Mathematics of Economics models (final report of the OTKA 108383 project)

The aim of this project was to study discrete economic models from the mathematical and computational perspective. We published results on the following topics: stable matchings and generalizations, Pareto-optimal matchings, popular matchings fair division, voting, mechanism design and complexity aspects of various game-theoretic notions. We have altogether more than 40 conference participations and we ourselves gave more than 35 conference talks on our work, including 11 invited talks but excluding conference talks delivered by our coauthors. We organized a conference and a workshop in December 2016 with the participation of distinguished colleagues from all over the world. The dissemination of our results for the wider public is served by two Élet és Tudomány papers and an interview on Radio Kossuth. (Note that acknowledgment for OTKA is not allowed in the above journal.)

Although the outcome of the project does not seem to be a potential source of *direct* profit for the economy, it is notable that our algorithm on the teachers' assignment problem (published in CEJOR) has been implemented at the Pavol Jozef Šafárik University. The corresponding integer program can be solved very quickly and it provides a better assignment of trainee teachers to schools than the local administration could compute ever before.

In what follows, we briefly survey the above mentioned topics of the project.

Stable matchings and generalizations

For an undirected graph G with vertex-preferences on the edges, a matching M is stable if every edge of G has a vertex u such that M contains an edge incident with u that is not worse than e for u. If G is a bipartite graph then the existence of a stable matching can always be found by the famous Gale-Shapley algorithm that itself is is a nontrivial consequence of Tarski's fixed point theorem.

A natural application of this kind of stability notion is the college admission problem, but there are less well-known applications as well. Network flows can be viewed as a generalization of bipartite matchings. The same idea allows us to generalize stable matchings to stable flows. Beyond describing basic structural properties of stable flows, we worked out new and simple algorithms to find them.

Stable matchings can also be viewed as common antichain of two posets, and from this perspective, it is interesting to study stable antichains. We proved that in a fairly complex model involving demands and weights there always exist a fractional stable antichain.

The stable roommates problem is the nonbipartite version of the stable matching problem. Here, a the existence of a stable matching is not guaranteed, but a so-called stable half matching always exists. The background of this problem has to do with the famous Scarf lemma. We studied Scarf's algorithm in the generalized hospital-resident problem where some residents are couples having conditions on their mutual placements and we also worked on fractional solutions of certain NTU-games.

For the bipartite version, the college admission problem version is a most important one. We have a result on stable matchings of teachers to schools where schools need a certain number of certain type lectures and individual teachers may be qualified to teach more subjects. Another result shows that if in the college admission problem there are upper and lower quotas for a nested system of colleges then a matroid-generalization of the stable marriage theorem allows us to find a stable assignment, if such assignment exists.

We also studied various other generalizations and extensions of the stable matching problem: uncertainty of preferences, choice functions instead of linear preferences for the agents, and the complexity of finding a matching that is stable and obeys certain covering constraints.

As we mentioned earlier, we also published a propagating paper in Élet és Tudomány on stable marriages.

Pareto-optimal matchings

If in our graph-model not all vertices have a preference on the neighbors then instead of stability, we often look for a Pareto-optimal (sometimes called efficient) matching. A matching M is Pareto-optimal if no Pareto-improvement for M exists where M' is a Pareto-improvement of M if no agent (i.e. vertex with preferences) is worse off by M' compared to M and at least one agents prefers M' to M.

We published results on Pareto-optimal b-matchings (also called as 'many-to-many' matchings) and for the generalization where there can be ties in the preferences. Another extension where we have results is the Pareto-optimal b-matching problem with lower quota requirements.

Popular matchings

Popular matchings serve as a voting-based alternative concept to stable matchings. It is a concept that offers an attractive trade-off between the size and the optimality of the matching with respect to agents' preferences. In short, a popular matching M guarantees that no matter what alternative matching is offered on the market, the majority of the agents will opt for M. Moreover, $|M|$ is relatively close to the size of the maximum matching in the market. The notion was first defined in cognitive science, where such a majority decision is a well-motivated potential focus of investigation. Our first paper focuses on two-sided markets where agents on one side have the usual, strict preference lists, while the other side either comprises of agents who might be indifferent between members on the first side, or it acts as a set of objects that do not care about which agent they are matched to. We show the striking difference between the two scenarios in terms of complexity: while the first case defines an NP-complete problem, the second case has a polynomial-time solution method. In our second paper on popular matchings, we investigate the case of manipulability. Given a bipartite instance and a specific edge of the graph, is it possible to compute a matching that contains the specific edge and it is popular as well. We present an algorithm that finds such a matching if any exists. As a byproduct, we establish a very strong structural connection between stable and popular matchings of the same bipartite instance.

Fair division and voting

In this project, we studied social choice problems related to fair division and elections. We introduced and analyzed directed acyclic graph games (DAG-games), a generalization a monotonic minimum cost spanning tree games. DAG-games model the cost sharing of infrastructural developments. Part of this result, namely the computation of nucleolus, relies on mathematical tools we developed in a former paper also under the present project.

Our team also worked on the apportionment problem, in which the seats of the Parliament has to be distributed among counties with different populations in a proportional way. Since fractional seats cannot be allotted divisibility issues may arise.

The Venice Commission gave a recommendation about the fairness of apportionments, that can be understood as a maximum departure limit from the average district sizes, as it was also interpreted in the new Hungarian regulation on the general elections. Following this concept, we designed an efficient algorithm for finding the allotment such that the differences from the average district size are lexicographically minimized. This result was inspired by some discrepancies in the draft version of the Hungarian Electoral Law - which were later corrected (the authors might like to think that they have to do something with this correction, although they got no formal or informal affirmation).

We analyzed the Papal conclave, introduced a spatial voting game and performed a computer simulation to find the most powerful actors (bishops) in the conclave.

We studied the cake cutting problem (the textbook example of a fair division problem) where players have different demands on the common inhomogeneous good. We propose a protocol for fair division and show that it is asymptotically optimal. For irrational demands, we also describe a protocol that is finite but can take arbitrarily long. This result won the best paper award at SAGT 2018. The topic of our second propagating paper is also the cake-cutting problem.

Mechanism design and complexity aspects

We studied the complexity of various problems in computational social choice, using the framework of parametrized complexity as well as classical complexity theory. We investigated several procedures from this point of view, e.g., campaign management in approval-based elections, allocation problems in serial dictatorship, and stable matching problems with constraints. We also initiated a new research direction to examine the complexity of control problems in the context of fair division; we focused mainly on obtaining an envy-free or a proportional allocation by deleting or adding the set of items that needed to be distributed (closing a problem that has been open for more than five years). Furthermore, we contributed to the newly appeared book edited by Ulle Endriss, titled Trends in Computational Social Choice, by providing a chapter that aims to serve as an entry-point for researchers of the comsoc area into the techniques of parametrized complexity.

Chess pairings in each round of a Swiss tournament are conducted by sophisticated matching methods, which are described in details in the FIDE guidebook as potentially exponential time heuristics. We show that the Dutch variant of the Swiss rule can be implemented by an efficient matching algorithm.

In the house-swapping problem, we look for a Pareto-optimal Pareto-improvement of a certain allocation of goods. We prove that the solution in case of divorcing and marrying agents is inapproximable but fixed-parameter tractable. We studied whether in the well-known serial dictatorship mechanism a particular agent must or may receive a certain object in the final assignment.

We proposed a network flow based algorithm to assign evaluators to research grant applications based on the procedure used in Slovakia came up with an integer programming approach to the practical placement of trainee teachers to schools. This latter result has been implemented and being used.