Final Report

CONSTITUTIVE MODELLING OF CELLULAR SOLIDS

Postdoctoral research grant

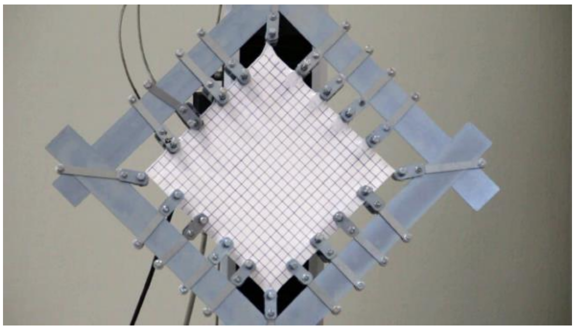
Principal investigator: **Dr. Attila KOSSA**
Project Identifier: **PD 108691**

Introduction

The present research was focused on the material modelling of cellular materials. The main objective of the research plan was to obtain novel accurate constitutive models for cellular solids including polymeric and metal foams as well. Based on literature surveys and scientific experiences, I've concluded at the beginning of the research period, that the constitutive characterization of polymeric foams is more important for the industry than the constitutive modelling of metal foams. Significant effort can be seen in developing novel accurate material models for polymer materials, including polymeric foams. Consequently, I've mainly focused on the constitutive modelling of cellular plastics. However, it is important to emphasize that the new scientific results may be used for other cellular solids. I would say that this research was productive and I am satisfied with the results. However, the research work cannot be considered final. There are still unanswered questions which should be solved in the future.

Scientific achievements

Novel test fixtures for foams

 **Figure 1:** Equi-biaxial compression fixture developed for polymer foams	 **Figure 2:** Equi-biaxial tension fixture developed for polymer foams

<u>Background</u>: The large strain elastic characterization of polymeric foam materials can be accurately described using compressible hyperelastic material model. It is a well-known phenomenon in hyperelasticity that fitting the model response to one material test may lead to extremely inaccurate results for other modes of

deformation. Test data for equibiaxial extension is crucial. However, achieving equibiaxial compression is not trivial. Most of the equibiaxial compression fixtures are designed for two-column testing machines.

New results: I've developed and designed a novel equibiaxial compression fixture for one-column testing machines (Fig. 1.). I've used this testing fixtures for the measurements carried out in this research. The equipment was published in the Journal of **Polymer Testing** (IF=2.350) [1]. The new fixture allows us to measure the very important equibiaxial compression responses of polymer foam materials. In addition to the compression fixture, I've developed an equibiaxial tensile fixture as well (Fig. 2.). The new devices can be easily attached to a single column testing system. Measuring the equibiaxial response of a particular foam material allows one to obtain more accurate material parameters for a hyperelastic foam material model, such as the Ogden-Hill compressible foam model.

Investigation of the effect of the skin layer of polymer foams

Background: On the surface of the closed-cell polymeric foams a so-called skin layer may be observed. The skin layer is a thin layer with modified material parameters that is created during the manufacturing process making the material structure highly non-homogeneous. In the skin layer, no cells are present, while in the transition layer between the skin layer and the core of the foam the material remains in a closed-cell structure, but with decreased cell-size. Therefore, the relative density of the skin layer and transition layer becomes higher. The mechanical behavior of polymer foams is highly influenced by structural inhomogeneity. The effect of skin layer on the overall mechanical behavior (tension and compression) was not taken into consideration or is often neglected in different discussions of the mechanical behavior of foams. This lack in the literature motivated to investigate the effect of the surface skin and transition layers.

New results: I've described the stress-stretch characteristic of the foam using the basic method of uniaxial compression and tension. The new results may help to estimate the degree of the inhomogeneity caused by these layers and this can lead to the development of more accurate constitutive model for FEA. As the results clearly show, the effect of skin layer is significant in all tests. In case of specimens with skin layer, stress values increase, thus the polymeric foam due to the skin layer effect becomes less compressible and less flexible. The initial slopes are higher, and the maximal tensile stretch is smaller, making the foam sheet with skin layer more fragile. On the other hand, the tensile strength increases, so the skin layer increases the strength of the foam sheet. This effect can be explained by the microstructural theory. In this theory, all the mechanical properties, that describe the mechanical behavior, can be expressed in the terms of the relative density. As the microscopic images show (Fig. 3.) skin layer's relative density is higher, leading to the increase of the elastic modulus and the gradient of the plateau.

The main scientific result is published in the Journal of **Cellular Plastics** (IF=2.057) [2].

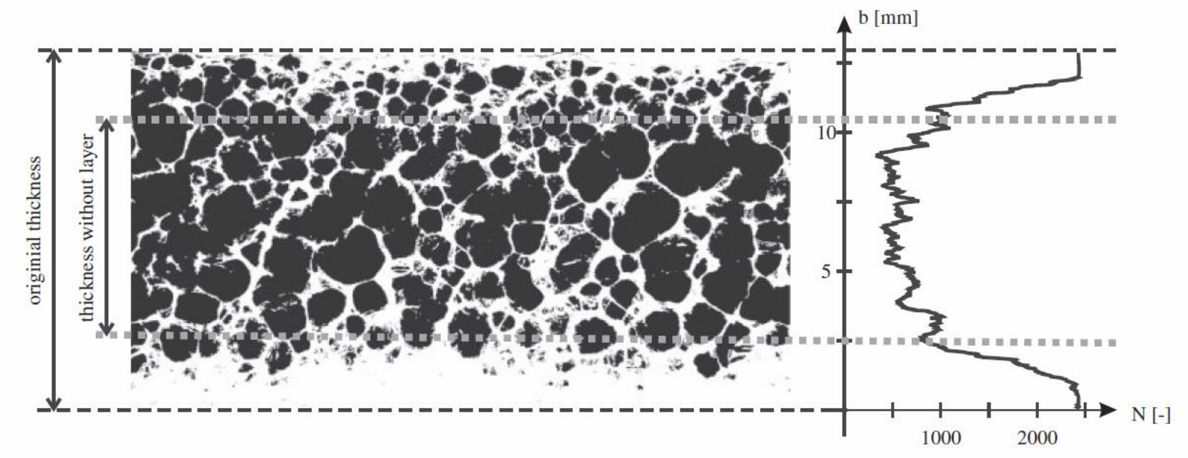


Figure 3: The material inhomogeneity caused by the skin layer on the surface and the cross-section of the foam sheet.

Analytical stress solutions for visco-hyperelastic polymer foams

Background: Polymer foams exhibit large deformations and displacements whereas the mechanical behavior shows time-dependent, i.e. viscoelastic properties. The viscoelastic property means that the mechanical behavior of the polymer foams is not only affected by the load but also by the loading rate. The viscoelastic behavior can be characterized by the so-called time-dependent material parameters and relaxation moduli which can be obtained via the stress-relaxation and the creep phenomena. The behavior of large elastic and viscoelastic materials can be described with adequate precision using the so-called visco-hyperelastic constitutive equation, which combine the hyperelastic and the viscoelastic material models. These complex material models are available in all commercial finite element software. In this approach, the time-dependent stress-relaxation phenomenon is modelled using the Prony-series representation, while for the long-term time-independent behavior a hyperelastic material model is adopted, which can be derived from the corresponding strain energy.

The most critical part during the material modelling of a particular foam material is to determine all the parameters in the constitutive equation, which should be obtained directly from experimental data using parameter-fitting algorithm. However, the visco-hyperelastic material law is defined in the form of a hereditary integral, therefore the general stress response function cannot be defined. The usually adopted algorithm to find the material parameters for a particular material is based on the stress-relaxation behavior and the long-term pure hyperelastic behavior, where the parameter-fitting of the long-term hyperelastic and the Prony parameters are separated ("separated fitting"). This approach involves significant errors into the fitting process, consequently, the fitted material parameters cannot accurately describe the overall visco-hyperelastic behavior, and the solution will be inaccurate (see Fig. 1.: separated fitting).

Using the analytically derived stress-response functions for the stress-relaxation loading case, which have not been provided in the literature before, the entire visco-hyperelastic material model could be fitted to the measurement data in one step. The lack of the closed-form stress solution for the visco-hyperelastic constitutive equation based on the hyperfoam model motivated to investigate analytically the material behavior of compressible polymer foams which also show time-dependent properties.

New results: I've obtained exact stress solutions for the ramp loading in case of uniaxial, equibiaxial and volumetric compression loading cases. The derived closed-form solutions were validated with numerical simulation using FEA. The results showed that the exact stress solutions give exactly the same results as the numerical analysis. Additionally, experimental work has been carried out, which showed that the derived analytical stress solutions provide a better approach for the material parameter identification. Consequently, these solutions can be utilized to improve the accuracy of the parameter-fitting algorithm by fitting all the hyperelastic and Prony-parameters in one step instead the separated-method, which is a commonly adopted approach in the engineering practice.

The main scientific results were published in the Journal of **Mechanics of Time-Dependent Materials** (IF=1.120) [4]. Related publications: [6], [7], [10], [13], [14].

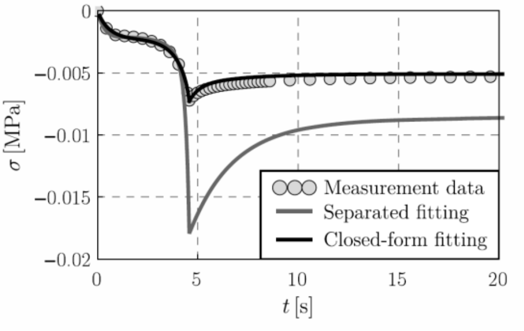


Figure 4: Accuracy of the closed-form fitting compared to the traditional method

Novel parameter fitting algorithm for compressible hyperelastic foam

Background: The finite strain elastic behavior of elastomeric polymer foams can be accurately captured using the Ogden-Hill hyperelastic material model based on the work of Ogden and Hill. This model is implemented as the "hyperfoam" hyperelastic model in the commercial finite element software, ABAQUS. A possible way to find the material parameters of a hyperelastic model is to fit the model responses of particular homogeneous deformations to experimental data. This is a widely used procedure for hyperelastic materials. However, the fitting of the hyperfoam model is more complicated than fitting of hyperelastic models proposed for incompressible materials. In the incompressible case, the transverse stretches can be easily calculated from the incompressibility constraint in uniaxial compression/tension or in other homogeneous deformations. However, for the hyperfoam model the transverse stretch, in general, cannot be obtained from the zero transverse stress constraint, even in uniaxial compression. Therefore, the parameter fitting procedure is not so trivial for this material model.

New results: I've developed a new parameter fitting strategy for the hyperfoam material model. The transverse stretch values of the experimental data are included in the quality function (error function) of the new method and, in addition, the zero transverse stress constraint was also enforced. Therefore, there is no need to solve the highly nonlinear zero transverse stress equation for the transverse stretch, which is the commonly adopted technique for this compressible hyperelastic model. Consequently, the new parameter fitting method can be used with any minimization solver to find the material parameters.

The main scientific results were published in the Journal of **Polymer testing** (IF=2.350) [3]. Related publications: [11].

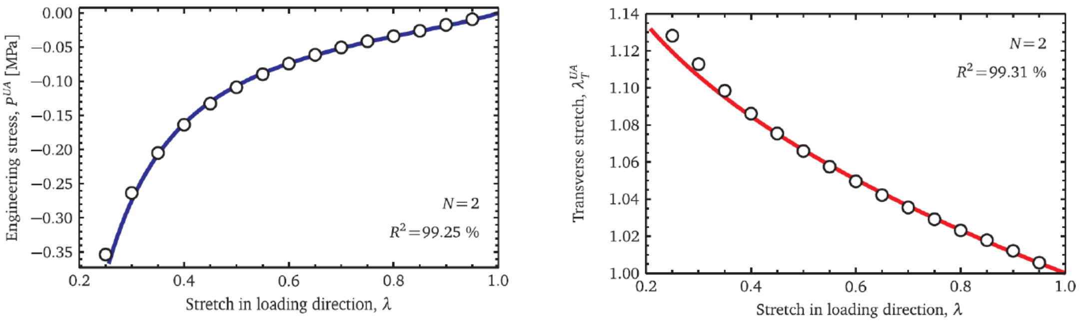


Figure 5: Model prediction for the stress and for the transverse stretch

Advanced constitutive modelling of thermoplastic foams

Background: The characterization of thermoplastics polymer foam materials is a challenging task for researcher in solid mechanics. These materials undergo large deformations during the forming process and they show elastic, viscous and plastic deformations as well. In addition, the material parameters are obviously temperature dependent. Consequently, constitutive modelling of these materials requires to take into account the viscoelastic effect with the yielding behavior in addition to the elastic contribution. The number of available models to perform this task is very limited. The only available material models implemented in the commercial finite element software ABAQUS is the "two-layer viscoplastic model". This model is mainly proposed for metals but it can be applied for other material as well.

New results: I've developed an algorithm to find the material parameters of this material model applied for polymeric foams. The results clearly show that the present constitutive model could accurately describe the mechanical behavior of thermoplastic foam materials (Fig. 6.). It should be emphasized that this part in the research is currently under development. I will improve this material model in the future in collaboration with my PhD student. Related publications: [12].

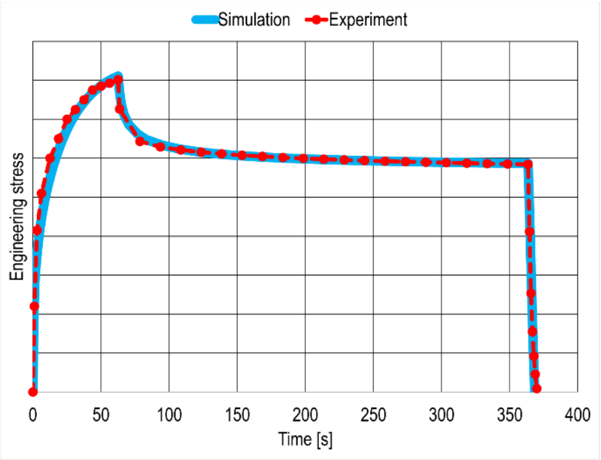


Figure 6: The two-layer viscoplastic model accurately captures the large strain elastic-plastic deformation and the relaxation as well

Other consequences of the research

In order to properly evaluate the outcome of this project, I would like to list some other achievements related to this project:

- I was the supervisor of a very talented student, who finished his BSc and MSc theses under my supervision [22,32]. Now, he is my ***PhD student*** and he is very active. His research topic (Developing viscous-elastic-plastic constitutive models at finite strain). I've involved him in the present OTKA research project and he is the coauthor some of the publication related to this project [2,3,4,6,7,10,12,13,15]
- Thanks to this research grant, I was able to visit the "*Danubia Adria Symposium on Advances in Experimental Mechanics*" conference series in 2014, 2015 and in 2016. I've been asked to be the ***member of the scientific committee*** of the "The Danubia Adria Society". Now I am a regular member of the scientific committee. http://das.tuwien.ac.at/about-das/scientific-committee/
- I've established ***new scientific partnerships*** in foreign countries.
- All the equipment bought in this research will be frequently used in lab tests corresponding to my future research works.
- During the project, I was the supervisor of 16 thesis works, where we investigated mechanical phenomena related to the constitutive modelling of cellular solids. The present OTKA grant is acknowledged in every thesis [18-33].

Publications related to the project

Papers in peer-reviewed international journals (with Impact Factor)

1. Kossa, A., 2015. A new biaxial compression fixture for polymeric foams. Polymer Testing 45, 47-51.
   DOI: 10.1016/j.polymertesting.2014.08.003. IF=2.350, SJR=0.947 Q1.
2. Berezvai, Sz., Kossa, A., 2016. Effect of the skin layer on the overall behavior of closed-cell polyethylene foam sheets. Journal of Cellular Plastics 52, 215-229.
   DOI: 10.1177/0021955X15575801. IF=2.057 (2015) , SJR=0.58 Q1 (2015).
3. Kossa, A., Berezvai, Sz., 2016. Novel strategy for the hyperelastic parameter fitting procedure of polymer foam materials. Polymer Testing 53, 149-155.
   DOI: 10.1016/j.polymertesting.2016.05.014. IF=2.350 (2015), SJR=0.947 Q1 (2015).

4. Berezvai, Sz., Kossa, A., 2016. Closed-form solution of the Ogden-Hill's compressible hyperelastic model for ramp loading. Mechanics of Time-Dependent Materials. Accepted, In Press. DOI: 10.1007/s11043-016-9329-5. IF=1.120 (2015), SJR=0.482 Q2 (2015).
5. Kossa, A., 2016. Closed-form stress solutions for incompressible viscohyperelastic solids in uniaxial extension. Submitted to Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik on July 31, 2016. Under Review.

Papers in peer-reviewed international journals (without Impact Factor)

6. Kossa, A., Berezvai, Sz., 2016. Visco-hyperelastic Characterization of Polymeric Foam Materials. Materials Today: Proceedings 3, 1003-1008. DOI 10.1016/j.matpr.2016.03.037. SJR=n.a.

International conference talks (plenary lectures & invited lectures)

7. Kossa, A., Berezvai, Sz., 2015. Visco-hyperelastic characterization of polymeric foam materials. 32nd International Danubia-Adria Symposium on Advances in Experimental Mechanics (DAS), Starý Smokovec, Slovakia, September 20-25, 2015.

International conference talks (session talks)

8. Kossa, A., 2014. Accurate hyperelastic model fitting procedure for polymeric foam materials. 14th European Mechanics of Materials Conference (EMMC), Gothenburg, Sweden, August 27-29, 2014.
9. Kossa, A., 2014. Effect of the modeling of lateral stretch in the parameter identification algorithm of hyperelastic foam materials. 31st International Danubia-Adria Symposium on Advances in Experimental Mechanics (DAS), Kempten, Germany, September 24-27, 2014.
10. Berezvai, Sz., Kossa, A., 2016. Nonlinear viscoelastic characterization of polymeric foams. VII European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS), Crete, Greece, June 5-10, 2016.
11. Kossa, A., 2016. On the constitutive modelling of elastomeric foams. 24th International Conference of Theoretical and Applied Mechanics (ICTAM), Montreal, Canada, August 21-26, 2016.
12. Kossa, A., Berezvai, Sz., 2016. Characterization of a thermoplastic foam material with the two-layer viscoplastic model. 33rd International Danubia-Adria Symposium on Advances in Experimental Mechanics (DAS), Portorož, Slovenia, September 20-23, 2016.

Papers in peer-reviewed Hungarian conference proceedings

13. Berezvai, Sz., Kossa, A., 2016. Memóriahabok mechanikai modellezése. XXIV. Nemzetközi Gépészeti Találkozó (OGÉT), Deva, Romania, April 21-24, 2016.

Hungarian conference talks (session talks)

14. Kossa, A., 2015. Polimer habok viszko-hiperelasztikus anyagmodellezése. XII. Magyar Mechanikai Konferencia (MAMEK), Miskolc, Hungary, August 25-37, 2015.
15. Berezvai, Sz., Kossa, A., 2016. Memóriahabok mechanikai modellezése. XXIV. Nemzetközi Gépészeti Találkozó (OGÉT), Deva, Romania, April 21-24, 2016.

International seminar talks

16. Kossa, A., 2015. Department of Mechanics, University of West Bohemia, April 24, 2015.
17. Kossa, A., 2016. Constitutive Modeling of Polymeric Foams and Finite Element Simulation of Chip Formation in Metal Cutting. Mathematics Colloquium at Department of Mathematics, Humboldt State University, Arcata, USA. April 28, 2016.

BSc and MSc theses, Hungarian

18. Koscsó Á.: Polimer habok cellaméretének hatása az eredő anyagi viselkedésre végeselemes analízis alkalmazásával. 2012.

19. Kovács A.: A cellaszerkezet geometriájának hatása polimer habok anyagi viselkedésére végeselemes analízis felhasználásával. 2012.
20. Hullár D.: Hiperelasztikus anyagmodell illesztés és végeselemes modellezés habszivacs alapanyag esetén. 2013.
21. Bertóti R.: Squash labda rugalmassági és dinamikus tulajdonságainak termomechanikai analízise. 2014.
22. Berezvai Sz.: Felületi bőrréteg eredő anyagi viselkedésre gyakorolt hatásának kísérleti és numerikus vizsgálata polimer hab alapanyag esetén. 2014.
23. Mester I.: Érintésmentes nyúlásmérő algoritmus fejlesztése videókép feldolgozási technika segítségével. 2014.
24. Csíkos D.: Polimer habok deformációinak modellezése kéttengelyű nyomókeretről atadódó terhelés esetén. 2015.
25. Szabó A.: Polimer habok hiperelasztikus anyagmodellezése kéttengelyű húzókereten végzett mérések alapján. 2015.
26. Hullár D.: Belső nyomással terhelt toroid gumimembrán deformációjának elméleti, numerikus és kísérleti vizsgálata. 2015.
27. Urbán L.: Mikrocelluláris polietilén-tereftalát termoformázásának numerikus szimulációja. 2015.
28. Propszt N.: Analytical, numerical and experimental investigation of the deformation of a hyperelastic spherical balloon. 2013.
29. Rádi F.: Constitutive modeling of a polymeric sponge material. 2014.
30. Rózsa Z.: Constitutive modeling of closed-cell polyethylene foam sheets. 2014.
31. Vitai L.: Modeling the material behavior of elastomeric foam sheets under impact loadings. 2015.
32. Berezvai Sz.: Visco-hyperelastic characterization of polymeric foams. 2016.
33. Lukács F.: Non-linear viscoelastic characterization of an earplug foam material. 2016.