Final report for OTKA K115518

In the below list of the results obtained in the project, we follow the structure of the research plan.

1. Classes of general algebras

In their 1984 seminal paper, Garcia and Taylor introduced the notion of interpretability of varieties and the lattice of interpretability types of varieties They formulated some difficult fundamental problems concerning the prime filters of this lattice. Since then, mostly partial results had been obtained in investigating those problems, by restricting them to idempotent or linearly defined varieties. For example, Valeriote and Willard proved that "n-permutability for some n" is prime in the lattice of interpretability types of idempotent varieties. Our result obtained in [GyMZ] gives sharp contrast to theirs: we proved that "n-permutability for some n" is not prime in the lattice of interpretability types of varieties. We also verified that for each $n>4$, n-permutability is not join-prime in the lattice of interpretability types of varieties.

The papers [Sz1], [Sz9], [Sz12] concern minimal varieties of arbitrary algebras: [Sz1] extends a result of Givant (1978), [Sz9] proves that every minimal abelian variety is either affine or strongly abelian, and [Sz12] proves that every minimal dual discriminator variety is minimal as a quasivariety. In [Sz3] and [Sz11] we investigate the relationship between some Mal'tsev conditions and compatible relations of algebras: The main result of [Sz3] is that an idempotent variety has a d-dimensional cube term if and only if its free algebra on two generators has no d-ary compatible cross, while [Sz11] extends Vaggione's result (2018) by showing how `ultralocal closure' can replace the usual local closure in some arguments that extend results, like the Baker-Pixley theorem, from finite to infinite algebras. In [Sz2] and [Sz6] Mal'tsev conditions are studied from a more practical point of view: [Sz2] produces a simple Maltsev condition which characterizes difference terms in the class of locally finite varieties, and [Sz6] explains why random search cannot be used to find finite algebras that distinguish between idempotent Mal'tsev conditions.

The main results of [Sz8] and [Sz10] are structure theorems for finite algebras. In [Sz4] and [Sz5] we applied structure theory to find a polynomial time algorithm for the subpower membership problem for finite algebras A with a cube term, if the centralizer of the monolith of every subdirectly irreducible algebra in HS(A) is abelian, or more generally, supernilpotent. The critical relations are the building blocks of the relational clone of a relational structure with respect to the relational operations intersection and direct product. In [KMZ2] we described the critical relations of the clones of the crown posets. One of the main tools in the proof is an earlier result of Demetrovics and Rónyai on crowns. By another use of their result, we proved that the subpower membership problem for any crown is polynomial-time solvable.

Global utility functions based on polynomial functions of lattices are useful in qualitative modelling of decisions and preference relations. In [MWZ1] we consider two types of such global utility functions, namely quasi- and pseudo-polynomial functions over distributive lattices. Given some partial information about the decision maker's preferences, finding a global utility function that matches this information gives rise to an interpolation problem. We present an algorithm that solves this interpolation problem for quasi- and pseudo-polynomial functions over finite distributive lattices. This algorithm has exponential running time, and we show that the bad running time is not avoidable (assuming P≠NP), since the quasi-polynomial interpolation problem is NP-complete if the number of attributes is at least four. We proved that categorically equivalent finite p-rings have isomorphic additive groups and that they also have the same number of generators. We also classified rings of order p^3 up to categorical equivalence [W7].

Polynomial functions over **Q**, that is, classical polynomials play the main role in [Cz2]. Namely, a cyclic polygon is a convex n-gon inscribed in a circle. If, in addition, one of its sides is a diameter of the circle, then the polygon is called *Thalesian*. Up to permutation, a Thalesian n-gon is determined by the lengths of its non-diametric sides. It is also determined by the distances of its non-diametric sides from the center of its circumscribed circle. In [Cz2], we proved that the Thalesian n-gon in general can be constructed with straightedge and compass neither from these lengths if $n \geq 4$, nor from these distances if $n \geq 5$.

2. Lattices associated with algebras

The quasiorders (preorders) on a set A form a complete lattice with respect to set inclusion, denoted by Quo(A). It was known by Takách's result, that Quo(A) is six-generated if $|A|$ is an accessible cardinal, and by Dolgos's more recent result, that Quo(A) is five-generated for a countable set A. In [K1] we generalized the latter result by proving that Quo(A) is five-generated if $|A|$ is an accessible cardinal. In [Cz8], we proved that if A is a finite set consisting of 2, 3, 5, 7, 9, or more than 10 elements, then Quo(A) is four-generated but not three-generated. Also, if A is countably infinite, then a four-generated sublattice contains all atoms of Quo(A). These statements improve a 1996 result of Chajda and Czédli where six generators were constructed, and the above result of T. Dolgos. In [CzK1], based on involved earlier constructions, we proved two new results in a concise way. Namely, assuming that $|A|$ is an accessible cardinal, we proved that Quo(A) is four-generated if $|A|>4$, and it is even (1+1+2)-generated for $|A|>55$ (and for some sporadic smaller values of $|A|$). We also proved that the lattice Tran(A) of all transitive relations on A is six-generated if $|A|$ is an accessible cardinal and $|A|>2$.

The conguence variety corresponding to a variety V is the lattice variety generated by all the congruence lattices of elements of V. The quasiorder variety of V is defined similarly. In [Gy2] we proved that if V is a locally finite congruence modular variety, then its congruence and quasiorder varieties coincide. A representation of a lattice with posets is an injective homomorphism from the lattice to the quasiorder lattice of a set such that each element of the image is antisymmetric. It is known that any lattice has such a representation. Also, the class of finite lattices having a representation with posets having no infinite chain, that is, posets satisfying ACC and DCC, is described, and coincides with that of the (McKenzie-) lower bounded lattices. In [Gy1], we gave an algorithmically checkable characterization for a finite lattice and a necessary condition for an arbitrary lattice to be representable with DCC posets.

In [KÁ1] and [KÁ2], we investigated the first-order definability in the *embeddability* and the *substructure* orderings of finite directed graphs. In [KÁ1], we described the power of the language of the *embeddability* ordering. Our description yields that, for example, if we allow the usage of a constant, a particular digraph, in our first-order formulas, then the full second-order language of digraphs becomes expressible. The limits of the expressive power of such languages are intimately related to the automorphism groups of the orderings. Such orderings, investigated previously in the line of this research, had all possessed automorphism groups either trivial or isomorphic to $\mathbf{Z}_2$. This pattern breaks in [KÁ2] for the *substructure* ordering. The automorphism group of the *substructure* ordering turned out to be far from trivial. In fact, the question what the automorphism group is stays unsettled in [KÁ2]. It was shown, nevertheless, that the automorphism group is finite. A conjecture has been presented, suggesting a 768-element group. In [KÁ2], with regard to definability in the *substructure* ordering, we lean on the result of [KÁ1] as the following was proven. With the addition of finitely many constants, the *embeddability* relation itself becomes definable in the *substructure* case.

In [H1], among other results, we showed that the invariance groups of lattice-valued functions depend only on the cuts of the function. In [H2] we proved that every isotone Boolean function is a lattice induced threshold function and vice versa. We generalized this result to functions on a k-element set. We also proved that there exists a lattice-valued function the cuts of which give all isotone Boolean functions. In [H3] we generalized the notion of invariance group using group actions of an abstract group. We characterized the Galois closures for both sides of the corresponding Galois connection and applied the results to known group actions, e.g. to the action of GL(n,2). In [H4], we characterized the lattice of cut-sets and the Dedekind-MacNeille completion of the set of images of a lattice-valued function by suitable concept lattices and gave a necessary and sufficient condition under which these lattices coincide. We gave necessary and sufficient conditions under which the lattice of cuts is completely distributive. In [H5], among other properties, we proved that for each positive integer n there exists an n-ary symmetric threshold function that is invariant to only permutation matrices. In [H6] we determined the first three largest numbers of subuniverses of n-element semilattices. We also described the n-element semilattices with these numbers of subuniverses.

For a bounded lattice L, the principal congruences of L form a bounded poset Princ(L). In 2013 Grätzer proved that every bounded poset can be represented in this way. Also, Birkhoff proved in 1946 that every group is isomorphic to the group of automorphisms of an appropriate lattice. In [Cz1], for an arbitrary bounded poset P with at least two elements and an arbitrary group G, we constructed a selfdual lattice L of length sixteen such that Princ(L) is isomorphic to P and the automorphism group of L is isomorphic to G. In [Cz3], we proved that every finite lattice L can be embedded in a three-generated finite lattice K. We also proved that every algebraic lattice with accessible cardinality is a complete sublattice of an appropriate algebraic lattice K such that K is completely generated by three elements. Note that ZFC has a model in which all cardinal numbers are accessible. Our results strengthen Crawley and Dean's 1959 results by adding finiteness, algebraicity, and completeness.

By 1991 and 2002 results of Freese, Grätzer, and Schmidt, every complete lattice A can be represented as the lattice Com(K) of complete congruences of a strongly atomic, 2-distributive, complete modular lattice K. Going much further in [Cz4], we represented three kinds of morphisms between two complete lattices A and A' by complete congruence lattices of two strongly atomic, 2-distributive, complete modular lattices K and K'. First, we represented an arbitrary map from A to A' that is 0-preserving, 0-separating, and preserves arbitrary joins by the extension map from Com(K) to Com(K'). Second, a {0,1}- and complete-meet-preserving map by the restriction map from Com(K) to Com(K').Third, a 0- and arbitrary join-preserving map by the composite of a map naturally induced by a complete lattice homomorphism from K to K' and the complete congruence generation in K'. (This paper deserved the Birkhäuser-Grätzer prize in 2016, see https://doi.org/10.1007/s00012-017-0459-7 for details.)

By lattice duality, slim planar semimodular lattices are the same entities as finite convex geometries of convex dimension at most 2. Hence these lattices are lattices *related* with frequently studied other structures. Their importance also comes from the following fact due to Grätzer and Knapp (2005): planar semimodular lattices and, in particular, rectangular lattices can easily be reduced to their "skeletons", which are slim. In 2009, Grätzer and Knapp proved that every planar semimodular lattice has a rectangular extension. In [Cz5], we proved that, under reasonable additional conditions, this extension is unique. This theorem naturally leads to a hierarchy of special diagrams of planar semimodular lattices. These diagrams are unique in a strong sense; we also explore many of their additional properties. We demonstrated the power of our new classes of diagrams in two ways. First, we proved a simplified version of our earlier Trajectory Coloring Theorem, which describes the inclusion con(p)⊇con(q) for prime intervals p and q in slim rectangular lattices. Second, we proved Grätzer's Swing Lemma for the same class of lattices, which describes the same inclusion more simply. The Swing Lemma, due to Grätzer for slim semimodular lattices and extended by Czédli, Grätzer, and Lakser for all planar semimodular lattices, describes the congruence generated by a prime interval in an efficient way. In [Cz6] we presented a new, direct proof of this lemma, which is shorter than the earlier ones. Also, motivated by the Swing Lemma and mechanical pinball games with flippers, we constructed an online game called Swing Lattice Game, which is available from Czédli's web page. Finite convex geometries are combinatorial structures. As mentioned earlier, they have connections to lattice theory. By a 2017 result of Richter and Rogers, they can be represented by the geometric convex hull operation acting on convex planar polygons. In [Cz7], we managed to replace these polygons by *almost circles* of arbitrary accuracy. That is, for every (small) positive epsilon, by convex sets including a disc and included in a (1+epsilon)-times bigger disc.

In [Cz9], after surveying some known properties of compact convex sets in the plane, we gave two rigorous proofs of the general feeling that supporting lines of these sets can be slide-turned slowly and continuously. This result plays a key role in geometric papers motivated by planar semimodular lattices; see [Cz11] (to be mentioned later) and two other papers outside the project (https://arxiv.org/pdf/1807.03443 and https://arxiv.org/pdf/1802.06457). Adaricheva and Bolat proved that if U and V are circles in a triangle, then one of these two circles is included in the convex hull of the other circle and two vertices of the triangle. In [Cz10], we gave a short new proof for this result, and we pointed out that a straightforward generalization for spheres fails. In [Cz11], we proved the converse of the Adaricheva-Bolat result (which is re-proved in [Cz10]) in the following sense. Assume that U is compact convex set in the plane. If for every V such that V is similar to U and for any triangle including both U and V we have that at least one of U and V is in the convex hull of the other and two appropriate vertices of the triangle, then U is a disk. The same (in fact, a stronger) statement holds with "isometric" instead of "similar". Originally, the Swing Lemma (Grätzer, 2015) describes how a congruence spreads from a prime interval to another prime interval in a slim, planar, semimodular lattice. In [Cz12], we generalized the Swing Lemma to planar semimodular lattices.

For bounded lattices L and L', let f: $L \to L'$ be a lattice homomorphism. Then the map Princ(f): Princ(L)→Princ(L'), defined by con(x, y) → con($f(x)$, $f(y)$), is a 0-preserving isotone map from the bounded ordered set Princ(L) of principal congruences of L to that of L'. In [Cz16], we proved that every 0-preserving isotone map between two bounded ordered sets can be represented in this way. Our result generalizes a 2016 result of Grätzer from {0,1}-preserving isotone maps to 0-preserving isotone maps. Observe that Princ mentioned in [Cz16] is a functor. Let A be a small subcategory of bounded posets and {0,1}-preserving isotone maps. In [Cz13], we proved that A is the Princ-image of a subcategory of the category of selfdual lattices with {0,1}-preserving lattice homomorphisms. Furthermore, we introduced the concept of cometic functors as an auxiliary tool to derive some families of maps from injective maps and surjective maps; this can be useful in various fields of mathematics, not only in lattice theory. Call a subposet Q of a finite distributive non-singleton lattice D *eligible* if 0, 1, and all join-irreducible elements are in Q. We say that an algebra A *represents* (Q,D) if there is an isomorphism between D and the congruence lattice of A such that Q corresponds to the set of principal congruences of A. In [Cz14] we proved that whenever D above has the property that each eligible subposet Q is representable by an algebra A (depending on Q), then D is planar and has at most one join-reducible coatom. In [Cz15], we gave the converse of the result of [Cz14]. In fact, we did even more by proving that whenever D is a non-singleton, finite, planar distributive lattice with at most one join-reducible coatom, Q is an eligible subposet of D, and G is a group, then there exists a *lattice* (not just an algebra) L such that L represents (Q,D) and the automorphism group of L is isomorphic to G.

Several papers (like the just-mentioned [Cz15]) in the literature represent posets and groups by lattices. In [Cz17], "representing" lattices with more ideals than filters were constructed. Let k be an infinite cardinal number, and let m=2^k, the size of the power set of a set of size k. Assume that P is a poset and G is a group such that their sizes, $|P|$ and $|G|$, are at most k. We proved the existence of a lattice L with the following four properties: (i) the poset Princ(L) of principal congruences of L is isomorphic to P, (ii) the automorhism group of L is isomorphic of G, (iii) L has m many ideals, but (iv) L has only k many filters. In [K2], we investigated |Con(L)|, when L is an n-element lattice. We proved that the third, fourth and fifth largest numbers of congruences of an n-element lattice are $5\cdot 2^{n-5}$ if $n \geq 3$, 2^{n-3} and $7\cdot 2^{n-6}$ if $n \geq 6$, respectively. Moreover, we determined the structures of the n-element lattices having $5\cdot 2^{n-5}$, 2^{n-3} and $7\cdot 2^{n-6}$ congruences, respectively, and also the structures of their congruence lattices. Since M. Kindermann's 1974 result, which characterizes polynomial functions on finite lattices as those multivariate monotone functions that preserve all (compatible) tolerance relations, tolerance relations have played an important role in lattice theory**.** [Cz18] introduces the concept of a *doubling tolerance T* on a modular lattice L of finite length as such a tolerance (relation) whose blocks are at most two-element. It appears that these tolerances give rise to an interesting lattice construction under the name *doubling*, and [Cz18] even exhibited an application of this construction for coalition lattices. As a continuation of [Cz18], [Cz19] investigates particular doubling tolerances under the name 2-uniform tolerances of lattices. A tolerance is called *2-uniform* if all of its blocks are two-element. [Cz19] characterizes permuting pairs of 2-uniform tolerances of lattices of finite length. In particular, any two 2-uniform congruences of such a lattice permute.

3. Semigroups and special classes of algebras

In [DTBSz1] we proved that the E-solid locally inverse semigroups are, up to isomorphism, the regular subsemigroups of lambda-semidirect products of completely simple semigroups by inverse semigroups where lambda-semidirect product is a version of the usual semidirect product within the class of regular semigroups. By extending the Margolis-Meakin expansion of an X-generated group G, in [BSz2] we introduced a transparent construction for the universal object among the X-generated F-inverse monoids, considered as unary inverse monoids, whose maximum group quotient is G. In [SzNSzM1] we describe the inverse monoids of partial permutations which arise as partial automorphism monoids of finite graphs, and we also characterize the inverse monoids which are isomorphic to partial automorphism monoids of finite graphs. Conjugation semigroups, where conjugation is a unary operation on a semigroup with properties stemming from the conjugation of quaternions, have appeared in category theory. In [BSZ3] we show that such unary operations of groups are in one-to-one correspondence with their endomorphisms into their centers, and we prove that cancellative conjugation semigroups are, up to isomorphism, the conjugation subsemigroups of conjugation groups.

In [SzN1], we studied locally injective maps (immersions) between finite-dimensional connected Delta-complexes by replacing the fundamental group of the base space by an appropriate inverse monoid. We showed how conjugacy classes of the closed inverse submonoids of this inverse monoid may be used to classify connected immersions into the complex. In [SzN2], we investigated algorithmic problems in inverse monoids. We proved that the class of finitely presented inverse monoids whose Schützenberger graphs are quasi-isometric to trees has a uniformly solvable word problem, furthermore, the languages of their Schützenberger automata are context-free. On the other hand, we show that there is a finitely presented inverse monoid with hyperbolic Schützenberger graphs and an unsolvable word problem.

4. Clones

We described the principal ideals of the minor quasiorder of functions in terms of colorings of partition lattices [W2]. For multisorted operations, we determined the minimal and the maximal elements of the minor quasiorder, we described its finite principal filters, and we characterized its order ideals in terms of relational constraints [W4]. We characterized lazy binary operations: we proved that lazy groupoids form 13 varieties, and we explicitly described the Cayley tables of the groupoids in these varieties [W6]. We classified undirected graphs with respect to the associative spectra of their graph algebras. We characterized antiassociative digraphs, and we also proved that the associative spectrum of any graph algebra is either constant or it grows exponentially [W8, W9]. We proved that every two-element algebra has the property that solution sets of systems of equations are exactly those sets of tuples that are closed under the centralizer of the clone of term operations of the algebra [W1]. We proved that in general for finite algebras this property is equivalent to polymorphism-homogeneity [W10].

For five of the six classes of maximal clones, it has been shown that their members are finitely generated. The only exception is the class of the clones of finite bounded posets. In his seminal paper, Tardos proved that in this class there exist non-finitely generated clones. It seems really difficult to give a reasonable description of the finite bounded posets with finitely generated clones. In [KMZ1] we investigated the clones of some posets in terms of the finite generability point of view. In [KMZ1], we presented a new family of finite bounded posets whose clones of monotone operations are not finitely generated. We also studied the clones of locked crowns (certain finite posets) from the finite generability point of view. Although our investigations have not been conclusive in this direction, they led to some general results on monotone clones. For example, we proved that if the clone of a finite bounded poset is finitely generated, then it has a three element generating set that consists of an ascending idempotent monotone operation and the 0 and 1 constant operations. We call a monotone operation ascending if it is greater than or equal to some projection.

We studied the monoidal intervals on finite sets, especially on sets with three or four elements. Our main goal is to give a complete description for the monoidal intervals on the set {0, 1, 2}. In [DM1] we investigated transformation monoids that are built up from inverse transformation monoids constructed from finite lattices by adding all the unary constant transformations. We gave a complete description for the corresponding monoidal intervals in the clone lattice. The results in [DMM1], [DM3], and [DMM2] are based on a detailed computer study. In these manuscripts, we determined various classes of finite monoidal intervals in the clone lattice. In [DM2] we studied a modified version of the classical table tennis (also called ping pong) as a mathematical game. The modification is about the counting of the points: a point is scored by the classical rules, but the result must be simplified (e.g., 6:3 must be simplified to 2:1). In [DM2] we gave some basic facts about this game called 'Arithmetical ping pong'.

Publications of the project

[Cz1] Gábor Czédli: *An independence theorem for ordered sets of principal congruences and automorphism groups of bounded lattices*, Acta Sci. Math. (Szeged) 82, 3-18 (2016)
http://www.math.u-szeged.hu/~czedli/publ.pdf/czedli_an-independence-theorem-for ordered-sets-of-principal-congruences-and-automorphism-groups-of-bounded-lattices.pdf
http://dx.doi.org/10.14232/actasm-015-817-8

[Cz2] Gábor Czédli: *Geometric constructibility of Thalesian polygons*, Acta Sci. Math. (Szeged) 83, 61-70 (2017)
http://www.math.u-szeged.hu/~czedli/publ.pdf/czedli_geometric-constructibility-of-thalesian-polygons.pdf
http://dx.doi.org/10.14232/actasm-015-072-8

[Cz3] Gábor Czédli: *Lattices embeddable in three-generated lattices*, Acta Sci. Math. (Szeged) 82, 361-382 (2016)
http://arxiv.org/abs/1512.03971
http://dx.doi.org/10.14232/actasm-015-586-2

[Cz4] Gábor Czédli: *Complete congruence lattices of two related modular lattices*, Algebra Universalis 78, 251-289 (2017)
https://tinyurl.com/czg137-pdf
http://dx.doi.org/10.1007/s00012-017-0457-9

[Cz5] Gábor Czédli: *Diagrams and rectangular extensions of planar semimodular lattices*, Algebra Universalis 77, 443-498 (2017)
http://arxiv.org/abs/1412.4453
http://dx.doi.org/10.1007/s00012-017-0437-0

[Cz6] Gábor Czédli and Géza Makay: *Swing lattice game and a direct proof of the swing lemma for planar semimodular lattices*, Acta Sci. Math. (Szeged) 83, 13-29 (2017)
http://arxiv.org/abs/1607.06809
http://dx.doi.org/10.14232/actasm-016-036-3

[Cz7] Gábor Czédli and János Kincses: *Representing convex geometries by almost-circles*, Acta Sci. Math. (Szeged) 83, 393-414 (2017)
http://arxiv.org/abs/1608.06550
http://dx.doi.org/10.14232/actasm-016-044-8

[Cz8] Gábor Czédli: *Four-generated quasiorder lattices and their atoms in a four-generated sublattice*, Communications in Algebra 45/9, 4037-4049 (2017)
https://tinyurl.com/czg134-pdf

[Cz9] G. Czédli and L. L. Stachó: *A note and a short survey on supporting lines of compact convex sets in the plane*, Acta Universitatis Matthiae Belii, Series Mathematics 24, 3-14, Online Edition: Issue 2016, 44-55 (2016)
https://actamath.savbb.sk/pdf/oacta2016006.pdf

[Cz10] G. Czédli: *An easy way to a theorem of Kira Adaricheva and Madina Bolat on convexity and circles*, Acta Sci. Math. (Szeged) 83, 703-712 (2017)
http://arxiv.org/abs/1610.02540
http://dx.doi.org/10.14232/actasm-016-307-7

[Cz11] G. Czédli: *Characterizing circles by a convex combinatorial property*, Acta Sci. Math. (Szeged) 83, 683-701 (2017)
http://arxiv.org/abs/1611.09331
http://dx.doi.org/10.14232/actasm-016-570-x

[Cz12] G. Czédli, G. Grätzer and H. Lakser: *Congruence structure of planar semimodular lattices: The General Swing Lemma*, Algebra Universalis 79:40 (2018)
https://tinyurl.com/czg142-pdf
https://doi.org/10.1007/s00012-018-0483-2

[Cz13] G. Czédli: *Cometic functors and representing order-preserving maps by principal lattice congruences*, Algebra Universalis 79:59 (2018)
https://tinyurl.com/czg144-pdf
https://doi.org/10.1007/s00012-018-0545-5

[Cz14] G. Czédli: *On the set of principal congruences in a distributive congruence lattice of an algebra*, Acta Sci. Math. (Szeged) 84, 357-375 (2018)
https://arxiv.org/abs/1705.10833
http://dx.doi.org/10.14232/actasm-017-538-7

[Cz15] G. Czédli: *Characterizing fully principal congruence representable distributive lattices*, Algebra Universalis 79:9 (2018)
http://arxiv.org/abs/1706.03401
http://dx.doi.org/10.1007/s00012-018-0498-8

[Cz16] G. Czédli: *Representing an isotone map between two bounded ordered sets by principal lattice congruences*, Acta Mathematica Hungarica 155/2, 332-354 (2018)
https://tinyurl.com/czg146-pdf
https://doi.org/10.1007/s10474-018-0844-5

[Cz17] G. Czédli and C. Muresan: *On principal congruences and the number of congruences of a lattice with more ideals than filters*, Acta Sci. Math. (Szeged) 85, 363-380 (2019)
http://arxiv.org/abs/1711.06394
https://doi.org/10.14232/actasm-018-538-y

[Cz18] G. Czédli: *Doubling tolerances and coalition lattices*, "Special Issue of the Journal of Multiple-Valued Logic and Soft Computing dedicated to Ivo G. Rosenberg", accepted (2021)
https://tinyurl.com/czgdtclat-pdf

[Cz19] G. Czédli: *Permuting 2-uniform tolerances on lattices*, submitted to "Special Issue of the Journal of Multiple-Valued Logic and Soft Computing dedicated to Ivo G. Rosenberg", submitted (2021)
http://arxiv.org/abs/1909.10845

[CzK1] G. Czédli and J. Kulin: *A concise approach to small generating sets of lattices of quasiorders and transitive relations*, Acta Sci. Math. (Szeged) 83, 3-12 (2017)
http://arxiv.org/abs/1609.01016
http://dx.doi.org/10.14232/actasm-016-056-2

[BSz1] M. B. Szendrei: *On products of inverse semigroups*, manuscript (2021)
http://www.math.u-szeged.hu/~bszendre/wreath-lambda_otka.pdf

[BSz2] K. Auinger, G. Kudryavtseva, M. B. Szendrei: *F-inverse monoids as algebraic structures in enriched signature*, Indiana University Mathematics Journal, accepted (2021)
https://arxiv.org/abs/1905.09256

[BSZ3] M. B. Szendrei: *Structure theory of regular semigroups*, Semigroup Forum 100, 119-140 (2020)
https://doi.org/10.1007/s00233-019-10055-8
https://arxiv.org/abs/1909.05280

[BSz4] M. B. Szendrei: *On the structure of cancellative conjugation semigroups*, manuscript (2021)
http://www.math.u-szeged.hu/~bszendre/SzM3.pdf

[DTBSz1] T. Dékány, M. B. Szendrei, I. Szittyai: *E-solid locally inverse semigroups*, Acta Sci. Math. 85, 381-411 (2019)
https://doi.org/10.14232/actasm-018-311-5
https://arxiv.org/abs/1807.01265

[D1] B. Billhardt, T. Dékány: *Embeddings of R-unipotent semigroups*, manuscript (2021)

[DM1] Miklós Dormán: *Transformation monoids with finite monoidal intervals*, Algebra Universalis, 77/2, 163-189, (2017)
http://www.math.u-szeged.hu/~dorman/Main/Publications/2017-D-tmfmi.pdf

[DM2] Csákány Béla, Dormán Miklós: *Aritmetikai pingpong*, Polygon, XXIV/1, (in Hungarian), (2016)
http://www.math.u-szeged.hu/~dorman/Main/Publications/2016-CD-app.pdf

[DM3] Dormán, M.: *Finite monoidal intervals II*,, manuscript, (2021)
http://www.math.u-szeged.hu/~dorman/Main/Publications/FMIs-II.pdf

[DMM1] Dormán, M., Maróti, M.: *Finite monoidal intervals*, manuscript, (2021)
http://www.math.u-szeged.hu/~dorman/Main/Publications/FMIs.pdf

[DMM2] Dormán, M., Maróti, M.: *Finite monoidal intervals III.*, manuscript, (2021)

[Gy1] G. Gyenizse: *On lattice representations with DCC posets*, Order 37, 31-43 (2020)
http://gllrumdsuhg.atw.hu/Order_lattices.pdf

[Gy2] G. Gyenizse: *Quasiorder lattices in congruence modular varieties*, Acta Scientiarum Mathematicarum (Szeged) 86/12, 3-10 (2020)
http://gllrumdsuhg.atw.hu/gumm_min_alg.pdf

[GyMZ] Gyenizse, G; Maróti, M; Zádori, L: *n-permutability is not join-prime for* $n \geq 5$, IJAC 30/8, 1717 - 1737 (2020)
https://doi.org/10.1142/S0218196720500605
https://www.worldscientific.com/doi/epdf/10.1142/S0218196720500605

[H1] E. K. Horváth, B. Seselja, A. Tepavcevic: *Cut approach to invariance groups of lattice-valued functions*, Soft computing 21/4, 853-859 (2017)
https://link.springer.com/article/10.1007/s00500-016-2084-3

[H2] E. K. Horváth, B. Seselja, A. Tepavcevic: *A note on lattice variant of thresholdness of Boolean functions*, Miskolc Mathematical Notes 17/1, 293-304 (2016)
http://real.mtak.hu/71435/1/1485.pdf

[H3] E. K. Horváth, R. Pöschel, S. Reichard: *Invariance groups of functions and related Galois connections*, Beitr Algebra Geom, (2020)
https://doi.org/10.1007/s13366-020-00525-4
https://link.springer.com/article/10.1007/s13366-020-00525-4

[H4] Eszter K. Horváth, Sándor Radeleczki, Branimir Seselja, Andreja Tepavcevic: *A note on cuts of lattice-valued functions, and concept lattices*, Fuzzy Sets and Systems, submitted (2021).

[H5] N. Bogya, E. K. Horváth: *Threshold functions and matrix groups*, manuscript (2021)

[H6] D. Ahmed, E. K. Horváth: *The first three largest numbers of subuniverses of semilattices*, Miskolc Mathematical Notes, accepted (2021)

[KÁ1] Á. Kunos: *Definability in the embeddability ordering of finite directed graphs, II*, Order 36/2, 291-311 (2019)
https://doi.org/10.1007/s11083-018-9467-2
online at Springer Nature Sharedit

[KÁ2] Á. Kunos: *Definability in the substructure ordering of finite directed graphs,* accepted in Order (2021)
https://doi.org/10.1007/s11083-020-09548-x
online at Springer Nature Sharedit

[KMZ1] Á. Kunos, M. Maróti, and L. Zádori: *On finite generability of clones of finite posets,* Order 36/3, 653-666, (2019)
https://doi.org/10.1007/s11083-019-09486-3
online at Springer Nature Sharedit

[KMZ2] Á. Kunos, M. Maróti and L. Zádori: *Critical relations of crowns in critical time of coronavirus depression*, Order, submitted (2021)
http://www.math.u-szeged.hu/~akunos/0_OTKA/critical_relations.pdf

[K1] J. Kulin: *Quasiorder lattices are five-generated*, Discuss. Math. Gen. Algebra Appl. 36, 59-70 (2016)
http://www.math.u-szeged.hu/~kulin/fajlok/kulin_quasiorder-lattices-are-five-generated.pdf

[K2] C. Muresan and J. Kulin: *Some extremal values of the number of congruences of a finite lattice*, Order 37, 445-460 (2020)
http://www.math.u-szeged.hu/~kulin/fajlok/muresan-kulin-1.pdf

[MWZ1] M. Couceiro, M. Maróti, T. Waldhauser, L. Zádori: *Computing version spaces in the qualitative approach to multicriteria decision aid*, Internat. J. Found. Comput. Sci. 30, 333–353, (2019)
www.math.u-szeged.hu/~twaldha/otka/otka115518/MWZ1.pdf
https://www.worldscientific.com/doi/abs/10.1142/S0129054119500084

[Sz1] Kearnes, K. A., Kiss, E. W., Szendrei, A.: *Varieties whose finitely generated members are free*, Algebra Universalis 79/3, 3:17, (2018)
http://arxiv.org/pdf/1508.03807v2
http://rdcu.be/Ip26
https://doi.org/10.1007/s00012-018-0480-5

[Sz2] Kearnes, K., Szendrei, A., Willard, R.: *Simpler Maltsev conditions for weak difference terms in locally finite varieties,* Algebra Universalis 78, 555-561 (2017)
http://www.math.u-szeged.hu/~szendrei/locfin-diffterm_final.pdf
https://doi.org/10.1007/s00012-017-0475-7

[Sz3] Kearnes, K. A., Szendrei, A.: *Cube term blockers without finiteness,* Algebra Universalis 78, 437-459, (2017)
http://arxiv.org/pdf/1609.02605
https://doi.org/10.1007/s00012-017-0476-6

[Sz4] Bulatov, A., Mayr, P., Szendrei A.: *The subpower membership problem for finite algebras with cube terms*, Logical Methods in Computer Science, 15/1:11 (2019)
http://arxiv.org/pdf/1803.08019v2
https://lmcs.episciences.org/5183

[Sz5] Mayr, P., Szendrei A.: *Algebras from congruences*, submitted, (2021)
http://arxiv.org/pdf/1910.00689

[Sz6] Bergman, C., Szendrei A.: *Random models of idempotent linear Maltsev conditions. I. Idemprimality*, Algebra Universalis, 81:9 (2020)
http://arxiv.org/pdf/1901.06316
https://rdcu.be/b1amk
https://doi.org/10.1007/s00012-019-0636-y

[Sz7] Ánh,P. N., Kearnes, K. A., Szendrei, Á.: *Commutative Rings Whose Principal Ideals Have Unique Generators*, Advances in Rings, Modules and Factorizations (Proceedings of the Conference on Rings and Factorizations, February 19-23, 2018, Graz, edited by Facchini A., Fontana M., Geroldinger A., Olberding B.), Springer Proceedings in Mathematics & Statistics, vol 321. Springer, Cham; pp. 1--9.
http://arxiv.org/pdf/1901.06304
https://doi.org/10.1007/978-3-030-43416-8_1

[Sz8] Kearnes, K. A., Szendrei, Á.: *Is supernilpotence super nilpotence?,* Algebra Universalis, 81:3 (2020)
http://arxiv.org/pdf/1907.03216
https://rdcu.be/b1PDK
https://doi.org/10.1007/s00012-019-0632-2

[Sz9] Kearnes, K. A., Kiss, E. W., Szendrei, Á.: *Minimal abelian varieties of algebras I*, Internat. J. Algebra Comput., online ready (2020).
http://arxiv.org/pdf/1912.05653
https://doi.org/10.1142/S0218196721500107

[Sz10] Kearnes,K. A., Meredith, C., Szendrei, Á.: *Neutrabelian algebras,* Algebra Universalis, 82:13 (2021)
http://arxiv.org/pdf/2007.07198
https://rdcu.be/cdNXu
https://doi.org/10.1007/s00012-020-00705-2

[Sz11] Kearnes, K. A., Szendrei, Á.: *Ultralocally closed clones,* Multiple-Valued Logic and Soft Computing, submitted (2021)
http://arxiv.org/pdf/2004.00228

[Sz12] Caicedo, X., Campercholi, M., Kearnes, K. A., Sánchez Terraf, P., Szendrei, Á., Vaggione, D.: *Every minimal dual discriminator variety is minimal as a quasivariety*, Algebra Universalis, accepted (2021)
http://www.math.u-szeged.hu/~szendrei/min-dd-var_posted.pdf

[SzN1] J. Meakin, N. Szakács, *Inverse monoids and immersions of cell complexes*, Internat. J. Algebra and Comput., accepted (2021)
arXiv:1709.03887

[SzN2] R. D. Gray, P. V. Silva, N. Szakács, *Algorithmic properties of inverse monoids with hyperbolic and tree-like Schützenberger graphs*, manuscript (2021)
arXiv:1912.00950

[SzNSzM1] R. Jajcay, T. Jajcayova, N. Szakács, M. B. Szendrei: *Inverse monoids and partial graph automorphisms*, Journal of Algebraic Combinatorics, online (2020)
https://doi.org/10.1007/s10801-020-00944-5
https://arxiv.org/abs/1809.04422v2

[W1] E. Tóth, T. Waldhauser: *On the shape of solution sets of systems of (functional) equations*, Aequationes Math. 91, 837–857 (2017)
www.math.u-szeged.hu/~twaldha/otka/otka115518/WT1.pdf
https://link.springer.com/article/10.1007/s00010-017-0499-2)

[W2] E. Lehtonen, T. Waldhauser: *Minor posets of functions as quotients of partition lattices,* Order 36, 23–41 (2019)
www.math.u-szeged.hu/~twaldha/otka/otka115518/WT2.pdf
https://link.springer.com/article/10.1007/s11083-018-9453-8

[W3] E. Lehtonen, T. Waldhauser, R. Pöschel: *Reflection-closed varieties of multisorted algebras and minor identities,* Algebra Universalis 79:70 (2018)
www.math.u-szeged.hu/~twaldha/otka/otka115518/WT3.pdf
https://link.springer.com/article/10.1007/s00012-018-0547-3

[W4] E. Lehtonen, T. Waldhauser, R. Pöschel: *Reflections on and of minor-closed classes of multisorted operations,* Algebra Universalis 79:71 (2018)
www.math.u-szeged.hu/~twaldha/otka/otka115518/WT4.pdf
https://link.springer.com/article/10.1007/s00012-018-0549-1

[W5] T. Waldhauser: *On eigenvectors of the Pascal and Reed-Muller-Fourier transforms,* Acta Cybernet. 23, 959–979 (2018)
www.math.u-szeged.hu/~twaldha/otka/otka115518/WT5.pdf
https://cyber.bibl.u-szeged.hu/index.php/actcybern/article/view/3987

[W6] B. Kaprinai, H. Machida, T. Waldhauser: *Lazy groupoids*, Semigroup Forum, 102, 160-183 (2020)
www.math.u-szeged.hu/~twaldha/otka/otka115518/WT6.pdf
https://link.springer.com/article/10.1007%2Fs00233-020-10128-z

[W7] K. Kaarli, T. Waldhauser: *On categorical equivalence of finite p-rings*, Communications in Algebra, 49, 1437-1450 (2021)
www.math.u-szeged.hu/~twaldha/otka/otka115518/WT7.pdf

https://www.tandfonline.com/doi/abs/10.1080/00927872.2020.1837850?journalCode=lagb20

[W8] E. Lehtonen, T. Waldhauser: *Associative spectra of graph algebras I. Foundations, undirected graphs, antiassociative graphs*, Journal of Algebraic Combinatorics, accepted (2021)
www.math.u-szeged.hu/~twaldha/otka/otka115518/WT8.pdf

[W9] E. Lehtonen, T. Waldhauser: *Associative spectra of graph algebras II. Satisfaction of bracketing identities, spectrum dichotomy*, Journal of Algebraic Combinatorics, submitted (2021)
www.math.u-szeged.hu/~twaldha/otka/otka115518/WT9.pdf

[W10] E. Tóth, T. Waldhauser: *Polymorphism-homogeneity and universal algebraic geometry*, Discrete Mathematics and Theoretical Computer Science, submitted (2021)
www.math.u-szeged.hu/~twaldha/otka/otka115518/WT10.pdf