Iterative dynamics of quantum systems: non-classical properties, applications for modelling physical systems and optical realizations

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Summary

We concentrated on the topics mentioned in the original research plan. We could successfully collaborate with the research group of Prof. Igor Jex in Prague, according to the aim of the project.

Each year we organized a 2 day Workshop in collaboration with the University of Pécs and the local Committee of the Hungarian Academy of Sciences, with around 50 paricipants from various research groups on Europe and Hundary. (http://tab.mta.hu/pecsi-teruleti-bizottsag/esemenyek/4th-work-meeting-quantum-optics-information, http://tab.mta.hu/pecsi-teruleti-bizottsag/esemenyek/5th-work-meeting-quantum-optics-information) In 2016 we organized (jointly with the Prague group) the Workshop of Quantum Simulation and Quantum Walks 2016, which is part of a series, with around 80 participants from all over the world (http://wqsqw2016.phys.cz/).

Two of the participating PhD students (Bálint Kollár and Zoltán Darázs) successfully defended their theses.

In the following we shortly describe our main findings related to each topic. The 22 published papers (21 articles in journals and 1 book) are listed at the end.

Randomization in quantum walks

Continuous-time quantum walks on dynamical percolation graphs

We studied the time evolution of continuous-time quantum walks on randomly changing graphs. At certain moments, edges of the graph appear or disappear with a given probability, as in percolation. We treated this problem in a strong noise limit. We focused on the case when the time interval between the subsequent changes of the graph tends to zero. We derived explicit formulae for the general evolution in this limit. We found that the percolation in this limit causes an effective time rescaling. Independently of the graph and the initial state of the walk, the time is rescaled by the probability of keeping an edge. Both the individual trajectories for a single system and average properties with a superoperator formalism were discussed. We gave an analytical proof for our theorem and we also presented results from the numerical simulations of the phenomena for different graphs. [1]

Percolation induced effects in two-dimensional coined quantum walks: analytic asymptotic solutions

We have studied discerete-time quantum walks on dynamical percolation graphs. We developed novel analytic methods based on the theory of random unitary operations which help us to determine explicitly the asymptotic dynamics of quantum walks on two-dimensional finite integer lattices with percolation. Based on this theory, we found new unexpected features of percolated walks like asymptotic position inhomogeneity or special directional symmetry breaking. [2]

Review of the results on discrete time quantum walks on percolation graphs

We have reviewed the literate and summarized our earlier results in a tutorial form and completed them on the topic of discrete time quantum walks on percolation graphs. We presented our method to solve the evolution on finite percolation graphs in the long time limit, applying the asymptotic methods concerning random unitary maps. We worked out the case of one-dimensional chains in detail and provided a concrete, step-by-step numerical example in order to give more insight into the possible asymptotic behavior. The results about the case of the two-dimensional integer lattice were also summarized, focusing on the Grover-type coin operator. [3]

Quantized recurrence time in iterated open quantum dynamics

We defined an expected return time to the original state in iterated open quantum dynamical systems in finite-dimensional Hilbert spaces. Starting from a pure state, the time evolution is induced by repeated applications of a superoperator (quantum channel) in each time step followed by a measurement to detect whether the system has returned to the original state. We proved that if the superoperator is unital in the part of the Hilbert space explored by the system, then the expectation value of the return time is an integer, equal to the dimension of this relevant Hilbert space. [4]

Generalized Kac lemma for recurrence time in iterated open quantum systems

We consider recurrence to the initial state after repeated actions of a quantum channel. After each iteration a projective measurement is applied to check recurrence. The corresponding return time is known to be an integer for the special case of unital channels, including unitary channels. We prove that for a more general class of quantum channels the expected return time can be given as the inverse of the weight of the initial state in the steady state. This statement is a generalization of the Kac lemma for classical Markov chains. [5]

Localization, delocalization, and topological phase transitions in the one-dimensional split-step quantum walk

Quantum walks are promising for information processing tasks because in regular graphs they spread quadratically more rapidly than random walks. Static disorder, however, can turn the tables: unlike random walks, quantum walks can suffer Anderson localization, with their wave function staying within a finite region even in the infinite time limit, with a probability exponentially close to 1. It is thus important to understand when a quantum walk will be Anderson localized and when we can expect it to spread to infinity even in the presence of disorder. In this work we analyze the response of a one-dimensional quantum walk—the split-step walk—to different forms of static disorder. We find that introducing static, symmetry-preserving disorder in the parameters of the walk leads to Anderson localization. In the completely disordered limit, however, a delocalization transition occurs, and the walk spreads subdiffusively to infinity. Using an efficient numerical algorithm, we calculate the bulk topological invariants of the disordered walk and find that the disorder-induced Anderson localization and delocalization transitions are governed by the topological phases of the quantum walk. [6]

Localization, delocalization, and topological transitions in disordered two-dimensional quantum walks

We investigate time-independent disorder on several two-dimensional discrete-time quantum walks. We find numerically that, contrary to claims in the literature, random onsite phase disorder, spin-dependent or otherwise, cannot localize the Hadamard quantum walk; rather, it induces diffusive spreading of the walker. In contrast, split-step quantum walks are generically localized by phase disorder. We explain this difference by showing that the Hadamard walk is a special case of the split-step quantum walk, with parameters tuned to a critical point at a topological phase transition. We show that the topological phase transition can also be reached by introducing strong disorder in the rotation angles. We determine the critical exponent for the divergence of the localization length at the topological phase transition, and find $\nu = 2.6$, in both cases. This places the two-dimensional split-step quantum walk in the universality class of the quantum Hall effect. [7]

Edge states and topological phases in quantum walks

Chiral symmetry in quantum walks based on the “gauge freedom”

We extended the concept of chiral symmetry from lattice Hamiltonians to discrete time quantum walks (DTQW), and provided the bulk--boundary correspondence for these systems. We have shown, that, as in 1-dimensional lattice Hamiltonians, chiral symmetry can provide topological protection to the energy of bound states at the left and right ends of an insulating bulk, with bulk winding numbers predicting the number of these bound states. For DTQWs, there are 2 bulk winding numbers, predicting the number of 0 and pi-quasienergy bound states: to obtain these, a single bulk timestep operator is not enough, rather, information from two time-shifted unitary timestep operators needs to be combined. [8]

Chiral symmetry in periodically driven systems

We generalized the concept of chiral symmetry, and the corresponding bulk--boundary correspondence, to periodically driven noninteracting lattice Hamiltonians. We have found compact expressions for the bulk topological invariant predicting the number of 0 (pi) quasienergy end states as winding numbers of the off-diagonal (diagonal) blocks of the time evolution operator from one chiral symmetric time to the next. This improves on our previous work on DTQWs, as this form of the 0 (pi) quasienergy winding number is well defined even if the bulk quasienergy gap at pi (0) quasienergy gap is closed. [9]

Scattering matrix approach to quantum walks

We generalized the concept of scattering matrices to DTQW's, and have shown that the scattering matrix theory of topological phases of lattice Hamiltonians applies to these systems as well. We thus obtained compact formulas for the topological invariants in one spatial dimension in all nontrivial symmetry classes, as matrix invariants of the scattering matrices at 0 and pi quasienergy.

We provided a schematic layout for an experiment to directly detect the bulk topological invariant. [10]

Hidden topological phases of the one-dimensional Hadamard quantum walk

We extended the concept of chiral symmetry from lattice Hamiltonians to discrete time quantum walks (DTQW), and provided the bulk--boundary correspondence for these systems. We have shown, that, as in 1-dimensional lattice Hamiltonians, chiral symmetry can provide topological protection to the energy of bound states at the left and right ends of an insulating bulk, with bulk winding numbers predicting the number of these bound states. [11]

Edge-state enhanced transport in a 2-dimensional quantum walk

We ask how to create an efficient transport channel from a fixed source site (A) to fixed target site (B) in a disordered two-dimensional discrete-time quantum walk by cutting some of the links. We show that the somewhat counterintuitive strategy of cutting links along a single line connecting A to B creates such a channel. The efficient transport along the cut is due to topologically protected chiral edge states, which exist even though the bulk Chern number in this system vanishes. We give a realization of the walk as a periodically driven lattice Hamiltonian and identify the bulk topological invariant responsible for the edge states as the quasienergy winding of this Hamiltonian. [12]

A Short Course on Topological Insulators: Band-structure topology and edge states in one and two dimensions (Lecture Notes in Physics, Springer)

This course-based primer provides newcomers to the field with a concise introduction to some of the core topics in the emerging field of topological band insulators in one and two dimensions. The aim is to provide a basic understanding of edge states, bulk topological invariants, and of the bulk--boundary correspondence with as simple mathematical tools as possible. We use noninteracting lattice models of topological insulators, building gradually on these to arrive from the simplest one-dimensional case (the Su-Schrieffer-Heeger model for polyacetylene) to two-dimensional time-reversal invariant topological insulators (the Bernevig-Hughes-Zhang model for HgTe). In each case the model is introduced first and then its properties are discussed and subsequently generalized. The only prerequisite for the reader is a working knowledge in quantum mechanics, the relevant solid state physics background is provided as part of this self-contained text, which is complemented by end-of-chapter problems. [13]

Topological bound states of a quantum walk with cold atoms

We suggest a method for engineering a quantum walk, with cold atoms as walkers, which presents topologically nontrivial properties. We derive the phase diagram, and show that we are able to produce a boundary between topologically distinct phases using the finite beam width of the applied lasers. A topologically protected bound state can then be observed, which is pinned to the interface and is robust to perturbations. We show that it is possible to identify this bound state by averaging over spin sensitive measures of the atom’s position, based on the spin distribution that these states display. Interestingly, there exists a parameter regime in which our system maps on to the Creutz ladder. [14]

Robustness of topologically protected edge states in quantum walk experiments with neutral atoms

By numerical simulations, we study the robustness of topologically protected edge states in the presence of decoherence in one- and two-dimensional discrete-time quantum walks. We also develop a simple analytical model quantifying the robustness of these edge states against either spin or spatial dephasing, predicting an exponential decay of the population of topologically protected edge states. Moreover, we present an experimental proposal based on neutral atoms in spin-dependent optical lattices to realize spatial boundaries between distinct topological phases. Our proposal includes also a scheme to implement spin-dependent discrete shift operations in a two-dimensional optical lattice. We analyze under realistic decoherence conditions the experimental feasibility of observing unidirectional, dissipationless transport of matter waves along boundaries separating distinct topological domains. [15]

Spectral flow and global topology of the Hofstadter butterfly

We study the relation between the global topology of the Hofstadter butterfly of a multiband insulator and the topological invariants of the underlying Hamiltonian. The global topology of the butterfly, i.e., the displacement of the energy gaps as the magnetic field is varied by one flux quantum, is determined by the spectral flow of energy eigenstates crossing gaps as the field is tuned. We find that for each gap this spectral flow is equal to the topological invariant of the gap, i.e., the net number of edge modes traversing the gap. For periodically driven systems, our results apply to the spectrum of quasienergies. In this case, the spectral flow of the sum of all the quasienergies gives directly the Rudner invariant. [16]

Detecting topological invariants in chiral symmetric insulators via losses

We show that the bulk winding number characterizing one-dimensional topological insulators with chiral symmetry can be detected from the displacement of a single particle, observed via losses. Losses represent the effect of repeated weak measurements on one sublattice only, which interrupt the dynamics periodically. When these do not detect the particle, they realize negative measurements. Our repeated measurement scheme covers both time-independent and periodically driven (Floquet) topological insulators, with or without spatial disorder. In the limit of rapidly repeated, vanishingly weak measurements, our scheme describes non-Hermitian Hamiltonians, as the lossy Su-Schrieffer-Heeger model. We find, contrary to intuition, that the time needed to detect the winding number can be made shorter by decreasing the efficiency of the measurement. We illustrate our results on a discrete-time quantum walk, and propose ways of testing them experimentally. [17]

Properties of quantum walks

Transport properties of continuous-time quantum walks on Sierpinski fractals

We modelled quantum transport, described by continuous-time quantum walks (CTQWs), on deterministic Sierpinski fractals, differentiating between Sierpinski gaskets and Sierpinski carpets, along with their dual structures. The transport efficiencies were defined in terms of the exact and the average return probabilities, as well as by the mean survival probability when absorbing traps are present. In the case of gaskets, localization can be identified already for small networks (generations). For carpets, our numerical results indicated a trend towards localization, but only for relatively large structures. The comparison of gaskets and carpets further implies that, distinct from the corresponding classical continuous-time random walk, the spectral dimension does not fully determine the evolution of the CTQW. [18]

Strongly trapped two-dimensional quantum walks

Discrete time quantum walks (DTQWs) are nontrivial generalizations of random walks with a broad scope of applications. In particular, they can be used as computational primitives, and they are suitable tools for simulating other quantum systems. DTQWs usually spread ballistically due to their quantumness. In some cases, however, they can remain localized at their initial state (trapping). The trapping and other fundamental properties of DTQWs are determined by the choice of the coin operator. We introduce and analyze a type of walks driven by a coin class leading to strong trapping, complementing the known list of walks. This class of walks exhibits a number of exciting properties with possible applications ranging from light pulse trapping in a medium to topological effects and quantum search. [19]

Iterative qubit dynamics

Exponential Sensitivity and its Cost in Quantum Physics

State selective protocols, like entanglement purification, lead to an essentially non-linear quantum evolution, unusual in naturally occurring quantum processes. Sensitivity to initial states in quantum systems, stemming from such non-linear dynamics, is a promising perspective for applications. Here we demonstrate that chaotic behaviour is a rather generic feature in state selective protocols: exponential sensitivity can exist for all initial states in an experimentally realisable optical scheme. Moreover, any complex rational polynomial map, including the example of the Mandelbrot set, can be directly realised. In state selective protocols, one needs an ensemble of initial states, the size of which decreases with each iteration. We prove that exponential sensitivity to initial states in any quantum system have to be related to downsizing the initial ensemble also exponentially. Our results show that magnifying initial differences of quantum states (a Schrödinger microscope) is possible, however, there is a strict bound on the number of copies needed. [20]

Measurement-induced chaos in an iterated Tavis-Cummings scheme

We proposed a cavity QED scheme based on the Tavis-Cummings model to realize a conditional, non-linear dynamics exhibiting complex chaos in an ensemble of two-level atoms. The atoms interact pairwise with the coherent cavity field, which is subjected to homodyne measurement after the interaction. One member of the pair of atoms is then measured and upon condition that it is found in its ground state the other member of the pair is kept, otherwise discarded. The quantum state of the resulting ensemble of atoms is related to the state of the original ensemble via a non-linear transformation. By iterating this process with the help of an optical conveyor belt, for example, one arrives at complex deterministic chaos, where the evolution of pure quantum states is sensitive to the initial conditions. We uncover basic features of this dynamics by exploring stability regions in the space of initial states and analyze the occurrence of stable periodic orbits in dependence of the parameters of the model. It is demonstrated that sensitivity to initial conditions can be applied to amplify small differences of quantum states. [21]

Publications

1. Zoltán Darázs, Tamás Kiss: Time evolution of continuous-time quantum walks on dynamical percolation graphs, J PHYS A-MATH THEOR 46: Paper 375305, 2013
2. B Kollár, J Novotny, T Kiss, I Jex: Percolation induced effects in two-dimensional coined quantum walks: analytic asymptotic solutions, NEW J PHYS 16: 023002, 2014

3. Kollar B, Novotny J, Kiss T, Jex I: Discrete time quantum walks on percolation graphs, EUR PHYS J PLUS 129: (5) , 2014
4. P. Sinkovicz, Z. Kurucz, T. Kiss, J. K. Asboth: Quantized recurrence time in iterated open quantum dynamics, Phys. Rev. A 91, 042108, 2015
5. P. Sinkovicz, T. Kiss, and J. K. Asbóth: Generalized Kac lemma for recurrence time in iterated open quantum systems, Phys. Rev. A 93, 050101(R), 2016
6. Tibor Rakovszky and Janos K. Asboth: Localization, delocalization, and topological phase transitions in the one-dimensional split-step quantum walk, Phys. Rev. A 92, 052311, 2015
7. Jonathan M. Edge and Janos K. Asboth: Localization, delocalization, and topological transitions in disordered two-dimensional quantum walks Phys. Rev. B 91, 104202 – Published 5 March 2015, Phys. Rev. B 91, 104202, 2015
8. János Asbóth, Hideaki Obuse: Bulk-boundary correspondence for chiral symmetric quantum walks, PHYSICAL REVIEW B 88, 121406(R), 2013
9. J. K. Asbóth, B. Tarasinski, P. Delplace: Chiral symmetry and bulk-boundary correspondence in periodically driven one-dimensional systems, PHYSICAL REVIEW B, 90, 125143, 2014
10. B. Tarasinski, J. K. Asbóth, and J. P. Dahlhaus: Scattering theory of topological phases in discrete-time quantum walks, Phys. Rev. A 89, 042327, 2014
11. Hideaki Obuse, Janos K. Asboth, Yuki Nishimura, and Norio Kawakami: Unveiling hidden topological phases of a one-dimensional Hadamard quantum walk, Phys. Rev. B 92, 045424 (2015), 2015
12. J. K. Asboth and J. M. Edge: Edge-state enhanced transport in a 2-dimensional quantum walk, Phys. Rev. A 91, 022324, 2015
13. JK Asbóth, L Oroszlány, A Pályi: A Short Course on Topological Insulators: Band-structure topology and edge states in one and two dimensions, Lecture Notes in Physics, Springer, 2015
14. Samuel Mugel, Alessio Celi, Pietro Massignan, János K. Asbóth, Maciej Lewenstein, and Carlos Lobo: Topological bound states of a quantum walk with cold atoms, Phys. Rev. A 94, 023631, 2016
15. Thorsten Groh, Stefan Brakhane, Wolfgang Alt, Dieter Meschede, Janos K. Asbóth, and Andrea Alberti: Robustness of topologically protected edge states in quantum walk experiments with neutral atoms, Phys. Rev. A 94, 013620, 2016
16. Janos K. Asboth, Andrea Alberti: Spectral flow and global topology of the Hofstadter butterfly, Phys. Rev. Lett. 118, 216801 (2017), 2017
17. Tibor Rakovszky, Janos K. Asboth, Andrea Alberti: Detecting topological invariants in chiral symmetric insulators via losses, Phys. Rev. B 95, 201407 (2017), 2017
18. Darázs Z, Anishchenko A, Kiss T, Blumen A, Mülken O: Transport properties of continuous-time quantum walks on Sierpinski fractals, PHYS REV E STAT NONLIN 90: (3) , 2014
19. B. Kollár, T. Kiss, and I. Jex: Strongly trapped two-dimensional quantum walks, Phys. Rev. A 91, 022308, 2015
20. Gilyén András, Kiss Tamás, Igor Jex: Exponential Sensitivity and its Cost in Quantum Physics, SCIENTIFIC REPORTS Volume: 6 Article Number: 20076, 2016
21. Juan Mauricio Torres, József Zsolt Bernád, Gernot Alber, Orsolya Kálmán, and Tamás Kiss: Measurement-induced chaos and quantum state discrimination in an iterated Tavis-Cummings scheme, Phys. Rev. A 95, 023828, 2017, 2017
22. Kornyik M, Michaletzky G.: On the moments of roots of Laguerre-polynomials and the Machenko-Pastur law, Annales Universitatis Scientiarium Budapestinensis Rolando Eötvös Nominatae Sectio Computatorica, 43:137-151, 2017, 2017