Global optimization methods for solving location problems

Final Report

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The main purpose of the project was to design reliable methods for some class of location problems, most of them being mixed integer nonlinear problems (MINLP). In order to reach the global optimum, rigorous methods were developed based on the Branch and Bound schema for each problem. Although these problems share some common properties, yet the main ingredients of the solution method are quite different for almost all problems at the end.
The research started quite well for the project, actually even before the project was initiated. This was due to that my daughter was born just when the project should have started, thus I have asked for a year of delay for maternal leave. I already had the experience that with a small baby it is not possible to do research efficiently. However we have started to work as soon as I applied for the project, and as a consequence, we had many papers done already in the first year. We were not so lucky later on, but step by step we made progress on almost all subjects. This is the reason why we have three papers in the first year, two papers in the second year, no one in the third year and one in the last one. We have one paper to be printed this year and three papers under revision or in final preparation.

Let us discuss one by one the problems which are solved during the project, but also the one which we could handle only partially for the moment.

1 Competitive Facility Location Problems on the Plane

The most important research questions in location nowadays are connected to the strategies of competing facilities. To determine "where" to locate their centres is a problem whose solution significantly affects the market share of a firm, and therefore its benefit in the future because the demand is spread among competitors according to the different market structures and costumer choices.

1.1 Multi-deterministic Costumer Selection Rule

One of the first research questions we discussed was about the proper choice for the costumer selection rule. In the literature, only the deterministic and probabilistic rules were well discussed, where costumers choose the best facility to be served by, or they split their demand probabilistically among all facilities proportionally with their attraction to each facility, respectively. For us it was clear, that there are many situations where a different costumer choice rule should be used, therefore we have investigated some well suiting rules for real-life problems and compared them to the existing ones. We have investigated the multi-deterministic rule, where from each chain only one facility (with the highest attraction) is chosen. Among those facilities, the demand is split probabilistically. It is based on the fact that many people like to visit more than one facility in general, but among those facilities offering the same, they choose the closest one. The new model was solved by an interval Branch and Bound method, that was able to solve problems up to 20.000 demand points, and also by an evolutionary method developed for this problem. The heuristic was so accurate, that it was able to find the global optimum in all verified cases. Compared to the probabilistic rule, we have seen that in most cases the optimum and also the optimizer is quite different, thus we have concluded that it is very important to choose the costumer choice rule as accurately as possible. The resulting paper [3] was published in the journal of Computers & Operations Research.

1.2 Partially Probabilistic Customer Choice Rule

We have investigated the use of another customer choice rule, called partially probabilistic, which is appropriate when there are many facilities with different owners (or offers). In such cases, costumers are not likely to split their demand for all facilities. Thus, it is natural to assume that there is a limit for the attraction; costumers have no interest in visiting those facilities having less utility then the given minimal attraction. The facilities fulfilling the minimal attraction condition split their demand proportionally to the attraction the costumers feel for them.

It may happen, that for a demand point no facility reaches the minimal attraction, resulting in the loss of its demand. Moreover, because of the condition for minimal attraction, the objective function is not continuous everywhere. Fortunately, we were able to compute appropriate bounds by interval-based inclusion functions, thus the interval Branch and Bound method was able to solve the problems up to 10.000 demand points. For larger problems, the UEGO method was able to solve the problem fast enough finding the verified global optimum in all the cases we could test it. The resulting paper [2] was also published in the journal of Computers & Operations Research.

1.3 Locating a new facility with closing and/or changing the quality of existing facilities

In a similar setting, we have solved another challenging problem from competitive facility location problems when the qualities of the existing facilities are considered as variables instead of fixed values. This setting was not examined in the plane yet, although it is natural to assume that the qualities of the existing facilities can be adjusted when a new facility is located. Additionally, we have allowed to close up some of the existing facilities, if necessary. We have solved this MINLP problem with an interval Branch and Bound method, where both the branching and bounding procedures are designed to suit the presence of binary variables, which decides that the facilities are open or closed. We have also adjusted the monotonicity test to handle the cases when integer variables are monotonous, and in most cases, it was working quite well. We could solve instances from small to medium sizes, and for larger instances, a smart heuristic procedure was built, which found a close approximation of the global optimum quite efficiently. The results were published in a short paper [5] in Intelligent Systems and Computing (Springer-Verlag), and the extended version with new results is under preparation for the Special Issue of the Journal of Global Optimization.

1.4 Huff-like Stackelberg location problems on the plane

.I was invited with some of my co-authors to publish a book chapter about Huff-like Stackelberg problems on the plane. The so-called Stackelberg location problems describe the location decisions of two competing firms that want to build new facilities. In a Stackelberg (or leader-follower) model the leader locates its facilities first, and once the locations of the new facilities are set, the follower locates its new facilities. Each firm has an objective function maximizing the market share or profit of its facilities which depends on the new facilities' locations as well as all existing facilities. Since both firms try to maximize their market share or profit, the leader has to take into account the action of the follower, leading to a bi-level optimization problem.
In the chapter, we have collected the possible models and their solution methods from the literature, and also described the open problems to be solved in the future. The chapter [4] was part of the book titled Spatial Interaction Models, as part of the Springer Optimization and Its Applications book series.

2 Location Problems on Networks

We were aiming to solve some challenging location problems on networks because in many real-life location problems, the assumptions that facilities can be located anywhere in the plane and a planar distance measure describe well real distances are purely wrong. In such cases taking the road or street network can give a much closer description to reality. As problems defined on networks need completely different methods to be solved, many such problems have been seldom studied. We have solved some of these during the project.

2.1 Stackelberg problem in a network

First, staying in the competitive environment, we have solved the Stackelberg problem in a network, maximizing the profit of the leader, where both the leader and the follower use the probabilistic customer choice rule for the fixed demand aggregated into demand points, the vertices. The problem differs from the previously solved Stackelberg problems not only by having the location space being a network but by the facilities taking operational costs into account. As a consequence, the objective function was even not necessarily continuous, as for a fixed leader, since there could be more than one optimal solution to the follower. Fortunately, the problem could be solved by an interval Branch and Bound method, which embeds another interval Branch and Bound method solving the follower problem. It means that a leader subproblem was described by a subedge for the leader and a set of subedges containing the optimal follower for the given subedge of the leader. Dividing a subproblem means dividing the subedge of the leader, and cloning the set of subedges of the follower for each new leader' subedge. For the bounding we could use both interval and DC-based bounds, allowing us to solve middle-sized problems. The results were published in the Journal of Global Optimization [6].

2.2 Stackelberg problem in a network with discrete quality variables

We also wanted to solve the problem with discrete quality variables, which also lead to a bi-level MINLP problem. Having quality variables involved, we also had to include operational costs on the qualities in the objective function. Unfortunately, it meant that most of our earlier bounding rules could not be used, and new ways had to be found. We have developed new bounding rules for the follower problem using the centered form and we were also able to modify our earlier DC bounds such that it can be used in some cases. The results obtained by the improved interval Branch and Bound method were disseminated on several conferences, lastly on the International Workshop on Urban Operations Research 2019 in Nagoya, Japan [8]. The paper is under revision by Kristóf Kovács before we submit it for publication in the near future.

2.3 p-facility Huff location problem on networks

We have also solved some rather classical problems on networks, like the p-facility Huff location problem. Huff(1964) was introducing the above mentioned probabilistic customer choice rule maximizing the market share of the facilities for fixed demand aggregated into demand points. This classical problem is well studied both in discrete and planar settings but has not been solved yet rigorously over networks. Thus, to fill this gap, we have investigated the classical p-facility Huff location problem on networks, which again led to an MINLP problem. In order to solve such a problem, we had to deal with the discrete decision (which edges to choose) together with the continuous variables (where to put a facility in a chosen edge). We designed a Branch and Bound method and proposed two approaches for the initialization and division of subproblems. The first one is based on the straightforward idea of enumerating every possible combination of p edges of the network as possible locations, and the second one defining sophisticated data structures that exploit the structure of the combinatorial and continuous part of the problem. Bounding rules are designed using DC (difference of convex) and Interval Analysis tools. The developed method was able to solve problems up to $p = 4$ facilities in reasonable time for medium-sized networks. The paper [1] was published in the European Journal of Operation Research.

2.4 Covering problem with continuous demand

Another classical problem was also studied by us, the covering location problem on networks. Contrary to those models already dealt with in the literature, we assumed that the demand is distributed along the edges of the network, which is a more realistic assumption for problems with networks representing high-density regions, such as cities. The problem is a challenging MINLP, in which combinatorial decisions (which edges of the network are to be selected to contain facilities) are coupled with continuous decisions (where to locate the facilities once the edges are chosen). We have solved this challenging problem using a special Branch and Bound method. For division, it was important to take into account the structure of the problem, since on a graph p edge-segment (or set of edges) defines a subproblem, which can be divided/defined in many ways. The introduction of the superset data structure developed by us allowed the method not to saturate memory with many small subproblems, so we could achieve good results on large graphs with a considerate number of facilities ($p < 5$). For the bounding, we made several quite different rules that worked very well complementing each other. The best results are achieved when the bounding rules are selected for the subproblems through a basic learning process. For a smaller graph, we also show the exponential explosion of time and memory requirements as a function of p. The article [9] appeared in the prestigious journal Omega.

2.5 Solving the 1-median problem on a network with continuous demand and demand surplus

Although this problem is not exactly the one which was aimed to be solved in the working plan, it fits well the research line and we found it more promising both in solution and applicability. In this problem, we seek the optimal location of only one facility, the median, where only a given fraction of the overall demand has to be satisfied. This is done in a discrete manner: the demand of an edge is either satisfied or not, thus apart from finding the optimal location of the median, we also have to decide which edges' demand should be satisfied. The objective is to minimize the sum of demand-weighted distances to the selected edges. Not surprisingly, it also leads us to an MINLP, for which we have designed a special Branch and Bound method. Straightforward bounding rules were useless, due to enormous overestimation. Fortunately, using either the exact or relaxed solution of the inner knapsack problem with appropriate coefficients, we were able to obtain sharp bounds on the objective function. From the computational results, it became clear that avoiding solving to optimality the knapsack problems in the early steps of the Branch and Bound is more efficient, though one needs to switch later in the tree to solve the knapsack problems to optimality to guarantee convergence. In a conference paper [7], we summarized our findings, that appeared in the Proceedings of the XIII Global Optimization Workshop. The full paper is still under revision.

3 Remaining problem

We were planning to study a less traditional location problem in data fitting. Namely, the problem in parameter selection, called overparameterization: given a data set and a parametric model different values for the parameters may be considered as plausible. It is meant in the sense that they all may fit very well the data set given, perhaps due to that there are too many parameters compared to the size of the data set. This problem is not yet solved, due to difficulties to handle the differences between candidate models. We aimed to find robust candidates, which never yield predictions too different than the others by variants of the minimax regret mathematical programs. Although the mathematical programs we were able to model, we could not develop solution techniques that are able to solve these MINLP problems.

4 Summary

We have addressed deterministic methods of Branch and Bound type for purely continuous and mixed integer nonlinear problems discussed above. These methods are quite demanding in terms of running times, and thus only applicable to problems in low dimension, as those appearing, for instance, in the above-mentioned location problems. The main difficulty of solving these problems comes from the mixed types of variables, which needs a special definition for the subproblems, and also special rules for the branching of the defined subproblems. These make an important role in the efficiency of the methods, and for each problem, these had to be selected depending on the properties of the problem and depending on the relative amount of integer variables and their connections to the continuous ones. We can state, that after having solved these hard problems, we gain quite a lot of knowledge about how to choose the appropriate data structure and bounding procedure for such problems, and thus, we are ready to take on a challenge for even harder problems.

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