CLOSING REPORT

Robust stability of dynamical systems

PI: Daniel Bachrathy

1 BACKGROUND & GOALS

The determination of the stability of systems with time delay and/or periodic excitation is of high importance for many industrial and research applications, like turning/milling processes, wheel shimmy, traffic jams and even for neural systems or human balancing. There are many time and frequency domain method to determine the stability, however, in some case, the computation of the stability boundary requires very high computational effort and unnecessarily high resolution, due to the dense and sharp line segments of the stability boundary (see Fig. 1). In these cases, the computation of only the lower envelope would be sufficient. Furthermore, the traditional methods are dealing with exact models, while in practice the parameters of the models are uncertain, moreover, they can change during the operation.

One of our goals was to establish a fast and reliable method, which can predict the robust stability, independent of the parameter fluctuations and apply it to the milling models, which has great practical importance.

The second goal was to create mathematically correct algorithms for the investigation of quasi-periodic systems. As a test case, the twin-spindle milling operation and the spindle speed variation in milling are used.

2 ACHIEVED GOALS

The project achieved goals in the *robust stability analysis* in two field.

Robust stability against time delay

In [1] we have developed a perturbation method for calculating the robust stability limits of delayed differential equations. The method eliminates the essential parameter sensitivity of the stability charts. The determined curve forms the lower envelope of the lobe structure, thus it can be used in case of inaccurate input parameters, such as: natural frequency or time-delay. In the proposed computation method, an additional perturbation parameter must be introduced, which increases the dimension of the parameter space by one, nevertheless, the applied self-developed Multi-Dimensional Bisection Method can overcome this problem without significant increase of computational time. In addition, in case of the turning process, the presented method also speeds-up the computation at lower spindle speed range, moreover, smaller resolution in the parameter space is sufficient (see Fig. 1). The robust stability limit can be determined analytically in closed form equations in special mechanical models, as it is shown in this paper for the delayed oscillator and for the single degree of freedom turning model with traditional process damping effect.

This method is improved for multi degree of freedom systems [24] both in time and frequency domain. It is applied in a cooperative work to a multi-cutter turning processes, too [9] (Fig.2).

The robustness against the time delay (that is the lover envelope of the stability diagram) is also demined for milling operation (Fig. 3), however, in this case the computation demand is significant.

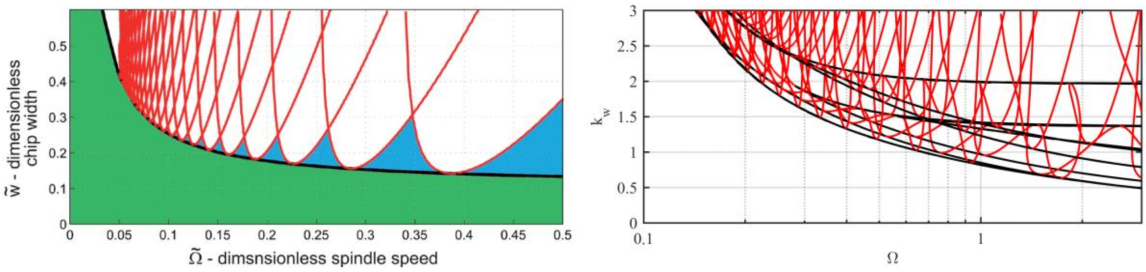

Fig.1 left: stability boundaries and the robust stability limit of the turning model.
Right: Stability boundaries of the two-cutter turning system (red lines) and its robust stability limits (black lines) for the dimensionless parameters

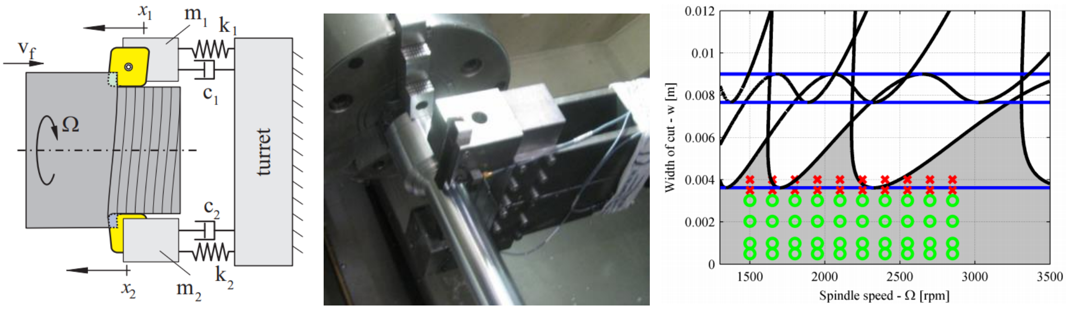

Fig.2 Mechanical model and the test rig of the two-cutter turning process and a corresponding measured stability chart.

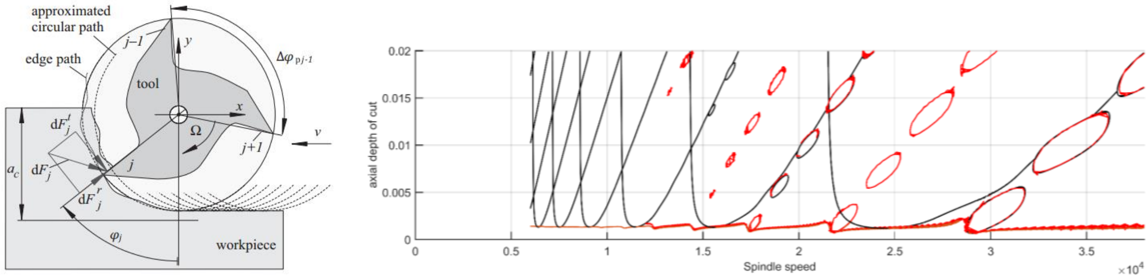

Fig.3 Mechanical model of a milling process and the corresponding stability boundaries (black lines) and its robust stability limits (red lines)

Robust stability against the dynamical system parameters

Due to imperfect measurements, noise, uncertain and varying operational conditions, the mathematical models provide a deficient representation of the system. This leads to the need for the adaptation of robust stability analysis methods, which guarantee stability against bounded uncertainties and perturbations. In [26], the previouse work of the PI: the extended multi-frequency solution is combined with the method of μ- analysis to determine robust stability boundaries for milling operations with uncertain dynamics. The measured FRFs can directly be applied, and no modal parameter fitting is required, which is an essential aspect for industrial applications. Several numerical issues were discussed in order to speed up the computation significantly. Based on a series of numerical tests, it can be stated that under optimized resolutions and settings, the computational time of the robust stability boundaries is only 2-5 times longer than the calculation of the conventional stability lobe diagram. The definition of stability radius in terms of the μ-values was found to be an effective method in the analysis of sensitivity of milling operations with respect to FRF-based uncertainties. When the measured frequency response functions are loaded with significant noise, then uncertainty can be characterized by the variance of the measurements. The proposed concept utilizes directly the uncertainty of the transfer function matrix, which can be calculated during the sequentially repeated modal experiments. The results were tested in a real case study and recommendations are given on the evaluation of the uncertainty envelope. The frequency response functions were measured in an industrial NCT EmR-610Ms type milling machine at the tool tip, and the variance of the measurements was calculated. The robust stability lobe diagrams were determined for uncertainty levels 1σ, 2σ and 3σ see (Fig. 4).

During the validation of the robust methods, many traditional stability chart were computed (see the dark grey line in Fig.4 right). For these computationally demanding problems the High Performance Computation (HPC) method were used (partially based on GPGPU hardware equipment).

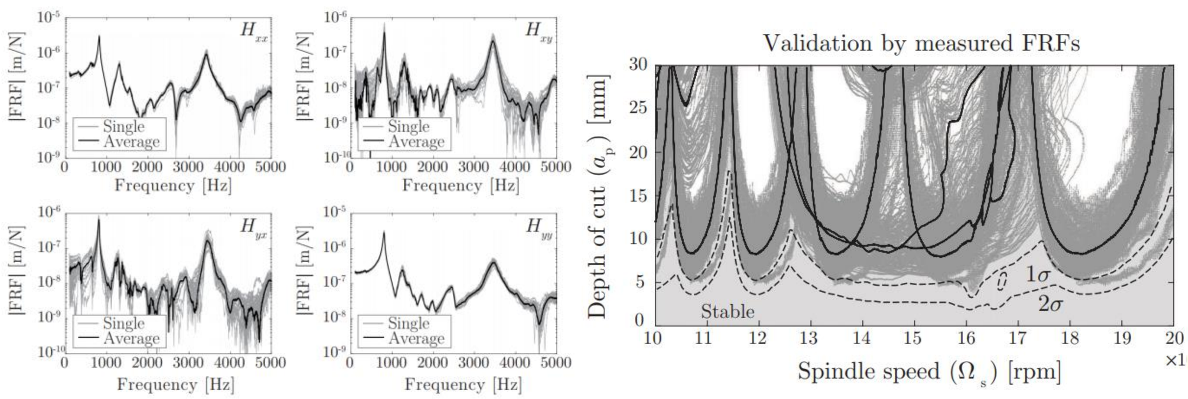

Fig.4 left: Average (black) and the uncertainty (grey) of the measured Frequency Response Function; right: corresponding stability boundary (black) and the determined robust stability boundary (dashed)

Stability of quasi periodic time-delayed systems

Fist the effect of wavy tool path on the stability properties of milling were investigated [17]. The mathematical model of such a milling operation was derived for a simple one DoF mechanical model, and it was shown that the model typically leads to a quasi-periodic parametric excitation in the DDE model. It was shown that the stability analysis of these systems is extremely time-consuming even if the most efficient methods (like the Semi-Discretization Method) are used for long periodic approximations of the quasi-periodicity. The combination of the SDM with the implicit subspace iteration method improves the efficiency of these calculations and makes it possible to construct stability charts for these cases in reasonable time. The analysis of the resulting stability charts showed that the application of a wavy tool path affects the stability properties in the domain of high cutting speeds only (see Fig. 5).

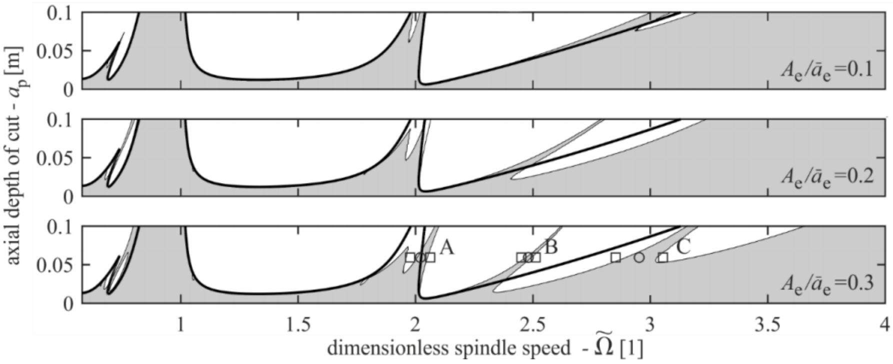

Fig.5 left: Effect of the amplitude of the wavy tool path (shaded region), thick lines indicate the stability boundaries corresponding to a straight tool path.

As a second case study, it is shown for that the commonly used Sinusoidal Spindle Speed Variation (SSSV) in milling can be described as a delay differential equation with modulated time-periodic coefficients [22]. It is shown that simplest analogue model of the SSSV in milling process which can already capture the emerging quasi-periodic phenomenon is the Delayed Mathieu Equation (DME) extended by a Frequency Modulated (FM) periodic coefficients and external sinusoidal forcing. Based on the series of Bessel functions of the modulated term, the infinite block Hill matrix in which all the elements are infinite Hill matrices themselves are determined. The stability boundary is approximated by the determinant of the truncated version and also the maximal value of the vibration displacement is approximated by L1 norm of the resultant coefficients. This could be important to predict the surface error of an SSSV milling processes.

The proposed algorithm is tested and compared with numerical time-domain simulations. The time signal in Fig.6a shows that after a transient phase the numerical simulation tends to the approximated solution. The corresponding spectrum contains all the components as assumed and some others belong to the transient part. The upper estimation provides a close approximation for the maximal value of the simulated quasi-periodic oscillations. The approximated maximal values in the function of δ and ε are plotted in Fig.6b by shaded color. Figure 1a presents the stability boundary by black lines where the determinant is zero.

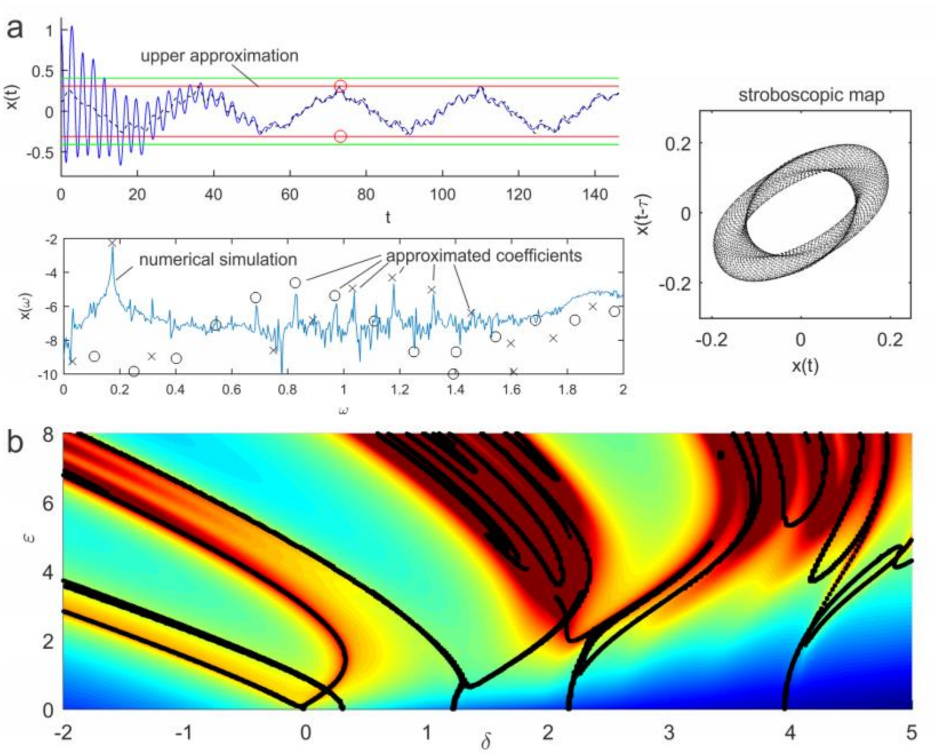

Fig. 6: a) time signal of a numerical simulation of the frequency modulated delayed Mathieu equation with quasiperiodic excitation, its stroboscopic map and the corresponding spectrum b) Colormap denotes the maximal position, the black lines denote the corresponding stability boundary

3 SIDE RESULTS

During the evaluation of the main research directions many associated problem must be solved. Most of them related to the measurement of the milling processes. Many of them represents a complete and independence research direction.

- We found that the forced vibration can significantly affect the stability measurements. These force vibration is closely related to the surface quality. Thus in publications [5,6,12,14,20], the surface modelling of a machined surface is investigated and measured.
- For the proper cutting force model an improved measurement method must be created. Several preparatory milling test were carried out [7,10]. The proposed method [23] the dynamic range of the Kistler force sensor were almost tripped, and the cutting force characteristic is determined in a single cutting test.
- Further problem was the accurate detection of the unstable behaviour of the milling process during the so-called chatter tests. A new method is introduced to identify the stability boundaries of cutting operations directly from vibration measurements to estimate the stability lobe diagram by means of the modulus of the dominant Floquet multiplier. The method is based on the investigation of the transient vibrations, which are generated by operational impact tests. The response of the system is captured by acceleration signals, from which the

perturbation about the periodic orbit can be separated by the use of a comb filter. With this quantitative measure of the machining process, the variation of the stability limit can also be traced without switching between stable and unstable cutting operations. The stability boundary can be determined precisely by means of interpolation between measurement points, furthermore, extrapolation can also be used to predict the distance from the stability boundary based on stable measurement points only. The efficiency and accuracy of the proposed methodology were validated by means of peripheral milling tests.

- It is also found during the cutting tests that the stochasticity in the material removal process is an inherent property. It is shown in [27,28] that the cutting force should be modelled as a stochastic process, driven by a Gaussian white noise (see Fig.7a). Furthermore, a possible explanation is given for the measurement difficulties of the theoretically predicted stability boundaries with the use of the stationary second moment of the stochastic model. The method predicts large amplitude vibrations near the stability boundary, presented by the stationary second moment above the stability chart in Fig.7b.

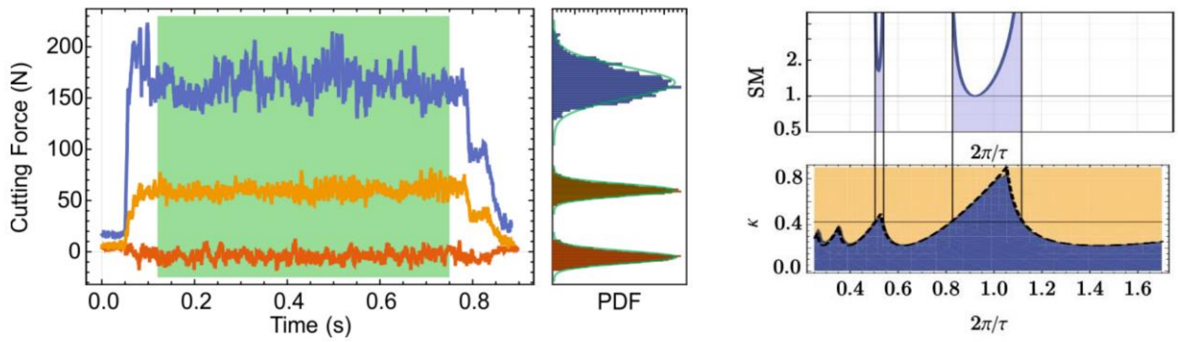

Fig. 7: a) Measured cutting force signal for turning: all the three force components show stochastic behavior presented by the probability density functions (PDF). b) The stable parameter domain (blue) and the dimensionless second moment (SM) along the spindle speed for a fixed chip width value

- The robust stability limit of a special neutral delayed dynamical system is analysed (as a mathematically similar problem to the quasi-periodic ones). In [11] numerous stability charts were computed via our HBDS method and plotted onto each other to get the robust stability region. This computation was extremely time consuming and achieved only by HPC GPU programming. We found out, that this set of boundary curves can be connected to form one surface by means of introducing an additional parameter. We deduced, that the special gradients of this surface determine the robust stability limits. The computation time for the corresponding implicit system of equations was one order magnitude faster than the brute-force method.

4 DISSEMINATION AND COLLABORATION AND IMPACT ON PI'S RESEARCH

The scientific work of the PI, an efficient algorithm in frequency domain (Harmonic Balance of Delayed Systems - HBDS) has been developed, which has been implemented for delayed linear time-periodic systems. This method is based on the Hill's infinite determinant method. Furthermore, the PI developed and updated the Multi-Dimensional Bisection Method (MDBM) for root-finding problems to accelerate the computations. This robust technique is able to find the submanifolds of the roots of a system of nonlinear equations with constraints, where the number of unknowns is larger or equal than the number of equations. The proposed method can be used for the determination of multiple solutions in the selected interval for any dimension and co-dimension. The recently optimized and updated freeware Matlab versions of these algorithms and the corresponding detailed user's guides can be found in the PI's homepage. The implemented methods already have great practical impact. There are many different users: - at least ten researchers from the Applied Mechanics Department - several national students - foreign researchers from Lappenranta (Finnland), Bristol (UK), Chemnitz (Germany), Ann Arbor (MI,USA), Elgoibar (Spain), Istanbul (Turkey) demonstrated by 15 dependent and 18 independent citations (based on google scholar), furthermore there are many other international researchers and students based on email feedbacks, a download metrics. Further preliminary collaborative results of the young researchers and the PI in the field of robust stability, stability of stochastic systems, changing dynamics and geometrical modelling is presented in the following subsections.

During the OTKA PD project eight journal papers were published, among which five were done in Q1 journals. The project facilitated the PI's collaborative work. We have a successive joint work with the

- Department of Mechanical Engineering, Lappeenranta University of Technology (Lappeenranta, Finland)
- Manufacturing Research Laboratory, Sabanci University (Istanbul, Turky)
- Nanjing University of Aeronautics & Astronautics (Nanking, China)

During these visits, the PI has presented the two above mentioned Matlab packages, which were used by the local researchers afterwards. Furthermore, these visits are initialized a common research leading to publications.

The project served as a good starting point for the PI's next NKFI projects. The PI won the FK-124462 and PD-124646 which started in December 2018. The aim of these new project is to describe cutting processes in a more complex way than ever with combined models, which enable the analysis of cross-effects of different physical phenomena onto each other and of uncertainties the processes are loaded with.

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