Open quantum system dynamics in the ultrastrong coupling regime

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Overview of the accomplished research

Equipments exploiting the working principles of quantum mechanics yield revolutionary applications. The necessary condition for this is the ability to manipulate quantum objects not only individually, but also to control their interaction by external means. Hybrid systems are of special interest since the coherent coupling amounts to a quantum interface between different degrees of freedom: radiation, mechanical motion, spin, or electric current, etc. Such quantum interfaces allow for extending the scope of applications by building composite quantum machines.

From a theoretical point of view, we aimed at extending our knowledge on hybrid interacting quantum systems in specific cases. One of the main subjects of investigation was focussed on the *strong coupling between a discrete quantum system and a continuum of modes*. The weak coupling of a discrete system to a continuum of modes, i.e., to a bath, is known to yield damping which can be accounted for by Markov approximation. We studied the non-perturbative, strong coupling limit where non-Markovian effects substantially different from relaxation occur. This is the core of the **magneto-mechanical system** realised by a magnetically trapped Bose-Einstein condensate of ultracold atoms (Sec. 1), where the hyperfine states of atoms (discrete degree of freedom) and the untrapped free motional states (continuum) are coupled in a way that can be controlled by external rf, microwave and optical radiation fields. We proposed an application to measure the noise spectrum in an external microwave field, which has been implemented in an experiment in collaboration with the group of Prof. J. Fortágh in Tübingen.

The other physical system where discrete-continuum coupling takes place is many-atom cavity QED, i.e., the description of **dissipative quantum critical phenomena** (Sec. 2). We developed the Keldysh-Green path integral formalism to fully describe the coupling of the continuum quasi-particle excitations of a trapped BEC with the photons of a high-finesse resonator. We revealed significant effects of the coloured noise on the quantum statistical features of the Dicke-type *quantum critical point*. Besides the continuous quantum phase transitions, we studied the critical behaviour analogous to a first-order phase transition in a driven-dissipative quantum system. This work was motivated by the experimental observation of co-existence of phases in the form of a quantum bistability. Working on the interpretation of measured data together with the experimental group of J. Fink and A. Wallraff at ETH Zürich, we proved that the observed phenomenon is a finite-size realisation of a first-order quantum phase transition.

The other subject was the extreme limit of interaction strengths, the so-called **ultrastrong coupling regime** (Sec. 3) where the controlled coupling reaches the value of the eigenfrequency of the bare systems. The components lose their individual properties completely and a quantum critical point appears. We studied this regime in the case of atoms and the radiation field, that can be treated within non-relativistic quantum electrodynamics. We gave a general framework on the basis of the Power-Woolley-Zienau gauge to describe the interaction the quantum electrodynamical radiation field with an *optically dense atomic ensemble* having substantial atom-atom interactions.

In the course of the project, the research program has been extended by studying **complex dynamical maps on qubits** (Sec. 4). This research line also led to the study of critical phenomena, as the complex dynamics associated non-linear maps can have the character of a first order phase transition, where the purity of the initial state plays the role of the control parameter. Applications of complex dynamical maps have been worked out in various examples, i.e., the efficient discrimination of non-orthogonal atomic states in the context of a cavity QED scheme. The principle of the *iterative non-linear protocols for quantum state* *discrimination* has been experimentally demonstrated in a collaborative work with the group of Peng Xue at the Beijing Computational Science Research Center, with the use of single-photon states and linear optical elements.

1 Hybrid mesoscopic systems with magneto-mechanical coupling

Bose-Einstein condensates of ultracold atoms can be used to sense fluctuations of the magnetic field by means of transitions into untrapped hyperfine states. It has been shown recently that counting the outcoupled atoms can yield the power spectrum of the magnetic noise. We calculated the spectral resolution function, which characterizes the condensate as a noise measurement device in this magneto-mechanical scheme. We used the description of the radio-frequency outcoupling scheme of an atom laser and took into account the gravitational acceleration. To provide insight into the outcoupling mechanism, we investigated an intuitive approach that takes into account the motion of the atoms in the gravitational field. We also presented a complete three-dimensional solution using the Green's function that takes into account the proper boundary conditions as well. The quantum-mechanical solution for the outcoupled matter wave allows for constructing the local, position-dependent spectral resolution function in both approaches, which can then be used to describe the detection process for arbitrary spatial resolution. Beyond the mainly linear dependence on the vertical size (the size in the direction of the gravitational field), the spectral resolution function exhibits a remarkable dependence also on the lateral extension of the condensate, which results from an interplay between the excitation of the different radial modes and the nonfactorizability of the BEC wave function. [1]

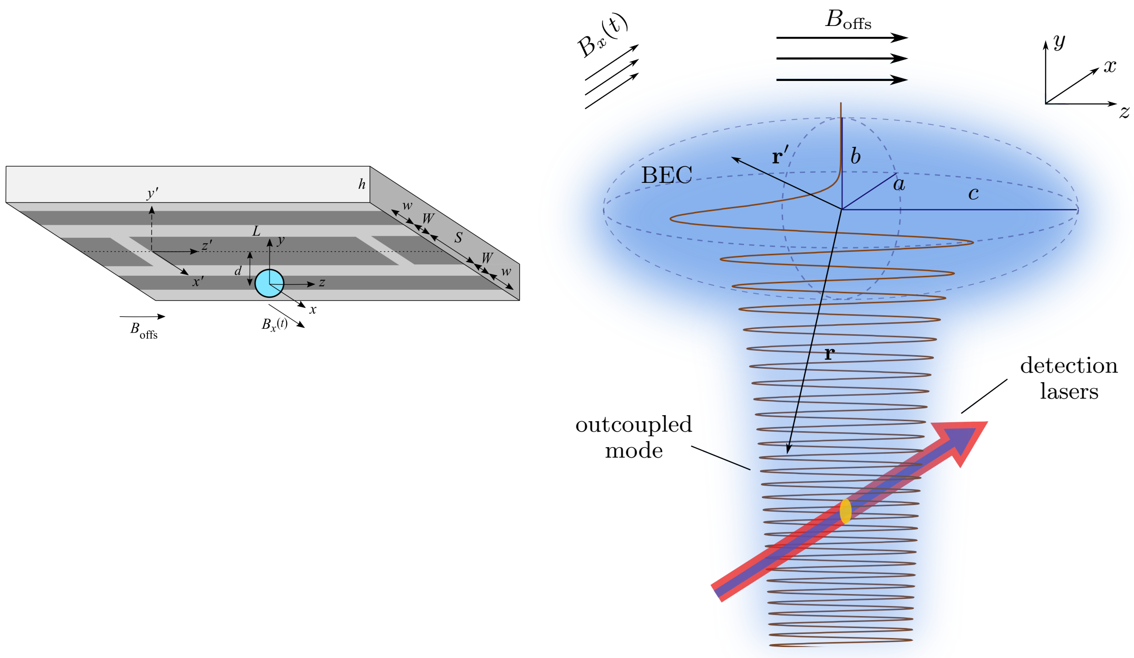

Figure 1: *Left: Sketch of the setup to sense the magnetic field with a Bose-Einstein condensate (BEC). The ultracold atom cloud is situated below a chip, which contains the electric circuit to trap magnetically the atoms, and also the resonant rf sources to generate microwave fields to excite magnetic dipole transition between the hyperfine states in the atoms of the BEC. Moreover, as illustrated in the figure, the microwave source can be an integrated coplanar waveguide resonator in which the magnetic field of a few quanta of microwave radiation is significantly enhanced to yield strong coupling with the BEC. Right: Sketch of the magneto-mechanically coupled Bose-Einstein condensate and the outcoupled mode for a monochromatic outcoupling microwave field.*

In collaboration with the experimental group of J. Fortágh at Tübingen University, we studied both theoretically and experimentally the magneto-mechanical interaction induced in a trapped Bose-Einstein condensate by a classical radio-frequency source. We performed the spectral analysis and the local measurement of intensity correlations of the microwave field using the ultracold quantum gas. The fluctuations of the electromagnetic field induce spin flips (hyperfine state transitions) in a magnetically trapped quantum gas and generate a multimode atom laser. The output of the atom laser was measured with high temporal resolution on the single-atom level, from which the spectrum and intensity correlations of the generating microwave field have been reconstructed in accordance with our recently proposed scheme. We gave the theoretical description of the atom-laser output and its correlations in response to resonant microwave fields and verify the model with measurements on an atom chip. The measurement technique is applicable for the local analysis of classical and quantum noise of electromagnetic fields, for example, on chips, in the vicinity of quantum electronic circuits. [2]

This method has been applied for the stationary microwave radiation field sustained by a coplanar waveguide resonator integrated on the atom chip. The measurement of the magnetic field of the resonator by means of counting the atoms that fall out of the condensate due to hyperfine transitions to non-trapped states was analysed in the ultra-low intensity limit. We determined the quantum efficiency of this detection scheme and showed that weak microwave fields at the single-photon level can be sensed with an integration time on the order of a second. [3]

2 Dissipative quantum critical phenomena

We studied the open-system realization of the Dicke model, where a bosonic cavity mode couples to a large spin formed by two motional modes of an atomic Bose-Einstein condensate. The cavity mode is driven by a high frequency laser and it decays to a Markovian bath, while the atomic mode interacts with a colored reservoir. We revealed that the soft mode fails to describe the characteristics of the criticality. We calculated the critical exponent of the superradiant phase transition and identified an inherent relation to the low-frequency spectral density function of the colored bath. We showed that a finite temperature of the colored reservoir does not modify qualitatively this dependence on the spectral density function. [4]

We investigated the quantum measurement backaction noise effects on the dynamics of an atomic Bose lattice gas inside an optical resonator. We described the dynamics by means of a hybrid model consisting of a Bose-Hubbard Hamiltonian for the atoms and a Heisenberg-Langevin equation for the lossy cavity-field mode. We assumed that the atoms are prepared initially in the ground state of the lattice Hamiltonian and then start to interact with the cavity mode. We showed that the cavity-field fluctuations originating from the dissipative outcoupling of photons from the resonator lead to vastly different effects in the different possible ground-state phases, i.e., the superfluid, the supersolid, the Mott and charge-density-wave phases. In the former two phases with the presence of a superfluid wavefunction, the quantum measurement noise appears as a driving term leading to depletion of the ground state. The timescale for the system to leave the ground state is presented in a simple analytical form. For the latter two incompressible phases, the quantum noise results in the fluctuation of the chemical potential. We derived an analytical expression for the corresponding broadening of the quasiparticle resonances. [5]

We studied quasiparticle scattering effects on the dynamics of a homogeneous Bose-Einstein condensate of ultracold atoms coupled to a single mode of an optical cavity. The relevant excitations, which are polariton-like mixed excitations of photonic and atomic density-wave modes, have been identified. All the first-order correlation functions have been presented by means of the Keldysh Green's function technique. Beyond confirming the existence of the resonant enhancement of Beliaev damping, we found a very structured spectrum of fluctuations. There is a spectral hole burning at half of the recoil frequency reflecting the singularity of the Beliaev scattering process. The effects of the photon-loss dissipation channel and that of the Beliaev damping due to atom-atom collisions can be well separated. We showed that the Beliaev process does not influence the properties of the self-organization criticality. [6]

We studied the spin-1 bilinear–biquadratic model on the complete graph of N sites, i.e. when each spin is interacting with every other spin with the same strength. Because of its complete permutation invariance, this Hamiltonian can be rewritten as the linear combination of the quadratic Casimir operators of SU(3) and SU(2). Using group representation theory, we explicitly diagonalized the Hamiltonian and mapped out the ground-state phase diagram of the model. Furthermore, the complete energy spectrum, with degeneracies,

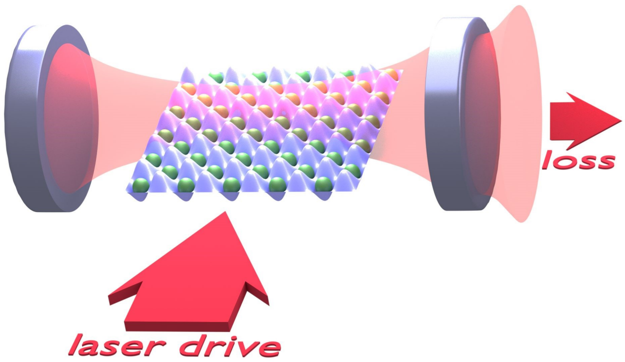

Figure 2: *Illustration of the coupled cavity Bose-Hubbard model setup. An atomic cloud is loaded into a square optical lattice, which is inside a single-mode high-Q Fabry-Pérot resonator. The period of the cavity mode is approximately equal to that of the optical lattice. The cavity is pumped from the side by light scattering off the atoms. The system is open: together with the external drive, the photons leak out from the cavity, resulting in heating and decoherence, or, in other terms, quantum measurement back-action since the out-coupled photons can be measured by classical detectors.*

was obtained analytically for any number of sites. [7]

We studied the ultrastrong coupling limit of cavity quantum electrodynamics in the case of the so-called circuit QED systems which comprise the interaction of field in microwave stripline resonators with artificial superconducting qubits. We collaborated with experimentalists with the aim of giving an interpretation for their recently measured data. We proved that the experiment had demonstrated in fact a first-order dissipative quantum phase transition in a driven circuit quantum electrodynamics system. Nonequilibrium phase transitions exist in damped-driven open quantum systems when the continuous tuning of an external parameter leads to a transition between two robust steady states. In second-order transitions this change is abrupt at a critical point, whereas in first-order transitions the two phases can coexist in a critical hysteresis domain. Here, in the specific experiment, the first-order transitions takes place when the photon blockade of the driven cavity-atom system is broken by increasing the drive power. The observed experimental signature is a bimodal phase space distribution with varying weights controlled by the drive strength. Our measurements show an improved stabilization of the classical attractors up to the millisecond range when the size of the quantum system is increased from one to three artificial atoms. The formation of such robust pointer states could be used for new quantum measurement schemes or to investigate multiphoton phases of finite-size, nonlinear, open quantum systems. [8]

We studied the time evolution of a quantum system undergoing a dissipative, first-order quantum phase transition. We considered the photon- blockade-breakdown phase transition, which takes place in the driven Jaynes-Cummings model with strong coupling between the two-level system and the harmonic oscillator. For a certain range of drive strength, the stationary solution corresponds to a bistability of classically distinguishable states. By unraveling the stationary solution into quantum trajectories, we resolved the nature of coexistence of phases. We showed that the bistability solution develops into a first-order phase transition in the thermodynamic limit. We calculated the finite-size scaling exponent numerically. Even in the thermodynamic limit, the stability of phases originates from the discrete spectrum of a small quantum system. The nonclassicality of the photon statistics leads to the possibility of switching between the phases. [9]

3 Fundamental modeling of ultrastrong-coupling candidate systems

We investigated the possibility of the Dicke-type superradiant phase transition of an atomic gas. We extended our recent work on the description of the ultrastrong coupling limit of the interaction between light and atoms within the regularized electric dipole gauge. As a new element, we took into account the short-range depolarizing interactions between atoms that approach each other as close as the atomic size scale. By using a mean field model, we found that a critical density does indeed exist, though the atom-atom contact interaction shifts it to a higher value than what can be obtained from the bare Dicke-model. We proposed that the system, at the critical density, transitions to the condensed rather than the "superradiant" phase. We demonstrated that criticality in a driven-dissipative system is strongly influenced by the spectral properties of the reservoir. [10]

We showed that the Power-Zienau-Woolley picture of the electrodynamics of nonrelativistic neutral particles (atoms) can be derived from a gauge-invariant Lagrangian without making reference to any gauge whatsoever in the process. This equivalence is independent of choices of canonical field momentum or quantization strategies. In the process, we emphasized that in nonrelativistic (quantum) electrodynamics, the all-time appropriate generalized coordinate for the field is the transverse part of the vector potential, which is itself gauge invariant, and the use of which we recommend regardless of the choice of gauge, since in this way it is possible to sidestep most issues of constraints. Furthermore, we pointed out a freedom of choice for the conjugate momenta in the respective pictures, the conventional choices being good ones in the sense that they drastically reduce the set of system constraints. [11]

4 Complex dynamical maps on qubits

A cavity quantum electrodynamics scenario was proposed for implementing a Schrödinger microscope capable of amplifying differences between nonorthogonal atomic quantum states. The scheme involves an ensemble of identically prepared two-level atoms interacting pairwise with a single mode of the radiation field as described by the Tavis-Cummings model. By repeated measurements of the cavity field and of one atom within each pair a measurement-induced nonlinear quantum transformation of the relevant atomic states can be realized. The intricate dynamical properties of this nonlinear quantum transformation, which exhibits measurement-induced chaos, allow approximate orthogonalization of atomic states by purification after a few iterations of the protocol and thus the application of the scheme for quantum state discrimination. [12, 13]

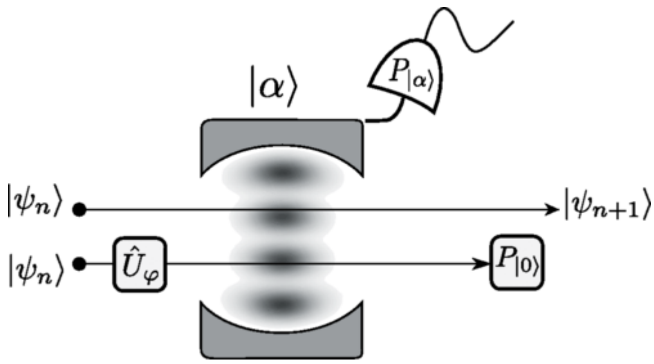

Figure 3: *Illustration of the scheme. Two two-level atoms in the same state interact with the cavity field prepared in a coherent state. Before the interaction, a unitary gate is applied to one of the atoms, and after the interaction and the projection of the field onto the initial coherent state, this same atom is projected onto its ground state. Finally, the other atom is left in a non-linearly transformed state.*

Quantum information processing can exploit the possibility to induce nonlinear maps on a quantum system by involving two or more identical copies of the given system in the same state. Such maps play a central role in distillation protocols used for quantum key distribution. We determine that such protocols may exhibit sensitive, quasi-chaotic evolution not only for pure initial states but also for mixed states, i.e. the complex dynamical behavior is not destroyed by small initial uncertainty. We show that the appearance of sensitive, complex dynamics associated with a fractal structure in the parameter space of the system has the character of a first order phase transition. The purity of the initial state plays the role of the control parameter and the dimension of the fractal structure is independent of the purity value after passing the phase transition point. The critical purity coincides with the purity of a repelling fixed point of the dynamics and we show that all the pre-images of states from the close neighborhood of pure chaotic initial states have purity larger than this. Initial states from this set can be considered as quasi-chaotic. [14]

We participated in a collaboration with the group of Peng Xue to experimentally realise a nonlinear quantum protocol for single-photon qubits with linear optical elements and appropriate measurements. Quantum nonlinearity was induced by postselecting the polarization qubit based on a measurement result obtained for the spatial degree of freedom of the single photon which played the role of a second qubit. Initially, both qubits were prepared in the same quantum state and an appropriate two-qubit unitary transformation entangles them before the measurement of the spatial part. We analysed the result by quantum state tomography of the polarisation degree of freedom. We demonstrated the usefulness of the protocol for quantum state discrimination by iteratively applying it to either of two slightly different quantum states which rapidly converge to different orthogonal states by the iterative dynamics. Our work creates an opportunity to employ effective quantum nonlinear evolution in quantum information processing. [15]

5 Further remarks

Apart from the main results presented in the previous sections, a number of five closely-related publications appeared during the project in journals with impact factor. These papers represent methodological advancements or special cases of the general principles presented in the main publications.

During the first three years of the project, we concluded most of these works. The last year has been left for the accomplishment of publications. We requested an additional fifth year in order to use the available budget to conference participations, as we had several invitations to present the project results. The pandemic crisis entirely obstructed these actions, however, at this point we prefer to close the project.

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