NKFIH K-109684 Final report
Stochastic processes on groups and graphs

The main focus of our research has been the study of group-invariant stochastic processes and unimodular random structures. Unimodular random graphs are the natural extension of transitive graphs to the disordered setting. One source of such graphs are the connected components in invariant percolation processes on Cayley graphs of groups. A second source are Benjamini-Schramm (i.e., local) limits of general finite (random or deterministic) graphs. A third source is to produce a new graph from an existing unimodular random graph via a local procedure, a so-called factor map. In our research, we have explored all these directions. Not only have we proved significant new results on processes on finite and infinite groups and graphs, but we have also used unimodular random graph and percolation ideas to understand important questions in group theory, with publications in *Ann Math*, *J of EMS*, and *Duke Math J*, and invited lectures at the ECM in Berlin and the ICM in Rio de Janeiro.

Regarding the exact questions posed in our project proposal 5 years ago, in some cases we have made real progress, one problem was solved by an other group of researchers, and in some cases we are still at an early stage in our investigations. The status of these questions will be detailed below. The group has had fluctuations in its research personnel, due to changes in scientific interest and the country of residence, but overall there appears to be healthy interaction and growth.

In the rest of this report, in Section 1, we summarize our results on spectral theory, that is, the interaction between eigenvalues and eigenvectors of natural operators (like the Laplacian) and algebraic and geometric properties of the underlying structure. In Section 2, we explore the spectral measure and entropy of stochastic processes other than just random walks, on finite graphs and their Benjamini-Schramm limits. Section 3 collects our advances on percolation type processes and factor graphs. Section 4 considers the time evolution of single- and multiparticle systems, such as random walks in group-invariant random environments, self-interacting random walks, and growth processes. The four sections have rich interaction with each other; in particular, several papers by our group will be mentioned in more than one section, and also most group members have papers in more than one section.

1. Spectral theory of graphs, processes and manifolds

Abért, Bergeron, Biringer, Gelander, Nikolov, Raimbault and Samet worked on understanding Benjamini-Schramm convergence in the realm of locally symmetric spaces and how it controls the the asymptotic behavior of **Betti numbers** and limit multiplicities. In the starting, quite substantial paper [2], among many results, they established that for a **higher rank semisimple Lie group** with property (T), every sequence of locally symmetric spaces Benjamini-Schramm converges to the corresponding symmetric space. The new tool leading to this surprising result was **invariant random subgroups**. They also analyzed how this convergence controls the relative Plancherel measure that lives on the unitary dual of the Lie group. This results in an array of uniform limit multiplicity results for arbitrary sequences of lattices instead of just covering towers. This paper was published in the *Annals of Mathematics*. The second paper [3] contains exotic constructions of invariant random subgroups in rank 1, that are not weak limits of induced invariant random subgroups coming from lattices.

Abért, Bergeron, Biringer and Gelander [4] worked on understanding Betti numbers of locally symmetric spaces for Benjamini-Schramm convergent sequences of non-positively curved manifolds with finite volume. While the higher rank case has been settled in the above Annals paper, it turned out that in rank one, the behavior is quite different. A general convergence result is false in dimensions 2 and 3. However, they show that it indeed holds in higher dimensions. To prove this, they had to combine a theorem of Elek on testing Betti numbers using a Poisson process with the technique worked out in the influential Ballman-Gromov-Schroeder paper.

Abért, Gelander and Nikolov [8] established a vanishing result for the **rank gradient** and the first torsion homology growth for right angled groups, that is, groups that admit a generating system where all generators commute with their neighbours. This applies to higher rank lattices, in particular, it lead to the first example of a cocompact lattice with vanishing rank gradient. The results, together with a famous conjecture by Bergeron and Venkatesh, and another by Gaboriau, imply that for hyperbolic 3-manifold groups, finite index subgroups admit only very complicated minimal generating sets.

Beyond eigenvalue multiplicities encoded by Betti numbers, Abért, Bergeron and Le Masson [5] investigated the asymptotic behavior of **eigenfunctions of the Laplacian on Riemannian manifolds**. This is an advanced theory, with deep results and conjectures. They showed that Benjamini-Schramm convergence provides a unified language for the level and eigenvalue aspects of the theory. As a byproduct they presented a mathematically precise formulation of Berry's conjecture for a compact negatively curved manifold and formulate a Berry-type conjecture for sequences of locally symmetric spaces. Before that, there was no exact formulation known. They proved weak versions of the above conjectures. Using ergodic theory, they also showed that a strong Berry conjecture would imply Quantum Unique Ergodicity.

When a group acts on a probability space, the Schreier graph of the action gives a measurable graph with some extra properties, called a **graphing**. Just like graphs and manifolds, graphings also have a spectral theory. And any invariant process on a transitive graph can be thought of as the automorphism group acting on a probability space, hence processes also have spectral measures, which are important objects. For instance, if the graphing has a spectral gap, then the action is strongly ergodic.

Backhausz and Virág [13] analyzed the spectral measure of invariant processes. They concentrated on factor of iid processes. They proved that a probability measure is the **spectral measure of a factor of iid process** on a vertex-transitive infinite graph if and only if it is absolutely continuous with respect to the spectral measure of the graph. Moreover, they showed that the set of spectral measures of factor of iid processes equals the $\bar{d}$-closure of factor of iid processes.

Bordenave, Sen and Virág [27] studied **mean quantum percolation**, the averaged spectrum of adjacency matrices of random graphs, and developed a technique to lower bound the mass of the continuous part of the spectral measure. It follows that the spectral measure of bond percolation in the two-dimensional lattice contains a non-trivial continuous part in the supercritical regime. The same result holds for the limiting spectral measure of a supercritical Erdős-Rényi graph and for the spectral measure of a unimodular random tree with at least two ends. They also gave examples of random graphs with purely continuous spectrum.

Kotowski and Virág [44] studied **1-dimensional Schroedinger operators** with random edge weights and their expected spectral measures near zero. They showed that the measure exhibits a spike, first observed by Dyson, when not assuming independence or any regularity of edge weights.

They also identified the limiting local eigenvalue distribution. It turns out that this is different from Poisson and also the usual random matrix statistics. As a striking application, they computed **Novikov-Shubin invariants** for various groups, including lamplighter groups and lattices in the Lie group Sol.

Abért [1] proved a strong approximation type result for **strongly ergodic** probability measure preserving actions, saying that under a spectral condition, a subgroup with small spectral index must also act strongly ergodically. This extends an early theorem of Shalom that was a starting point for the recent progress on expander graph constructions.

Fraczyk [38] proved a new inequality on the difference between the **spectral radius** of the Cayley graph of a group and the spectral radius of a Schreier graph of the group, for any subgroup. This improves on previous work of Abért, Glasner and Virág. As an application, he extended Kesten's theorem on spectral radii to uniformly recurrent subgroups and gave a short, elegant proof that the result of Lyons and Peres on cycle density in **Ramanujan graphs** holds on average. Namely, he showed that for an infinite Ramanujan graph, the time spent in short cycles by a random walk is sublinear in the length of the walk. Note that the Lyons-Peres paper is quite long and highly technical.

One of the spectral theory goals in our proposal was the **Bilu-Linial conjecture**: for any finite graph there is a lift that is a Ramanujan graph. Shortly after, this was basically proved by Marcus, Spielman and Srivastava. Nevertheless, comparing roots of graph polynomials for lifts plays an important role in Csikvári's work on matchings [31], explained in more detail in the next section.

2. Graph polynomials, Benjamini-Schramm convergence, entropy

Since the k^{th} moment of the spectral measure of the Laplacian of a random walk encodes the k-step return probability, which is determined by the $k/2$-neighbourhood of the starting point, it is easy to show that, for a Benjamini-Schramm convergent sequence of bounded degree finite graphs, the uniform distribution on the eigenvalues converges to the spectral measure of the limiting random rooted graph: the spectral measure satisfies **locality**. Instead of the eigenvalues, which are the roots of the characteristic polynomial of the transition matrix, one can study the uniform measure on the roots of any graph polynomial. These measures encode a lot of information (such as entropy) about statistical physics models associated to the graph polynomial, and Benjamini-Schramm locality is closely related to thermodynamic convergence.

Abért, Csikvári, Frenkel and Kun [7] introduced the **matching measure** of a finite graph as the uniform distribution on the roots of the matching polynomial of the graph. They proved Benjamini-Schramm locality for these measures on bounded degree graphs, and deduced the convergence of the normalized matching entropy (i.e., the normalized logarithm of the number of *all* matchings).

Csikvári has proved several deep results on the **number of matchings** in finite graphs. He proved Friedland's Lower Matching Conjecture [31]. This conjecture gives a lower bound on the number of matchings of a given size in regular bipartite graphs, generalizing a famous theorem of Schrijver on the number of perfect matchings. Before this paper, the best known result was an averaged version due to Gurvits. Note that the methods also give a new, simple proof for the known theorems of Gurvits and Schrijver. In a companion paper [32], he also gave stronger bounds for vertex-transitive bipartite graphs, and showed that in this setting, the perfect matching entropy per site is continuous with respect to Benjamini-Schramm convergence. That this does not hold for general (non-vertex-transitive) regular bipartite graphs was proved earlier in [7].

Beyond bounded degrees, Csikvári, Frenkel, Hladky and Hubai [35] proved that in a suitable range and with suitable normalization, **chromatic entropy** converges along convergent sequences of dense graphs, extending the earlier analogous result of Csikvári-Frenkel for graphs with bounded degree. Later, in a breakthrough paper, Frenkel [40] extended this result to **graphs with intermediate density**, and laid the foundations of a tentative new limit theory for such graphs, interpolating between the Benjamini-Schramm limit theory for bounded degree graphs and the Lovász et al. limit theory for dense graphs — an important well-known problem.

Matching and chromatic polynomials have some useful special properties. Csikvári [33] introduced and studied a new graph polynomial in this family, the **co-adjoint polynomial**.

Finer properties of the **set of roots** of a graph polynomial are also worth studying. Csikvári et al. [11] determined all graphs with only integer matching polynomial roots in various graph classes such as traceable graphs, regular graphs, claw-free graphs. Bencs and Csikvári [23] proved a new zero-free region for the partition function of the hard-core model, that is, the independence polynomials of graphs with largest degree D.

Csikvári et al. [29] established an **entropy-maximization** property of the complete graph for some hard-constraint statistical physics models, such as the Widom-Rowlinson model and the hard-core model. On the other end, Csikvári and Lin [36] proved a lower bound on the entropy of such models on any bipartite graph; these are special cases of the famous **Sidorenko conjecture**. (Also conjectured by Erdős and Simonovits in a different form.) Recently, Csikvári and Szegedy [37] studied a class of determinant inequalities closely related to Sidorenko's conjecture. Their main result can be interpreted as an **entropy inequality for Gaussian Markov random fields**.

Csikvári wrote a survey [34] on statistical properties of matchings in large and infinite graphs.

Mészáros [48] studied the distribution of the **sandpile group of random d-regular graphs** and established the limiting distributions for these groups. It turns out that for the directed model it follows the so-called Cohen-Lenstra heuristics, while for the undirected model, the limit is given by Clancy, Leake and Payne. As an application, the open question of Frieze and Vu whether the adjacency matrix of a random regular graph is almost surely invertible follows.

A similar type of entropy locality was proved in Fraczyk [39]: for any infinite sequence of torsion-free lattices in a higher rank Lie group with Kazhdan's property (T), the sequence of normalized **first mod-2 Betti numbers** of the quotients vanishes in the limit.

A great example of the applicability of Benjamini-Schramm locality is the following. Beringer and Timár [25] used the locality of the size of the largest matching (which follows, e.g., from [7]) and Azuma-Hoeffding type concentration arguments to prove most of the claims that Liu, Slotine and Barabási (Nature, 2011) predicted about **network controllability** based on simulations.

3. Percolation and other factor of iid processes

A central question in statistical physics is to understand the geometry of critical systems: shape of clusters, decay of correlations, behaviour under small perturbations. Studying stochastic processes on Cayley graphs of groups adds to these questions a focus on the relationship between the behaviour of stochastic processes and the geometric and algebraic properties of the underlying group. Another related point of view is to ask which spatial processes can be constructed from an iid process with a local algorithm, i.e., which processes are factors of iid. We already saw examples of such results in earlier sections: Backhausz and Virág [13] characterized the spectral measures of factor of iid processes on transitive graphs, Csikvári and Szegedy [37] established entropy inequalities for Gaussian Markov random fields.

A great success of our project, bringing together probabilistic and algebraic points of views, is the recent work of Hutchcroft and Pete [43]: using percolation theoretic tools, they solve an almost 20-year-old question of Gaboriau in the area of measure-preserving actions of groups, which Pete learnt from Abért. The result is that **infinite Kazhdan (T) groups have cost 1**, which means that, for any $\epsilon > 0$, one can construct an invariant bond percolation that is a spanning graph of the entire group and has average degree at most $2 + \epsilon$. It remains open if such a construction is possible in a factor of iid manner, but the above mentioned results of Backhausz and Virág [13] show that this would be a really hard task.

Cost also appears in the work of Abért and László Tóth [9], who analyze the **rank gradient** of finitely generated groups w.r.t. sequences of subgroups of finite index that do not necessarily form a chain, by connecting it to the cost of a probability measure preserving action. The connection is made by the notion of **local-global convergence**, introduced by Hatami-Lovász-Szegedy.

With two substantial papers [41, 42], Garban and Pete have completed their project started with Schramm in 2007 on the existence and some conformal and geometric properties of the **scaling limits of near-critical and dynamical percolation and the minimal spanning tree on the planar triangular lattice**. The near-critical version is the first construction of a non-trivial continuum renormalization flow for a planar statistical physics model, while the dynamical version is the first construction of a time-homogenous Markov process with Smirnov's famous conformally invariant scaling limit of critical percolation as the stationary distribution. Moreover, a lot of new information is gained on the discrete models themselves, and the techniques developed have already been applied to a wide range of models, by leading figures of the area like Wendelin Werner. One application is an appendix written by Pete to a work on probabilistic combinatorics [10], where he proved a large deviation result for near-critical planar percolation on finite boxes.

Timár [54] proved the **indistinguishability of trees** in uniform and minimal random spanning forests on non-amenable groups, answering old questions of Benjamini, Lyons, Peres and Schramm.

In a striking paper [57], Timár gave a surprising negative answer to a question of Itai Benjamini. He first showed that any amenable Cayley graph can be embedded in the Euclidean space $\mathbb{R}^3$ as a **factor of a Poisson point process**, then using this, he constructed an isomorphism-invariant partition of $\mathbb{R}^3$ into connected infinite indistinguishable pieces such that the adjacency graph of the pieces is the 3-regular tree. In the proof, he needed that any one-ended amenable Cayley graph has an **invariant one-ended spanning tree**, which he proved in [55]. Another technique he developed for the proof is also interesting in a more general context, which he explained in [56].

A well-known open problem about Bernoulli percolation on transitive graphs is the Benjamini-Schramm **locality of the critical density**, which can also be viewed as the uniformity of near-critical behaviour. Since for locality questions the natural generality is often unimodular random graphs, Beringer, Pete and Timár [26] studied the natural generalizations of three usual notions of critical density from transitive graphs (where they all agree) to the setting of unimodular random graphs, and found that locality completely fails in this generality, for any notion of critical density.

Locality and spectral ideas also play some role in the work of Bartha and Pete [21], who found an interesting dichotomy for **noise sensitivity of bootstrap percolation** on finite graphs: complete occupation is noise sensitive on boxes of the square lattice, while insensitive on random regular graphs (except for one quite interesting case of the parameters, still under investigation).

Ráth and Valesin [53] compared a correlated percolation model on $\mathbb{Z}^d$ (the stationary distribution of the **spread-out voter model**) to Bernoulli percolation, and found that the percolation thresholds in the two models are close to each other if the radius of the spread is big enough. This is another manifestation of locality.

Ráth [52] derived a simple formula characterizing the distribution of the size of the connected component of a fixed vertex in the **Erdős-Rényi random graph** which allowed him to give elementary alternative proofs of some results about the susceptibility in the subcritical graph and the central limit theorem for the size of the giant component in the supercritical graph.

Crane, Freeman and Bálint Tóth [30] investigated the growth of clusters in the **forest fire model** of Ráth and Tóth, a dynamical mean-field model of self-organized criticality. They proved precise limiting results on the evolution using Smoluchowski-type coagulation equations.

Generalizing results of Backhausz, Szegedy and Virág on pairwise spin correlation bounds for factor of iid processes on regular trees, Backhausz, Gerencsér, Harangi, and Vizer [12] took two sets of vertices that are separated appropriately, and gave a bound on the correlation of any two functions of these two families of random variables. This gives a new, quantitative proof of a result of Pemantle, namely that **factor of iid processes on the tree have trivial one-ended tail**. Recall here the surprising result of Lyons-Nazarov that the unique invariant random perfect matching on the 3-regular tree, whose tail has full information, is nevertheless a factor of iid process, hence the one-ended tail triviality cannot be much strengthened.

Abért and Biringer [6] introduce and study **Benjamini-Schramm convergence in the realm of Riemannian manifolds**. They prove the starting results. It turns out that the unimodularity condition is very natural in this category: it asks that the lift to the unit tangent bundle be invariant under the geodesic flow.

Bencs and László Tóth [24] studied **invariant random subgroups of groups acting on rooted trees**. They mainly focused on weakly branch groups. They gave a characterization of invariant random subgroups of the iterated wreath product of alternating groups. They also proved that every weakly branch group has continuum many distinct atomless ergodic IRS's. This extends a result of Benli, Grigorchuk and Nagnibeda who exhibited a group of intermediate growth with this property.

Finally, going against the usual locality mantra, Medvedev and Pete [47] established the surprising phenomenon that, for First Passage Percolation with **heavy-tailed edge-weights** on near-critical random graphs, adding a few edges dramatically speeds up the process.

4. Random walks and multi-particle systems

Random walks on transitive graphs in deterministic or in invariant random environment (RWRE) are central objects of probability and mathematical physics. We have already mentioned the works of Virág et al. [27, 44] on the spectral theory of random Schroedinger operators, and the work of Fraczyk [38], which may be considered as understanding the spectral gap of random walks on quasi-crystals. In this section, we will discuss our non-spectral results (except that in Bálint Tóth's results below some of the proof techniques are actually spectral). Furthermore, in multi-particle systems, there can be extra symmetries making some of the questions similar to questions of random walks on groups; e.g., Bálint Tóth's interchange process is an interacting particle system on the one hand, and a random walk on the symmetric group on the other hand. Also, following special particles in interacting particle systems on $\mathbb{Z}$, for instance, is similar to studying random walks in random environment on $\mathbb{Z}$. In turn, random walk in random environment results on $\mathbb{Z}$ may have implications for random walks on groups, as shown in [44].

Bálint Tóth and Gady Kozma [45] proved a **central limit theorem for random walks in doubly stochastic random environment on $\mathbb{Z}^d$**, i.e., with a divergence-free drift field. The method of „relaxed sector condition” (a sophisticated functional analytic approach) developed in earlier work of Tóth and collaborators is applied and the CLT is obtained for the so-called annealed (averaged) setting. This result settles a problem which evaded many attempts in the last three decades. Then, Tóth [58] proved the **quenched version of the central limit theorem** for the same problem. That is, the result is proved for almost all environments, which is considerably more subtle. The proof of quenched tightness contains a refined argument leading to a robust diffusive heat kernel bound in a non-reversible (non-self-adjoint) context. This seems to be the first one of its kind, and may be applicable for a large class of random environments on groups.

A famous concrete example of a divergence-free drift field is **random walk on the randomly-oriented Manhattan lattice**. Every line in $\mathbb{Z}^d$ is assigned a uniform random direction, and the random walk on this directed graph chooses uniformly from the d legal neighbours at each step. The above general theorems show that the model is diffusive in four and more dimensions. Ledger, Tóth and Valkó [46] prove that it is superdiffusive in two and three dimensions.

Tóth wrote a survey [59] on **normal and anomalous diffusion** of two types of random motions with long memory. The first class is random walks on Z^d in divergence-free random drift field, modelling the motion of a particle suspended in time-stationary incompressible turbulent flow. The second class is self-repelling random diffusions, where the diffusing particle is pushed by the negative gradient of its own occupation time measure towards regions less visited in the past.

Bartha and Telcs [22] considered **simple random walk on the Penrose tiling**, the most famous quasi-crystal. They showed that for almost every Penrose tiling (with respect to an arbitrary invariant measure on the space of tilings) the path distribution of the walk converges weakly to that of a non-degenerate Brownian motion.

Balázs and Nagy [17] identified the ballistic (or diffusive) limit of the **second class particle** in practically *any* nearest-neighbour one-dimensional asymmetric (symmetric, resp.) particle system with an established hydrodynamic limit. This is done by a substantial generalisation of Ferrari and Kipnis' 1995 argument. In another paper [16] motivated by the behaviour of second class particles, they extended recurrence results on **branching-annihilating random walks** to cases where the jump rates have non-local dependence on the configuration of the process.

Balázs, Nagy, Bálint Tóth and István Tóth [18] investigated the hydrodynamic behaviour of an interacting particle system with a flux that changes convexity. This phenomenon gives rise to **coexistence of shock and rarefaction waves** in various combinations. This model is the first one for which such exotic behaviour is rigorously established.

Balázs, Rassoul-Agha and Seppäläinen [19] investigated the Doob-transform of a **directed RWRE**, demonstrate **KPZ-scaling** and connect the emerging harmonic function to Busemann-type limits and the quenched **large deviations** rate function of the walk.

Balázs and Bowen considered **product form blocking measures** in the general framework of nearest neighbor asymmetric one dimensional misanthrope processes. When applied to simple exclusion and zero range, they derived a **fully probabilistic proof of the Jacobi triple product**, a famous identity that mostly occurs in number theory and the combinatorics of partitions.

Balázs also initiated [28] an application of interacting particle system results to explain **geomorphologic phenomena** of hill slope evolution in nature.

Partially motivated by Diffusion Limited Aggregation on the one hand, and by Conformal Loop Ensembles on the other, Pete and Wu [51] constructed and studied an **aggregation process of chordal Schramm-Loewner SLE(κ) excursions** in the unit disk that is invariant under all conformal self-maps of the disk.

Balázs and Folly [15] extended the well-known analogy of reversible Markov chains and **electric resistor networks** to the **non-reversible case**. To this order, they introduced a new electrical component and then extended some of the classical results in this new setup.

Pete and Ray [50] modified the construction of Carmesin, Federici, and Georgakopoulos to get a **transient Gromov-hyperbolic** unimodular random graph that has **no transient subtrees** and that has the **Liouville property** for harmonic functions. Such graphs must be far from transitive, hence it is somewhat surprising that a unimodular example exists.

Finally, regarding random walks on the symmetric group, we have two results. Balázs and

Szabó [20] studied effectiveness of **card-shuffling methods** for only randomizing certain subsets of practical interest of the permutation group of the deck. Patkó and Pete [49] determined the **mixing time of an interchange process** on S_{n+m}, generated by the transpositions given by the edges of the dumbbell graph (K_n and K_m joined by a single edge). Motivated by these results, they formulated a conjecture on when such processes show the **mixing time cutoff phenomenon**.

Hivatkozások

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