Final research report
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Tamás Kis

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1 Overview

In our research project we have made some progress in three problem areas:

1. scheduling problems with non-renewable resource constraints,
2. vehicle routing problems,
3. resource-constrained shortest path problems.

All of these problems involve scheduling and resource constraints in a broad sense. The primary focus was approximability and the development of efficient (exact) methods. A common algorithmic approach in both the approximation algorithms and the exact methods is linear programming. It seems that using linear programs in approximation algorithm for machine scheduling problems with non-renewable resources is currently the only viable approach. Likewise, in exact methods for the resource constrained shortest path problem, our results show that linear programming based branch-and-bound deserves more attention than it has gained in the past 20 years.

We have also gained a lot of insight into the design of polynomial time approximation schemes for scheduling problems (PTAS), and that of fully polynomial time approximation schemes (FPTAS). While there is a fairly good understanding of when there is a chance to devise an FPTAS for an optimization problem, the same is not true for PTAS-es. Simply put, if there is a dynamic program of pseudo-polynomial time for an optimization problem, and some technical conditions hold, then the existence of an FPTAS is guaranteed, which is a well-known result of G. Woeginger. Such a strong result is not known for PTAS-es. In the course of our work, we tried to make some progress in this direction as well (at least on our limited area of research), although our work is inconclusive yet.

2 Main results

2.1 Scheduling with non-renewable resources

In machine scheduling problem besides the machines (which are renewable resources of unit capacity, see below), we may have additional renewable and non-renewable resources. Renewable resources can model the processors, machines, workforce, etc., whereas non-renewable resources are the raw materials, energy, money, those things which are consumed by the jobs. In mathematical terms, the main difference between the two type of resources is how the corresponding resource constraints are modeled. For a renewable resource, there is an upper bound on the total amount that can be used simultaneously at any moment of time. That is, suppose we have a set $\mathcal{J}$ of jobs, each having a processing time p_j, and resource requirement a_{ij} from each resource i. For a renewable resource i we have a time dependent capacity b_{it} at any moment of time, and the resource constraints take the following form:

$$\sum_{j \in \mathcal{J}\,:\, S_j \le t < S_j + p_j} a_{ij} \le b_{it}, \quad \forall t,$$

where S_j is the starting time of job j in some schedule S (for simplicity, we consider non-preemptive models). For a machine, $b_{it} = 1$ or 0, i.e., the machine is either available or not at any time t, and $a_{ij} \in \{0, 1\}$.

In contrast, for a non-renewable resource i, there is an initial stock b_{i1} available at the beginning of the time horizon, and there are further supplies $b_{i,\ell}$ at dates u_ℓ with $0 = u_1 < u_2 < \cdots < u_q < u_{q+1} = \infty$, and the resource constraints take the form

$$\sum_{j \in \mathcal{J}\,:\, S_j < u_{k+1}} a_{ij} \le \sum_{\ell=1}^{k} b_{i,k}, \quad k = 1, \ldots, q.$$

In the course of the project, we have investigated the *complexity* and the *approximability* of machine scheduling problems with additional non-renewable resources and for various objective function and processing environments.

Our results are summarized in Table 1. In the first column the problem is specified with the standard $\alpha|\beta|\gamma$ notation, where the α field is the processing environment (1 denotes a single machine, and P is a parallel machine environment, while Pm indicates a fixed number of machines), β is the set of additional constraints, e.g., nr means that there are additional non-renewable resources, $q = 2$ or $q = const$ stipulates that the number of supplies is 2 or a constant, and the γ field is the objective function, where $C_{\max}$ is the maximum job completion time which is to be minimized, $\sum w_j C_j$ is the weighted sum of job completion times, and $L_{\max}$ is the maximum lateness (with respect to given due-dates of the jobs).

As can be seen from the table, we have an almost complete picture on the approximability of the makespan ($C_{\max}$) objective in single and parallel machine environments. The only open question is if there is an FPTAS for $1|nr = 1, q = const|C_{\max}$ or for $Pm|nr = 1, q = const|C_{\max}$? On the other hand, we have a number of results for special cases of single machine scheduling with non-renewable resources and the weighted completion time objective ($1|nr = 1|\sum w_j C_j$). A major open question is the approximability status of the general case with a single non-renewable resource without any further restriction. We only know that this problem does not admit an FPTAS, but for instance it is not known whether an approximation algorithm exists with constant approximation factor.

On top of approximability results, we have devised practical algorithms as well. In [Györgyi and Kis, 2018a], we describe a branch-and-cut method for $1|nr|L_{\max}$, i.e., for minimizing the maximum lateness on a single machine. To this end, we derived new valid inequalities for the convex hull of feasible solutions, and assessed their merits in solving the problem by branch-and-cut.

Table 1: Summary of approximablity and complexity results for machine scheduling with non-renewable resources.

Problem	Result	Publication
$1\|nr=1,q=2\|C_{\max}$	FPTAS	[Györgyi and Kis, 2015b]
$1\|nr=const,q=2\|C_{\max}$	no FPTAS[a]	[Györgyi and Kis, 2015b]
$1\|nr,q=2\|C_{\max}$	no PTAS	[Györgyi and Kis, 2015a]
$1\|nr=const,q=const,r_j\|C_{\max}$	PTAS[b]	[Györgyi and Kis, 2015a]
$Pm\|nr=const,r_j\|C_{\max}$	PTAS	[Györgyi and Kis, 2017]
$Pm\|nr=const,p_j=a_j\|L_{\max}$	PTAS	[Györgyi and Kis, 2017]
$P\|nr=1\|C_{\max}$	PTAS	[Györgyi, 2017]
$P\|nr=const,q=2\|C_{\max}$	no PTAS[a]	[Györgyi and Kis, 2017]
$1\|nr=1,q=2\|\sum w_jC_j$	FPTAS	[Kis, 2015]
$1\|nr=1\|\sum w_jC_j$	no FPTAS	[Kis, 2015]
$1\|nr=1,p_j=a_j=\bar{a}\|\sum w_jC_j$	polytime	[Györgyi and Kis, 2018c]
$1\|nr=1,p_j=w_j=1\|\sum w_jC_j$	polytime	[Györgyi and Kis, 2018c]
$1\|nr=1,a_j=w_j=1\|\sum w_jC_j$	polytime	[Györgyi and Kis, 2018c]
$1\|nr=1,w_j=\bar{w},p_j=a_j\|\sum w_jC_j$	polytime	[Györgyi and Kis, 2018c]
$1\|nr=1,a_j=\bar{a},p_j=w_j\|\sum w_jC_j$	polytime	[Györgyi and Kis, 2018c]
$1\|nr=1,w_j=p_j=a_j\|\sum w_jC_j$	no FPTAS	[Györgyi and Kis, 2018c]
$1\|nr=1,q=2,w_j=p_j=a_j\|\sum w_jC_j$	weakly NPH	[Györgyi and Kis, 2018c]
$1\|nr=1,q=2,p_j=1,w_j=a_j\|\sum w_jC_j$	weakly NPH	[Györgyi and Kis, 2018c]
$1\|nr=1,w_j=p_j=a_j\|\sum w_jC_j$	2-approx	[Györgyi and Kis, 2018c]
$1\|nr=1,q=const,w_j=p_j\|\sum w_jC_j$	PTAS	[Györgyi and Kis, 2018c]

[a]if nr is a constant of at least 2
[b]jobs may have release dates

We have also taken the viewpoint of algorithmic mechanism design for a variant of the problem, where there are no machines, only a set of precedence constrained tasks group into jobs, and the jobs compete for non-renewable resources. The system-wise objective is to minimize the maximum job tardiness, but the jobs are selfish, and they may lie about their true due-dates. We have devised a centralized mechanism without payments which ensures that for each job the best option is to tell its true due-date, they can get no advantage by telling a false due-date, see [Egri and Kis, 2018].

2.2 Resource constrained shortest path problems

In the resource constrained shortest path problem we have a directed graph with a designated source and sink node. Each arc e has a *length* $\ell(e) \in \mathbb{Q}$ (any rational number, can be positive and negative as well), and also an r-dimensional vector of resource requirements $r(e) \in \mathbb{Q}^r_{\geq 0}$ (non-negative weights). In addition, for each resource $i = 1, \ldots, r$, there is a bound U_i. The objective is to find a shortest length $s{-}t$ path P, which respects the resource constraints, i.e., $\sum_{e\in P} r_i(e) \leq U_i$ for each $i = 1, \ldots, r$. In [Horváth and Kis, 2016] we have devised a number of techniques to accelerate LP-based branch-and-bound methods with the aim of finding optimal, or suboptimal solutions quickly. Among others, we have developed a new variable-fixing method and a primal heuristic, and two new classes of valid inequalities (cutting planes) that can be used in search-tree nodes in the course of branch-and-bound. In the computational tests, the former two ingredients have proved to be the most useful. As a bye-product, we have found that most of the benchmark instances widely used in the literature even today can be solved by mere pre-processing, using only old, well-known techniques.

In [Horváth and Kis, 2018a] we have carried-out a more theoretical investigation of the resource constrained shortest path- and related problems. Our objective was to find a so-called multi–criteria approximation scheme, in which a small violation of the resource constraints is permitted, but the solution found must be at least as good as an optimal feasible solution. Our scheme works only if the number of resource constraints is a constant. Note that such a scheme is known for the resource constrained spanning-tree, and for the resource constrained matroid basis problem, under the same restriction. We have also shown that a multi-criteria approximation scheme exists for all the above problem only if the number of resources is a constant.

2.3 Scheduling of chains on a single machine

In [Horváth and Kis, 2018b], we consider a scheduling problem where a set of unit-time jobs have to be sequenced on a single machine without any idle times between the jobs. Preemption of processing is not allowed. The processing cost of a job is determined by the position in the sequence, i.e., for each job and each position, there is an associated weight, and one has to determine a sequence of jobs which minimizes the total weight incurred by the positions of the jobs. In addition, the ordering of the jobs must satisfy the given chain precedence constraints. In this paper we investigate the polyhedron associated with a special case of the problem where each chain has length two. We show that optimizing over this polyhedron is strongly NP-hard, however, we present a class of facet-defining inequalities along with a polynomial-time separation procedure. We generalize these results to the case of chains with lengths at most two. Finally, we present our computational results that show that separating these inequalities can significantly speed-up a linear programming based branch-and-bound procedure to solve the problem.

2.4 Vehicle routing

We have worked on two different vehicle routing problems. In the integrated vehicle and crew scheduling problem, one has timetable (e.g., daily schedule of the public buses of a town), which should be served by a fleet of vehicles, and by a set of drivers. Firstly, the number of vehicles plus drivers must be minimized, and the second objective is to minimize the total working hours of both the drivers and the vehicles, while a number of work regulations must be satisfied. We have devised a new modeling of the problem for scheduling the vehicles and the drivers simultaneously, and implemented an exact solution method for find strong lower and upper bound for the problem [Horváth and Kis, 2017]. In fact, previously no exact method was published for this complex problem, and we compared our results to the best heuristics in the literature.

In [Györgyi and Kis, 2018d] an online vehicle routing problem is studied, which has been recently proposed in the literature. In that problem, there is a fleet of vehicles, and the transportation requests are not known in advance, but they arrive over time. In addition, the requests themselves carry probabilistic information in that initially they specify only a large time window in which the service should be started, but they give the starting and ending locations precisely. Later, a definitive narrower time window arrives for the same request specifying the desired pickup time of the transportation service. If no vehicle can serve the request in the desired time window, then a penalty must be paid. The objective is to minimize the total penalty cost plus the deadhead cost (travelling empty between requests). We have developed a new algorithm, which seems to significantly outperform the published method on most benchmark instances.

2.5 Further results

We have also worked on the theoretical background of solving integer programs with linear constraints and objective function. In a joint work with E. Balas, we have investigated the relationship between various classes of cutting planes, some old ones, and a new one [Balas and Kis, 2016]. Among other results, we have shown that the class of Lift-and-Project cuts is the same as the recently proposed Generalized Intersection Cuts of Balas and Margot. We have also investigated the connection between the former two classes of cuts, and simple intersection cuts, known from the 70's. It turned out that simple intersection cuts form a sublass of Lift-and-Project cuts if the convex body from which the cuts are generated is not a stripe (defined by two parallel hyperplanes). In addition, we have given some conditions under which Lift-and-Project cuts represent simple intersection cuts. Moreover, we have studied the relation to the corner polyhedron, and among other results, we have given a necessary and sufficient condition for a convex polyhedron to be equal to the intersection of all its corner polyhedral relaxations.

We have written a book chapter in the upcoming book entitled Math for Digital Factories [Kis and Drótos, 2017]. There are no new results, but we give an overview of two important subfields by summarizing our previous work, and some progress since then.

Currently, we are working on a new method for a resource constrained scheduling problem, which can also be interpreted as a rectangle packing problem. That is, given a rectangle shaped bin with fixed width, but variable height, and we have a number of rectangles that should be packed into the bin. The rectangles cannot be rotated and their sides are parallel to that of the bin. The objective is to find a packing of the rectangles into the bin such that the height of the bin is minimized [Madarasi and Kis, 2017].

3 Achievements

Péter Györgyi has received a second Prize Award on the Best PhD Student Paper Competition at the conference PMS 2018 for the paper [Györgyi and Kis, 2018b] (see `http://www.pms2018.ing.uniroma2.it/PhDWinners.html`) .

4 Outlook to future work

Currently, we are working on approximation algorithms for machine scheduling problem with the total weighted completion time objective. In [Györgyi and Kis, 2018c] the approximability of a number of special cases is investigated, but we want to fully explore the approximability of the this class of problems. Moreover, disjunctive programming is a very good approach for several practical problems, and we want to continue the line of work of [Balas and Kis, 2016].

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