensional trigonometric and Walsh-Fourier series. The Fejér and Cesàro summability methods are investigated. We prove that the maximal operator of the summability means is bounded from the corresponding classical or martingale Hardy space to the L_p space for some $p > p_0$. For $p = 1$, we obtain a weak type inequality by interpolation, which ensures the almost everywhere convergence of the summability means.

In another paper, we generalized the atomic decomposition of the Hardy spaces for the L_1 space.

We continued also our research about the variable Lebesgue spaces. In a paper, we summarized the results of the variable Lebesgue spaces. We have investigated the boundedness of the maximal operators in variable Lebesgue spaces. We have generalized the Besicovitch's covering theorem for the so-called γ-rectangles and we have introduced a more general maximal operator. The strong- and weak type inequalities have been proved for this maximal operator. Applying these results, we have investigated a general multi-dimensional integral operator V_T. Under the condition that the kernel function of V_T is in a suitable Herz space, we have gotten several convergence theorems about norm and almost everywhere convergence and convergence at Lebesgue points. As special cases, three multi-dimensional integral operators, the θ-summation of Fourier transforms and Fourier series and the discrete wavelet transforms have been considered.

Let $p(.)$ be a variable exponent function satisfying the globally log-Hölder continuous condition and $0<q<=\infty$. Let $H_{p(.)}$ and $H_{p(.),q}$ be the variable Hardy- and Hardy-Lorentz spaces defined via the radial grand maximal function. We characterized $H_{p(.)}$ and $H_{p(.),q}$ by means of the Littlewood-Paley g-function or the Littlewood-Paley g_λ^*-function via first establishing a Fefferman-Stein vector-valued inequality on the variable Lorentz space $L_{p(.),q}$. Moreover, the finite atomic characterizations of $H_{p(.)}$ and $H_{p(.),q}$ are also obtained. As applications, we then established a criterion on the boundedness of sublinear operators from the variable Hardy and Hardy-Lorentz spaces into a quasi-Banach space. Applying this criterion, we showed that the maximal operators of the Bochner-Riesz and the Weierstrass means are bounded from the variable Hardy and Hardy-Lorentz spaces to the $L_{p(.),q}$ spaces. As consequences, some almost everywhere and norm convergences of these Bochner-Riesz and Weierstrass means are also obtained. These results are also proved for anisotropic Hardy and Hardy-Lorentz spaces defined by a general expansive matrix A on R^n. We published 4 papers on this topic.

In this topic Kristóf Szarvas has got the PhD degree. The title of his dissertation is: „Változó indexű Lebesgue-terek és alkalmazásuk a Fourier-analízisben". Amongst others, he generalized the classical Hardy-Littlewood maximal operator for the so-called gamma-rectangles. With the help of the generalization of numerous previous results, he proved that the generalized maximal operator (under some certain conditions) is bounded on the variable Lebesgue spaces.

We worked also in martingale theory. We introduced the generalized multi-parameter martingale BMO spaces. The atomic decomposition of the multi-parameter martingale Hardy-Lorentz space $H_{p,q}^s$ is given. With the help of this, the dual space of $H_{p,q}^s$ is characterized as the generalized BMO space. As an application, John-Nirenberg inequality is generalized for multi-parameters.

Let (Ω,F,P) be a probability space and φ be a Musielak--Orlicz function. We proved that the Doob maximal operator is bounded on the Musielak—Orlicz space L_φ. Using this and extrapolation method, we then established a Fefferman--Stein vector-valued Doob maximal inequality on L_φ. As applications, we obtained the dual version of the Doob maximal inequality and the Stein inequality for L_φ, which are new even in weighted Orlicz spaces. We established the atomic characterizations of five martingale Musielak--Orlicz Hardy spaces. From these atomic characterizations, we further deduced some martingale inequalities between different martingale Musielak--Orlicz Hardy spaces, which essentially improve the corresponding results in Orlicz space case and are also new even in weighted Orlicz spaces. By establishing the Davis decomposition, we obtained the Burkholder--Davis--Gundy inequality associated with Musielak--Orlicz functions. Finally, using the previous martingale inequalities, we proved that the maximal Fejér operator is bounded from H_\varphi to L_\varphi, which further implies some convergence results of the Fejér means; these results are new even for the weighted Hardy spaces. We also proved the boundedness of the Cesaro and Riesz maximal operators from the variable Hardy spaces and Hardy–Lorentz spaces to the variable Lorentz spaces. As a consequence, we got theorems about almost everywhere and norm convergence of the Cesaro and Riesz means. We published 2 papers on this topic.

We worked also in wavelet theory. In a paper of length pp. 70, we investigated the connection between multi-dimensional summability theory and continuous wavelet transform. We considered two types of θ-summability of Fourier transforms, the circular and rectangular summability. Norm and almost everywhere convergence of the θ-means are shown for both types. The inversion formula for the continuous wavelet transform is usually considered in the weak sense. Here the inverse wavelet transform is traced back to summability means of Fourier transforms. Using the results concerning the summability of Fourier transforms, norm and almost everywhere convergence of the inversion formula are obtained for functions from the L_p and Wiener amalgam spaces. The points of the set of the almost everywhere convergence are characterized as the Lebesgue points. Using summability theory, we obtained restricted convergence of the inverse continuous wavelet transform at Lebesgue points for functions from the L_p and Wiener amalgam spaces.

We considered conditions on the coefficients of a Walsh multiplier operator that imply the operator is bounded on certain dyadic Hardy spaces Hp, $0 < p < \infty$. In particular, we were interested in two classical coefficient conditions, originally introduced for the trigonometric case, the Marcinkiewicz and the Hörmander-Mihlin conditions. They are known to be sufficient in the spaces Lp, $1 < p < \infty$. We studied the corresponding problem on dyadic Hardy spaces, and found the values of p for which these conditions are sufficient. Then, we dealt with the cases of H1 and L1 which are of special interest. Finally, based on a recent integrability condition for Walsh series, a new condition is provided that implies that the multiplier operator is bounded from L1 to L1, and from H1 to H1. We note that existing multiplier theorems for Hardy spaces give growth conditions on the dyadic blocks of the Walsh series of the kernel, but these growth are not computable directly in terms of the coefficients.

The Lagrange interpolatory polynomials do not define a uniformly convergent approximating process for arbitrary continuous functions. However relaxing certain conditions we can define better tools. For example the Fejér polynomials interpolating the function on the Chebyshev nodes do converge for any continuous function. Bernstein also constructed several processes of this type. We defined some processes for barycentric interpolation based on equidistant node-system which is convergent for any continuous function. As far as we know this is the first process of this type. We also prove an upper estimate for the rate of convergence. It turns out that the results are very similar to the ones known for the Bernstein process obtained from the classical Lagrange interpolation.

In the paper „Some new results on orthogonal polynomials for Laguerre type exponential weights", we proved some results on the root distances and the weighted Lebesgue function corresponding to orthogonal polynomials for Laguerre type exponential weights. We used it to obtain the Lebesgue constants for weighted Lagrange interpolation for different node systems. Moreover, we proved general lower estimations. In the paper „On Hermite Fejer interpolation on Laguerre nodes", we investigated the weighted Hermite Fejér interpolation and obtain some convergence theorems using proper function classes. At the same time, we prove convergence results for Hermite interpolation, too. In this topic, László Szili has obtained the degree of habilitation. The title of his dissertation is: „Egyenletesen konvergens polinomapproximációs eljárások".

The Zernike polynomials form a complete orthogonal system on the complex unit disk. These functions (i.e. the coefficients of the Zernike expansion) are widely applied in medical engineering to describe optical behaviour of the human eye (the cornea). Utilizing the Blaschke functions and representations of the Blaschke group, we may transform the original Zernike basis to acquire new orthogonal systems on the disk. The congruence transformations on the Poincaré and the Cayley–Klein models of the Bolyai–Lobachevsky hyperbolic geometry (closely related to Blaschke functions) play an important role. We published the details of this construction of a complete orthogonal system, named rational Zernike functions, together with a new hyperbolic implementation of the Nelder–Mead simplex method in the Cayley–Klein model. These new systems could be practically useful: by the congruence transformations, one may transform the aforementioned Zernike aberration coefficients for scaled and rotated pupils. Two conference talks were also given to present these ideas. A conference paper has been also submitted and accepted. This paper focuses and summarizes the main mathematical ideas of the past years of the research group related to cornea modelling.

Furthermore, we carried out investigations related to matrix norms, and a generalization of eigenvectors. A common way to define a norm of a matrix is to take the supremum of the fraction of the vector norms of the matrix-vector product and the nonzero vector, with respect to a given vector norm, i.e. the least upper bound for the norm of the vectors of the transformed unit sphere. We examined the above mentioned fraction, defining induction curves and surfaces, we have shown that there exist some vectors, such that this fraction is independent of the applied p-norm (and are not eigenvectors). These are to be called p-eigenvectors. Exact solutions were constructed for some simple matrices. A paper has been submitted and accepted introducing the basics of this novel research area, and a conference talk was also given. We continued these investigations, and recently a method for constructing p-eigenvectors in an efficient way was also found, the relation to regular eigenvectors was discussed via permutations of the original matrix. Exact solutions were formulated in case of 3x3 diagonal matrices. We have provided examples for cases where infinitely many p-eigenvectors exists, and under certain mild assumptions the cardinality of p-eigenvectors was estimated. Induction sets of 3 dimensions, together with the corresponding p-eigenvectors were visualized as a further novelty. The results of this research conducted together with Zsolt Németh have been summarized in a paper already accepted, but not yet published. Further open questions were formulated, e.g. the characterization of matrices where the upper estimate for the p-eigenvectors is sharp.

The use of orthogonal transformation in signal processing has a long history. Besides the classical systems like trigonometric, and polynomial systems there have many other systems been used including such recent ones as wavelets. The choice of a particular system is influenced by several points of view. One of them is the type and properties, like periodicity of the signal. Another issue is the objective of the signal processing. It may be compression, approximation, feature extraction etc. In our research we put the emphasis on human medical signals and their transformation by a proper function system. Issues like the optimal choice of the system, approximation properties, compression ratio, detection and classification problems were addressed.

One of the foci of our research activity was projections by means of rational orthogonal systems, with emphasis on applications. In particular, we were interested in optimalization according to conditions, aspects originated from specific real problems. These problems were connected to human ECG processing, including compression of the signal and classification of heartbeats. We constructed adaptive rational transformations, the efficiency of which were tested and proved on data bases that considered as standards in this field. The same topic in a more general framework was also discussed. We investigated also the application of rational variable projection methods for a specific ECG signal processing problem: heartbeat classification. An adaptive transformation method involving an orthogonal transformation with the rational Malmquist-Takenaka systems is developed and applied to the heartbeats of the ECG signals in order to extract morphological descriptors for arrhythmia type classification. Several questions have been examined during the development, including the proper system identification method for the inverse poles, the proper representation of the rational systems, the proper set of descriptors utilized as feature vector, and connecting signal processing problems. We investigated the modelling of the QRS complexes of ECG heartbeats with rational functions. A rational model with a new formalism is introduced that allows the geometric interpretation of QRS complexes. We explored the connection between certain medical descriptors, so-called fiducial points, of the QRS complexes and the characteristic points of the corresponding rational model curve. This model not only provides an analytic way to determine the fiducial points of the QRS complexes, but also a synthesis method that can be considered as a new concept for system identification method of the rational orthogonal systems. We have developed the theory of a new optimization method called generalized vairable projection. Compared to the original variable projection method the dimension of the subspace is not fixed a priori, a dimension type free parameter is added. Then we dealt with the resulting nonlinear optimization problem of the free parameters. To this order, based on the well-known particle swarm optimization (PSO) algorithm, we developed the multi-dimensional hyperbolic PSO algorithm. The main motivation was to increase the adaptivity while keeping the computational complexity manageable. The case of rational systems has been worked out completely. We also extended the least square fitting of rational functions to the variable projection problem. We constructed a gradient based optimization algorithm for rational functions. As a case study the problem of electrocardiogram (ECG) signal compression was discussed. By means of comparison tests performed on the PhysioNet MIT-BIH Arrhythmia database, we demonstrated that our method outperforms other transformation techniques. One of the key issues in such application is the proper localization of the poles. In connection with this problem we have made quantitative investigation in order to determine the effect of perturbation of the locations of poles with respect to the degree of approximation of the signal. In another application, we worked on the problem of nonlinear spline regression, where the optimal position of the knots is to be determined. The optimization strongly depends on the initial estimation of the free parameters. To this end, we designed a fast and efficient knot prediction algorithm, which utilizes the numerical properties of first order B-splines. In order to evaluate the performance of our method, we approximated one dimensional discrete time series, and performed an extensive comparative study using both synthetic and real data. It is worth noting that the case of rational systems is not just an ordinary special case. Its importance is justified by the wide range of their applications, like filter design, system and control theory etc.

We developed an off-line supervised detection of epileptic seizures in long-term Electroencephalography (EEG) records. In order to characterize seizure patterns, we designed a sparse version of rational decomposition, which was combined with other features like the Local Gabor Binary Patterns. For the test of the algorithm we used the PhysioNet CHB-MIT Scalp EEG database. The experiments show that the proposed technique outperforms other dedicated methods by achieving the overall sensitivity of 70.4% and the overall specificity of 99.1%. In another our research for epileptic seizure detection using only one EEG channel we viewed the sampled signal as piece-wise linear function. Then we developed the technique to reduce its complexity while preserving the information relevant in EEG processing. The method we worked out is a so-called hybrid one. Both in the time and frequency domains we took the space of piecewise linear functions as the subspace for model reduction. The aim of the reduction was to obtain simple representation that still contains the relevant information. In the time space we developed a modified half-wave method while in the frequency space we applied orthogonal projection using the Franklin system. The tests show that our method performs better than the existing ones.

Conference talks:

Weisz Ferenc: Two-dimensional Fourier transforms and Lebesgue points, Lecture at the 11th Joint Conference on Mathematics and Computer Science, Eger, Hungary, May 20–22, 2016, 2016

Weisz Ferenc: Lebesgue points of two-dimensional Fourier transforms, Lecture at the Time-Frequency Analysis and Related Topics, June 6 - 10, Strobl, Austria, 2016, 2016

Weisz Ferenc: Generalizations of Lebesgue points for two-dimensional functions, Lecture at the VII Jaen Con- ference on Approximation Theory, July 3 - 8, ´ Ubeda, Ja´ en, Spain, 2016, 2016

Fridli Sándor: Sufficient conditions for the integrability of dyadic maximal Walsh series, Lecture at the 11th Joint Conf. on Math. and Comp. Sci., May 20-22, 2016, 2016

Bognár,Gergő, Gilián Zoltán, Fridli Sándor: Orthogonal transformations in ECG processing, Lecture at the BJMT Applied Mathematical Conference, Győr, June 1-3, 2016, 2016

Vértesi Péter: The Bernstein Erdos Conjecture for certain Haar (Tchebycheff) Systems, Lecture at the VII Jaen Con- ference on Approximation Theory, July 3 - 8, ´ Ubeda, Ja´ en, Spain, 2016, 2016

Szarvas Kristóf: Convergence of integral operators in variable Lebesgue spaces, Lecture at the 11th Joint Conference on Mathematics and Computer Science, Eger, Hungary, May 20–22, 2016, 2016

Szarvas Kristóf: Variable Lebesgue spaces and integral operators, Conference poster at the Time-Frequency Analy- sis and Related Topics, June 6 - 10, Strobl, Austria, 2016, 2016

Kovács Péter: Optimization problems in signal compression, Lecture at the 11th Joint Conference on Mathematics and Computer Science, Eger, Hungary, May 20–22, 2016, 2016

Kovács Péter, Schipp Ferenc: Model reduction and its applications, Lecture at the BJMT Applied Mathematical Conference, Győr, June 1-3, 2016, 2016

Kovács, P.: Rational Variable Projection Methods in Signal Processing, Lecture at the 16th International Conference on Computer Aided Systems Theory (EUROCAST), Las Palmas de Gran Canaria, Spain,, 2017

F. Weisz: Multi-parameter martingales. Lecture at the Central South University, Changsha, China. 2016

F. Weisz: Multi-parameter martingales and applications., Lecture at the Wuhan University, Wuhan, China. 2016

F. Weisz: Multi-parameter martingales and some applications in Fourier analysis, Lecture at the Multi-parameter martingales and some applications in Fourier analysis. Normal University, Beijing, China, 2016

F. Weisz: Kétdimenziós Fourier-transzformáltak Lebesgue-pontjai., Lecture at the Mathematical Institute, University of Pécs, Hungary, 2016

F. Weisz: The inverse of the continuous wavelet transform. Lecture at the 16th International Conference on Computer Aided Systems Theory (EUROCAST), Las Palmas de Gran Canaria, Spain,, 2017

F. Weisz: Fourier-transzformáltak és Lebesgue-pontok. Lecture at the Rényi Institute of Mathematics, Budapest, Hungary, 2017

F. Weisz: Rectangular summability and Lebesgue points of higher dimensional Fourier transforms., Lecture at the VIII Jaen Conference on Approximation Theory, July 2 - 7, Ubeda, Jaen, Spain, 2017

F. Weisz: Convergence of rectangular summability and Lebesgue points of higher dimensional Fourier transforms., Lecture at the 6th Workshop on Fourier Analysis and Related Fields, Pécs, August 24-31, 2017

Bognár Gergő, Fridli Sándor,: Heartbeat Classication of ECG Signals Using Rational Function Systems, Lecture at the 16th International Conference on Computer Aided Systems Theory (EUROCAST), Las Palmas de Gran Canaria, Spain,, 2017

Fridli Sándor: Approximation problems in ECG signal processing, Lecture at the VIII Jaen Conference on Approximation Theory, July 2 - 7, Ubeda, Jaen, Spain,, 2017

Gergő Bognár, Sándor Fridli, Péter Kovács, Ferenc Schipp: ECG processing by rational systems, IEEE 30th Neumann Colloquium, Budapest, Hungary, 2017.11.24-2017.11.25.

Zoltán Fazekas, Levente Lócsi, Alexandros Soumelidis, Ferenc Schipp (with Zsolt Németh): Rational Zernike functions capture the rotations of the eye-ball, The 20th European Conference on Mathematics for Industry (ECMI 2018), Budapest, June 18–22, 2018.

Levente Lócsi, Zsolt Németh, Ferenc Schipp: The Blaschke group and rational Zernike functions, Numbers, Functions, Equations (NFE 2018), Hajdúszoboszló, August 26 – September 1, 2018.

K. Szarvas: The boundedness of the Cesaro means in variable dyadic Hardy spaces, Conference on Numbers, Functions, Equations 2018, August 26 - September 1, Hajdúszoboszló, Hungary, 2018.

K. Szarvas: Cesaro means in variable dyadic Hardy spaces. 12th Joint Conference on Mathematics and Computer Science, Cluj-Napoca, June 14-17, 2018.

K. Szarvas: The boundedness of the Cesaro means in variable dyadic martingale Hardy spaces. Harmonic Analysis and Applications, June 4-8, Strobl, Austria 2018.

F. Weisz: Summability of trigonometric and Walsh-Fourier series. Central South University, Changsha, China, 2017.

F. Weisz: Hardy spaces and summability of trigonometric and Walsh-Fourier series. Normal University, Beijing, China, 2017.

F. Weisz: Summability in variable Hardy and Hardy-Lorentz spaces. Harmonic Analysis and Applications, June 4-8, Strobl, Austria 2018.

F. Weisz: Variable Hardy and Hardy-Lorentz spaces and applications in Fourier analysis. 12th Joint Conference on Mathematics and Computer Science, Cluj-Napoca, June 14-17, 2018.

F. Weisz: Variable Hardy spaces and summability. IX Jaen Conference on Approximation, July 8th-13th, Ubeda, Jaen, Spain, 2018.

F. Weisz: Variable Hardy spaces and applications in Fourier analysis. Numbers, Functions, Equations 2018, August 26 - September 1, Hajdúszoboszló, Hungary, 2018.

F. Weisz: Laudation to Sándor Fridli (The scientific work of Prof. Sándor Fridli). Numbers, Functions, Equations 2018, August 26 - September 1, Hajdúszoboszló, Hungary, 2018.

L. Lócsi: Induction curves and p-eigenvectors (an introduction), Conference on Harmonic Analysis and Related Fields, Visegrád, June 11–13, 2019.

Zs. Németh and F. Schipp and F. Weisz: Hyperbolic transformations of Zernike functions and coefficients. Computer Aided Systems Theory - EUROCAST, Las Palmas, Gran Canaria, Spain, 2019.

K. Szarvas: Mixed Hardy spaces and martingale inequalities. Conference on Harmonic Analysis and Related Fields, Visegrád, June 11-13, 2019.

P. Vértesi: About the ODRA 1304. ELTE IK, 2019.

P. Vértesi: A joint work with Laszlo Szili. Conference on Harmonic Analysis and Related Fields, Visegrád, June 11-13, 2019.

F. Weisz: Variable spaces, martingales and applications in Fourier analysis. Matematikai Tudományok Osztálya Fourier analysis and applications című tudományos ülése, May 15, 2019.

F. Weisz: Generalization of Lebesgue points and Walsh-Lebesgue points. Conference on Harmonic Analysis and Related Fields, Visegrád, June 11-13, 2019.

F. Weisz: Laudation to Ferenc Schipp (The scientific work of Prof. Ferenc Schipp). Conference on Harmonic Analysis and Related Fields, Visegrád, June 11-13, 2019.

F. Weisz: Hardy spaces spaces with variable exponents and Fourier analysis. Barcelona Analysis Conference, June 25-28, 2019.

F. Weisz: Hardy spaces with variable exponents and applications in summability. X Jaen Conference on Approximation, June 30th- July 05th, Ubeda, Jaen, Spain, 2019.

F. Weisz: Variable Hardy spaces and Fourier analysis. IWOTA 2019, International Workshop on Operator Theory and its Applications, Lisbon, Portugal, July 22-26, 2019.

F. Weisz: Lebesgue points of two-dimensional functions and summability. City University of Hongkong, China 2019.

F. Weisz: Summability and Lebesgue points of two-dimensional Fourier transforms. University of Macau, China 2019.

F. Weisz: Hardy spaces with variable exponents and applications in Fourier analysis. International Conference on Function Spaces and Geometric Analysis and Their Applications, September 30-October 04, Nankai University, Tianjin, China 2019.

F. Weisz: The generalizations of Lebesgue points and summability. Normal University, Beijing, China 2019.