Final report on the project K-108384:
'A categorical study of quantum symmetries and their applications'

The subject of our fifty-four months long project was a study of *quantum symmetries* applying the language and tools of *category theory*. As it had been envisaged in the research plan, we investigated various algebraic structures which generalize Hopf algebras (and hence groups in particular); and their appearance as symmetries of quantum field theories.

Below we summarize the results achieved in each work package (WP) of the project. In the list of references, an asterisk distinguishes those publications which resulted from the current project.

WP1. Interpreting weak (Hopf) bialgebras as (Hopf) bimonoids in appropriate duoidal categories. Proving a duality between groupoids with finitely many objects and cosemisimple pointed weak Hopf algebras.

Classical *bialgebras* — say, over a field k — can be described as monoids in the monoidal category of k-coalgebras; which are in fact the comonoids in the category of k-vector spaces. Equivalently, bialgebras are comonoids in the monoidal category of k-algebras; that is, of the monoids in the category of k-vector spaces. The notions of a monoid in the category of comonoids; and of a comonoid in the category of monoids coincide in any braided (so in particular symmetric) monoidal category and they are termed *bimonoids*. So bialgebras are bimonoids in the symmetric monoidal category of vector spaces over the field k.

For *weak bialgebras* [1], however, the unit is no longer a coalgebra map and the counit is no longer an algebra map. So for a long time no interpretation of weak bialgebras as bimonoids in any braided monoidal category had been available. The key observation in [2*] is that it is possible to identify weak bialgebras with bimonoids; though not in a braided monoidal category but more generally, in a *duoidal* category [3]. A duoidal category possesses two different but compatible monoidal structures. The monoid and comonoid structures of a bimonoid in a duoidal category are defined with respect to these different monoidal structures. The duoidal category in which a weak bialgebra can be seen as a bimonoid is the category of bimodules of the algebra $R\otimes R^{\mathsf{op}}$, where R is the so-called base algebra of the weak bialgebra. Both monoidal products are module tensor products over $R \otimes R^{\mathsf{op}}$ but different actions are used in both cases. A weak bialgebra was proven to be a *weak Hopf algebra* (in the sense of [1]) if and only if as a bimonoid in the duoidal category of $R \otimes R^{\mathsf{op}}$-bimodules it satisfies the Hopf condition in [4] (i.e. it is a *Hopf monad* in the sense of [4]).

The above interpretation of weak bialgebras as bimonoids naturally leads to a definition of morphisms between them (which need not be homomorphisms of algebras). A functor to the category of weak bialgebras (and these morphisms between them), from the category of small categories with finitely many objects (and functors as morphisms between them) was obtained by taking the free vector space spanned by the morphisms in a category. This functor was proven to possess a right adjoint, given by taking the grouplike elements in a suitable sense. This adjunction was proven to restrict to the full subcategories of groupoids and of weak Hopf algebras, respectively. Moreover, it induces equivalences between the category of small categories with finitely many objects and the category of pointed cosemisimple weak bialgebras; and between the category of small groupoids with finitely many objects and the category of pointed cosemisimple weak Hopf algebras. This extends the well-known duality between groups and pointed cosemisimple Hopf algebras.

This was part of the PhD research of López-Centella at the University of Granada, Spain, jointly supervised by Böhm and Gómez-Torrecillas.

[1] Böhm, Gabriella; Nill, Florian; Kornél Szlachányi: *Weak Hopf algebras. I. Integral theory and C*-structure*, J Algebra 221 no. 2 (1999) 385-438.

[2*] Böhm, Gabriella; Gómez-Torrecillas, José; López-Centella, Esperanza: *On the category of weak bialgebras*, J Algebra 399 (2014) 801-844.

[3] Aguiar, Marcelo; Mahajan, Swapneel: *Monoidal functors, species and Hopf algebras*, Vol. 29 CRM Monograph Series. American Mathematical Society, Providence, RI, 2010.

[4] Chikhladze, Dimitri; Lack, Stephen; Street Ross: *Hopf monoidal comonads*, Theory Appl Categ 24 no. 19 (2010) 554-563.

WP2. A unified description of various quantum symmetries as Hopf monads.

In the above summarized research package WP1 and the resulting paper [2*] we obtained a description of weak Hopf algebras as Hopf monoids in well-chosen duoidal categories. In the paper [5] (prior to the current project) some other quantum symmetries — like small groupoids and Hopf algebroids over central base algebras — were identified with Hopf monoids in suitable duoidal categories.

The starting point of the research resulting in the paper [6*] was the observation that these duoidal categories are, in fact, endohom categories of map monoidales (aka map pseudo monoids) in monoidal bicategories. The aim was to take a next step beyond duoidal categories, and give a common description of more kinds of quantum symmetries as Hopf monads on monoidales in monoidal bicategories. Several equivalent characterizations of classical Hopf algebras (among bialgebras) were extended to this level of generality.

In particular, Hopf monads in general do not possess antipode morphisms. But in many known examples there are antipodes. Their existence was explained by the natural Frobenius structure of the base monoidale in these examples. The theory worked out was shown to cover — in addition to the weak Hopf algebras in [2*] and the small groupoids and the Hopf algebroids of [5] — also Hopf monoids in braided monoidal categories and the Hopf monads of [7] on autonomous monoidal categories.

After the completion of the paper [6*], a new quantum symmetry structure was proposed in [8] under the name *Hopf category*. Motivated by that, in [9*] a new family of monoidal bicategories was constructed whose Hopf monads include Hopf categories, *Hopf polyads* in [10] and *Hopf group algebras* in [11].

This highly successful unified treatment of quantum symmetries as Hopf monads in various monoidal bicategories was the subject of an invited plenary talk by Böhm at the conference 'Category Theory 2015'. Also based on this research, an approach to the most easily accessible quantum symmetries via Hopf monads was summarized (in a form digestible also by students) in a textbook by Gabriella Böhm, entitled *"Hopf Algebras and Their Generalizations from a Categorical Point of View"*. It was accepted for publication by Springer Verlag, it is to appear in the *Lecture Notes in Mathematics* series before the end of 2018.

[5] Böhm, Gabriella; Chen, Yuanyuan; Zhang, Liangyun: *On Hopf monoids in duoidal categories*, J Algebra 394 (2013) 139-172.

[6*] Böhm, Gabriella; Lack, Stephen: *Hopf comonads on naturally Frobenius map-monoidales*, J Pure Appl Algebra 220 no. 6 (2016) 2177-2213.

[7] Bruguières, Alain; Lack, Steve; Virelizier, Alexis: *Hopf monads on monoidal categories*, Adv Math 227 no. 2 (2011) 745-800.

[8] Batista, Eliezer; Caenepeel, Stefaan; Vercruysse, Joost: *Hopf Categories*, Algebr Represent Theory 19 no. 5 (2016), 1173-1216.

[9*] Böhm, Gabriella: *Hopf polyads, Hopf categories and Hopf group monoids viewed as Hopf monads*, Theory Appl Categ 32 no. 37 (2017) 1229-1257.

[10] Bruguières, Alain: *Hopf polyads*, Algebr Represent Theor 20 no. 5 (2017) 1151-1188.

[11] Turaev, Vladimir G.: *Homotopy field theory in dimension 3 and crossed group-categories*, preprint available at `http://arxiv.org/abs/0005291`.

WP3. Finding the right notion of weak (Hopf) bimonoids in duoidal categories.

A *bimonoid* in a duoidal category [3] is an object equipped with a monoid structure with respect to one of the monoidal structures, and with a comonoid structure with respect to the other monoidal structure; in such a way that the multiplication and the unit are comonoid morphisms; equivalently, the comultiplication and the counit are monoid morphisms (see WP1 for the particular case of bimonoids in braided monoidal categories). In the student-project of Chen — supervised by Böhm and resulting in the paper [12*] — the compatibility conditions between the monoid and comonoid structures were *weakened* analogously to the axioms of weak bialgebra [1]. The most important results on weak bialgebras — such as the structure of the base algebras and the behavior of the representations — were proven to extend to this generalization.

[12*] Chen, Yuanyuan; Böhm, Gabriella: *Weak bimonoids in duoidal categories*, J Pure Appl Algebra 218 no. 12 (2014) 2240-2273.

WP4. Isolating from Van Daele and Wang's definition of multiplier weak Hopf algebra the structure which is present without the antipode. With a study of its base algebras, finding a non-unital generalization of separable Frobenius algebra.

The most well-known examples of Hopf algebras are the linear spans of (arbitrary) groups. Dually, also the vector space of linear functionals on a finite group carries the structure of a Hopf algebra. In the case of infinite groups, however, the vector space of linear functionals with finite support possesses no unit. Consequently, it is no longer a Hopf algebra but, more generally, a *multiplier Hopf algebra* [13]. Replacing groups with finite groupoids, both their linear spans and the dual vector spaces of linear functionals carry weak Hopf algebra structures [1]. Finally, removing the finiteness constraint in this situation, both the linear spans of arbitrary groupoids, and the vector spaces of linear functionals with finite support on them are examples of *weak multiplier Hopf algebras* in [14,15].

In the papers [14,15] only weak multiplier *Hopf* algebras are considered without looking for the more general structure of weak multiplier bialgebra. But if considering monoids instead of groups, their linear spans (and vector spaces of functionals in the finite case) are only bialgebras, no longer Hopf algebras. Similarly, the linear spans of small categories with finitely many objects (and the vector spaces of functionals in the case when also the number of arrows is finite) are only weak bialgebras but not weak Hopf algebras. To be able to describe the analogous structures associated to categories without any finiteness assumption, in [16*] the notion of *weak multiplier bialgebra* was worked out (which contains as a particular case non-weak multiplier bialgebras, neither defined before).

In this non-unital — 'multiplier' — setting, the (separable Frobenius) base algebras of a weak bialgebra are replaced by certain distinguished (non-unital) subalgebras of the multiplier algebra of a weak multiplier bialgebra. Under a mild (fullness) assumption on the (multiplier valued) comultiplication, these subalgebras were proven to carry coseparable co-Frobenius coalgebra structures. This can be seen as the appropriate non-unital generalization of separable Frobenius algebra in the sense that the canonical epimorphisms to the module tensor products over them split. An appropriate category of modules over a regular weak multiplier bialgebra with a full comultiplication was proven to be monoidal admitting a strict monoidal and faithful (in some sense forgetful) functor to the category of firm bimodules over the base algebra. Some other representation categories in the same setting were investigated, and related to each other, in [17*] and [18*].

While it was proven in [13] that a multiplier Hopf algebra is a Hopf algebra if and only if it possesses a unit element, no analogous result is available in the weak case; a weak multiplier Hopf algebra with a unit element is not known to be a weak Hopf algebra. Such a statement was proven in [14,15] only under the additional condition that also the opposite algebra is a weak multiplier Hopf algebra; and this assumption was needed in [14,15] also to prove several other expected results. But this is a rather strong restriction, not even satisfied by weak Hopf algebras in general. So one of our aims in [16*] was to find the appropriate intermediate case, for which the expected properties can be proven, but which also contains all weak Hopf algebras. A satisfying answer was found by considering regular weak multiplier bialgebras equipped with an antipode map in a suitable sense.

The paper [16*] contains some results from the PhD research of López-Centella at the University of Granada, Spain, supervised jointly by Böhm and Gómez-Torrecillas.

[13] Van Daele, Alfons: *Multiplier Hopf algebras*, Trans. Amer. Math. Soc. 342 no. 2 (1994) 917-932.

[14] Van Daele, Alfons; Wang, Shuanhong: *Weak Multiplier Hopf Algebras. Preliminaries, motivation and basic examples*, in: Operator Algebras and Quantum Groups. W. Pusz and P.M. Sołtan (eds.), Banach Center Publications (Warsaw), vol. 98 (2012), 367-415.

[15] Van Daele, Alfons; Wang, Shuanhong: *Weak Multiplier Hopf Algebras. The main theory*, Journal für die reine und angewandte Mathematik (Crelle's Journal) Published Online: 2013-07-23, DOI: https://doi.org/10.1515/crelle-2013-0053.

[16*] Böhm, Gabriella; Gómez-Torrecillas, José; López-Centella, Esperanza: *Weak multiplier bialgebras*, Trans Amer Math Soc 367 (2015) no. 12, 8681-8721.

[17*] Böhm, Gabriella: *Comodules over weak multiplier bialgebras*, Int J Math 25 1450037 (2014).

[18*] Böhm, Gabriella: *Yetter-Drinfeld modules over weak multiplier bialgebras*, Israel J Math 209 no. 1 (2015) 85-123.

WP5. (Weak) multiplier bialgebras in braided monoidal categories.

In all of the papers [13,14,15,16*], (weak or not) multiplier (Hopf or not) bialgebras only were considered on vector spaces. But there are many other interesting situations; like modules over commutative rings, graded vector spaces, Hilbert spaces, and so on. All of these settings can be treated simultaneously, and a deeper insight can be gained by studying various multiplier structures in unspecified braided monoidal categories.

The novelty of the approach in [19*] is a concise formulation of the axioms of *multiplier bimonoid* in a braided monoidal category in terms of a generalized version of the *fusion morphism* of [20] — avoiding any reference to multipliers. This approach allows for a smooth treatment of modules and comodules; constituting monoidal categories in both cases. A conceptual explanation of this monoidal structure was found via the study of a functor induced by a multiplier bimonoid, by observing that it carries a structure generalizing that of a bimonad (aka opmonoidal monad).

Multiplier Hopf monoids were introduced in [21*] as multiplier bimonoids whose fusion morphisms are invertible. In the category of vector spaces over the complex numbers, multiplier Hopf algebra of [13] was re-obtained by that. It was shown that the key features of multiplier Hopf algebras (over fields) remain valid in this more general context. Namely, for a multiplier Hopf monoid, the existence of a unique antipode was proved in an appropriate, multiplier-valued sense which was shown to be a morphism between some twisted multiplier bimonoids. For a regular multiplier Hopf monoid (whose twisted versions are multiplier Hopf monoids as well) the antipode was proved to factorize through a proper automorphism. Under mild further assumptions, duals in the base category were shown to lift to the monoidal categories of modules and of comodules over a regular multiplier Hopf monoid. Finally, the so-called Fundamental Theorem of Hopf modules was proved which states an equivalence between the base category and the category of Hopf modules over a multiplier Hopf monoid.

As recalled above (see WP1), a bialgebra over a field or, more generally, a bimonoid in a braided monoidal category, is an object carrying a monoid and a comonoid structure subject to compatibility conditions that can be interpreted as saying that a bimonoid is a monoid in the category of comonoids; equivalently, it is a comonoid in the category of monoids. A multiplier bialgebra over a field [16*] or, more generally, a multiplier bimonoid in a braided monoidal category [19*], is a generalization which is no longer a monoid or a comonoid in the base category. However, in [22] a monoidal category was constructed, whose objects are certain non-unital algebras (say over a field), and in which the comonoids include the multiplier Hopf algebras of [13]. The aim of [23*] was to generalize and strengthen this result. Namely, under mild assumptions (involving a class $\mathcal{Q}$ of regular epimorphisms) we constructed a category M of certain semigroups in a braided monoidal category C. We described multiplier bimonoids in C (whose structure morphisms lie in $\mathcal{Q}$) as certain comonoids in M. Defining the morphisms between such multiplier bimonoids as the morphisms between the corresponding comonoids in M, we obtained a category of multiplier bimonoids in C.

Note that the results of [23*] only tell us how *certain* multiplier bimonoids in a braided monoidal category C can be seen as *certain* comonoids in an appropriately constructed monoidal category M. In [24*] we applied the methods of [25] to prove a bijection between *arbitrary* multiplier bimonoids in C and *arbitrary* simplicial maps from the Catalan simplicial set to a suitable simplicial set constructed for this purpose. With its help, multiplier bimonoids can be regarded as (co)monoids in something more general than a monoidal category (namely, the simplicial set itself). We analyzed the particular simplicial maps corresponding to that class of multiplier bimonoids which can be regarded as comonoids in [23*].

Finally in [26*] *weak multiplier bimonoids* in braided monoidal categories were introduced and studied. For that a further generalization of the fusion morphisms — used in [19*] in the non-weak case — was needed. Under some assumptions, the so-called base object of a regular weak multiplier bimonoid was shown to carry a coseparable comonoid structure; hence to possess a monoidal category of bicomodules. In this case, appropriately defined modules over a regular weak multiplier bimonoid were proven to constitute a monoidal category with a strict monoidal forgetful type functor to the category of bicomodules over the base object.

Braided monoidal categories considered in WP5 include various categories of modules or graded modules, the category of complete bornological spaces, and the category of complex Hilbert spaces and continuous linear transformations, see [26*].

[19*] Böhm, Gabriella; Lack, Stephen: *Multiplier bialgebras in braided monoidal categories*, J Algebra 423 (2015) 853-889.

[20] Street, Ross: *Fusion operators and cocycloids in monoidal categories*, Appl Categor Struct 6 no. 2 (Special Issue on Quantum Groups, Hopf Algebras and Category Theory, ed. A. Verschoren, 1998) 177-191.

[21*] Böhm, Gabriella; Lack, Stephen: *Multiplier Hopf monoids*, Algebr Represent Theory 20 no. 1 (2017) 1-46.

[22] Janssen, Kris; Vercruysse, Joost: *Multiplier bi- and Hopf algebras*, J Algebra Appl 9 no. 2 (2010) 275-303.

[23*] Böhm, Gabriella; Lack, Stephen: *A category of multiplier bimonoids*, Appl Categor Struct 25 no. 2 (2017) 279-301.

[24*] Böhm, Gabriella; Lack, Stephen: *A simplicial approach to multiplier bimonoids*, Bull Belgian Math Soc Simon Stevin 24 (2017) 107-122.

[25] Buckley, Mitchell; Garner, Richard; Lack, Stephen; Street, Ross: *The Catalan simplicial set*, Math Proc Camb Phil Soc 158 (2015) 211-222.

[26*] Böhm, Gabriella; Gómez-Torrecillas, José; Lack, Stephen: *Weak multiplier bimonoids*, Appl Categor Struct 26 no. 1 (2018) 47-111.

WP6. Analysis of skew-monoidal categories.

Skew monoidal categories (SMC) are monoidal categories with non-invertible coherence morphisms. As shown in our paper [27] preceding the current project, bialgebroids over a ring R can be characterized as the closed skew monoidal structures on the category of R-modules in which the unit object is the regular module. This offers a new approach to bialgebroids and Hopf algebroids.

Little is known about skew monoidal structures on general categories. In [28*] we analyzed the one-object case: the *skew monoidal monoids* (SMM). It was shown that each SMM possesses a dual pair of bialgebroids describing the symmetries of its (co)module categories. These bialgebroids are submonoids of their own base and are rank 1 free over the base on the source side. Various equivalent definitions of SMM were presented, the structure of their (co)module categories was studied and the possible closed and Hopf structures on a SMM were discussed.

The main object of study in [29*] was the structure of the category of modules over a SMC which, in case of bialgebroids, is known to be a monoidal category equipped with a monadic strong monoidal functor to the category of bimodules. Whether analogous structures exist for SMC-s is a nontrivial problem illustrated by the fact that for the category of comodules over an SMC even the existence of a reasonable tensor product is an open question. Working in the framework of enriched categories we presented a construction of a (skew monoidal) tensor product of modules and of a monadic skew monoidal forgetful functor. We also gave conditions for the category of modules to be monoidal and the forgetful functor to be strong monoidal. In formulating these conditions a notion of 'self-cocomplete' subcategories of presheaves appears to be useful which provides also some insight into the problem of monoidality of the skew monoidal structures found in [30] on functor categories.

In our research plan for the project we had also planned to prove appropriate weakenings of Mac Lane's coherence theorem on skew-monoidal categories. This problem, however, was solved in the meantime by other authors in [31-33]. So we focussed on the other (not completely unrelated) problems described above.

[27] Szlachányi, Kornél: *Skew-monoidal categories and bialgebroids*, Adv in Math 231 no 3-4 (2012) 1694-1730.

[28*] Szlachányi, Kornél: *Skew monoidal monoids*, Comm Algebra 44 no 6 (2016) 2368-2388.

[29*] Szlachányi, Kornél: *On the tensor product of modules over skew monoidal actegories*, J Pure Appl Algebra 221 (2017) 185-221.

[30] Altenkirch, Thorsten; Chapman, James; Uustalu, Tarmo: *Monads need not be endofunctors*, In: Foundations of software science and computational structures, pp 297-311, Lecture Notes in Comput Sci 6014, Springer 2010.

[31] Lack, Stephen; Street, Ross: *Triangulations, orientals, and skew monoidal categories* Adv in Math 258 (2014) 351-396.

[32] Bourke, John; Lack, Stephen: *Free skew monoidal categories*, J Pure Appl Algebra available online at `https://doi.org/10.1016/j.jpaa.2017.12.006`.

[33] Uustalu, Tarmo: *Coherence for skew monoidal categories*, Electron Proc Theor Comput Sci 153 (2014) 68-77.

WP7. Quantum theory and local causality.

Quantum theory is usually thought to be non-local. Most derivations of the contradiction between causality and the quantum description of nature are based on the putative implication of Bell's inequalities by various causality requirements, because these inequalities exclude the strong correlation present in quantum theories.

It was shown in our papers [34,35] prior to the current project that the derivation of these inequalities from common cause principles is impossible in quantum theories. In this project, in the papers [36*] and [37*], we showed that Bell's local causality is valid in a general class of quantum field theories, no contradiction occurs. For the precise statement, first we formulated Bell's notion of local causality in algebraic (classical and quantum) field theories. Then in case of local primitive causality, which is a regular assumption in algebraic field theories, we proved that Bell's local causality holds if the corresponding local (abelian or non-abelian) von Neumann algebras are atomic. In classical theories without local primitive causality, state extension procedure from a Cauchy surface algebra to the quasilocal one may correspond to a causal Markov process. In that case Bell's local causality is shown to hold if the local (abelian) algebras are finite dimensional.

Péter Vecsernyés, together with Gábor Hofer-Szabó finished a short book [38*] which is an extended review of their publications in recent years about the verified peaceful coexistence of causality and quantum theory. It was published in February 2018 by Springer Verlag in the *Brief Review Book Series*.

[34] Hofer-Szabó, Gábor; Vecsernyés, Péter: *Noncommuting local common causes for correlations violating the Clauser–Horne inequality*, J Math Phys 53 (2012) 122301.

[35] Hofer-Szabó, Gábor; Vecsernyés, Péter: *Noncommutative common cause principles in algebraic quantum field theory*, J Math Phys 54 (2013) 042301.

[36*] Hofer-Szabó, Gábor; Vecsernyés, Péter: *On the concept of Bell's local causality in local classical and quantum theory*, J Math Phys 56 (2015) 032303.

[37*] Hofer-Szabó, Gábor; Vecsernyés, Péter: *A generalized definition of Bell's local causality*, Synthese, 193 (2016) 3195-3207.

[38*] Hofer-Szabó, Gábor; Vecsernyés, Péter: *Quantum Theory and Local Causality*, ISBN 978-3-319-73932-8, Springer, 2018.

WP8. Dynamical description of measurements in quantum mechanics.

A measurement in standard quantum theory is 'described' by an instant non-unitary, non-linear jump into an eigenstate of the measured quantity with probability — meaning in fact relative frequency — equal to the expectation value of the corresponding spectral projection in the identically prepared states of repeated experiments.

One may try to interpret the measurements as very fast processes; so that non-unitarity and non-linearity arise as an effective behavior of the unknown unitary dynamics of the interaction of the measured quantity with the measuring device. The probabilistic nature may be attributed to the unknown initial state of the measuring device. Keeping these assumptions in [39*] a two-step dynamical model was constructed for selective measurements in arbitrary finite dimensional quantum mechanics. The first one of these steps is the non-selective measurement, or M-decoherence of a self-adjoint observable M described by a semigroup of completely positive maps generated by the linear, deterministic first order Lindblad differential equation for the state. The second step is a process from the decohered state resulting from the first step to an M-pure state, which is described by an effective non-linear 'randomly chosen' toy model dynamics: the pure states arise as asymptotic fixed points, and their emergent probabilities are the relative volumes of their attractor regions.

[39*] Vecsernyés, Péter: *An effective toy model in $M_n(\mathbb{C})$ for selective measurements in quantum mechanics*, J Math Phys 58 (2017) 102109.

WP9. Superselection sectors and phase structure in quantum chains.

Phases of algebraic field theoretical models can be defined as partitions of the pure states of the observable algebra A of the model, given by equivalence classes of local observable algebra extensions in the irreducible representations of A obtained from the pure states. Although formerly in [40] we derived the general structure of phases of Hopf spin chains, there are popular particular models where the explicit construction can be useful and interesting.

In the MSc thesis project [41*] of Megyeri — supervised by Vecsernyés — the phase structure of the pure translation invariant states of the spin one-half XYZ-chain was described. Since this model can be rewritten as a Hopf spin chain based on the group algebra H of the finite group $\mathbb{Z}_2 \times \mathbb{Z}_2$, the equivalence classes of phases correspond to the cohomology classes of the so-called bosonic intermediate Δ-cocycles of the Drinfeld double of H, which were explicitly presented. The publication of these results is delayed by Megyeri's moving to the UK for a PhD student position.

In [42*] the superselection sectors — that is, the localized and transportable amplimorphisms — of the observable algebras of various Hubbard quantum chains were classified. Unlike in Hopf spin chains, here the observable algebra is not an UHF, but only an AF C^*-algebra. This fact is reflected on the superselection sectors: they are not characterized by the monoidal category of representations of a finite dimensional (quasitriangular) Hopf algebra but by that of the compact group $U(1)$.

[40] Müller, Volkhard F; Vecsernyés, Péter: *Phase structure of G-spin models*, to be published.

[41*] Megyeri, Balázs; Vecsernyés, Péter: *Phase structure of the spin* 1/2 *XYZ-chain*, to be published.

[42*] Barankai, Norbert; Vecsernyés, Péter: *Algebraic quantum field theory of the Hubbard chain*, to be published.