

Final Report of OTKA project no. K106118

Preface

The original research plan has proposed 3 topics - one for every year of the project period. As the project has progressed, some closely related investigations were found to be interesting to be included in the project. As a result, three additional topics were also considered as being part of the OTKA project. In the following parts the results are discussed divided by the topics.

1st topic: Spectral filtering of GOCE SGG data [7], [12], [8]

The GOCE satellite observes gravity gradients with unprecedented accuracy and resolution. The GOCE observations are reliable within a well-defined measurement bandwidth, the 5-100 mHz. Thus different filters were defined, which suppress the signal and the noise in the stop-band, i.e. outside of the 5-100 mHz interval (the suppression was defined by 40 dB), and keeps the signal possible untouched in the pass-band, within 5-100 mHz (the pass-band ripple was defined not to exceed 0.025 dB).

Within the frame of the OTKA project different finite (FIR) and infinite impulse response (IIR) filters have been designed to obtain the demanded pass. Note that in the case of the IIR filter, in order to avoid nonlinear phase response, the filtering sequence should be applied back and forth. According to that, the filter specifications are redefined for that case by a suppression of 80 dB in the stop band, and an threshold for the ripple of 0.05 dB in the pass band.

The most effective filter among the 4 tested filters was the IIR filter. It has been designed using the elliptic approximation. This type of approximation results also in equiripple magnitude response. A cascade of a 5-order low pass and a 9-order high pass filter is designed. Exhaustive time and frequency domain investigations prove that the proposed infinite impulse response filter can be a real competitor of the existing solution of the filtering problem.

2nd topic: Alternative method for GOCE SGG processing [5]

In the second year of the OTKA project the main focus was on developing a computationally efficient method for GOCE Satellite Gravity Gradiometry (SGG) adjustment. Based on preliminary investigations, the choice for the purpose was the Semi-Analytical approach. The Semi-Analytical approach reduces the computational time drastically from some months to some minutes, and the required CPU from a supercomputer to a regular PC. Even though the Semi-Analytical approach has been implemented years ago for SGG data, no methodology for band limited measurements has been developed before, making it applicable for the GOCE. This has been implemented now. For the filtering of the gravity gradients to restrain to the information in the Measurement Bandwidth (MBW), an IIR filter (developed a year earlier) has been used. Due to the ill-posed condition of the

normal matrix of the adjustment, regularization of that has been implemented, determining an optimal regularization factor for each gravity gradients. The Semi-Analytical adjustment is implemented for GOCE SGG only so far, thus no full gravity field solution can be achieved, which could be done only by the inclusion of GOCE Satellite-to-Satellite Tracking (SST) data as well. The SST would be needed for recovering the long-wavelength gravity signal.

With the use of the developed band limited Semi-Analytical method, tests on the signal content of the filtered SGG data in the spherical harmonic (or Legendre) domain has been performed, i.e. the propagation of the effect of the filtering (in the time-domain) to the spherical harmonic domain was investigated. More specifically, it was determined that which spherical harmonics are affected by the spectral filtering, how the signal content of the observations is changed by degrees and orders of the spherical harmonics. By defining the signal content of the observed SGG as a function of degree and order, degrees for need of additional gravity data and of regularization in the processing sequence can be identified. As for the results, it was found that the filtering of the observations along the orbit demolishes the long-wavelength information up to degree 40, with the exception of the V_{xz} gradient, which is gaining the full power around degree 70. Basically, the signal content was found to be significantly influenced by the noise content of the long-wavelength observations, which has unlikely leaked into the middle- and short-wavelength signal. This effect is more emphasized for the (near-)zonals. At the other end of the MBW, no actual degree of signal loss could be detected. The signal is gradually vanishing until the high degrees. According to these findings, for SGG-only solutions Kaula regularization of the (near-)zonals is suggested. Furthermore, a Kaula regularization on the short-wavelength coefficients is suggested to be applied, starting from degree and order of approximately 150.

3rd topic: Application for GOCE gravity information for regional gravity field modelling [16], [17]

The last year of the project was concentrating on regional use of GOCE-borne gravity field information. An updated gravity field model has been determined for Hungary using deflections of the vertical, gravity anomalies, quasigeoid heights by GPS/levelling and surface gravity gradients. The solution methodology is least-squares gravity field parameter estimation using Spherical Radial Base Functions (SRBF) and regularization by Variance Component Estimation (VCE). For reference gravity field the latest GOCE DIR R5 model has been combined with the EGM2008 model and has been evaluated in comparison with the EGM2008 and EIGEN-6C4stat models to assess the performance of our regional gravity field solution. The relative contribution of each regional dataset to the final model has been evaluated. According to our tests, the resulted gravity model may be considered as the most accurate Hungarian geoid model up to date.

4th topic: Error propagation of rotation of SGG observations [13], [7], [8]

The observations of SGG are performed along the satellites orbit in a gradiometer-fixed reference frame. The error in this reference frame is similar along two perpendicular axes of the gradiometer, $10\text{-}12 \text{ mE/s}^2/\sqrt{\text{Hz}}$, and slightly less accurate along the third one, about $20 \text{ mE/s}^2/\sqrt{\text{Hz}}$. Normally, in Earth sciences some Earth-fixed reference frames are applied. Thus the transformation

of the gradiometer-fixed observables to Earth-fixed is unavoidable. The error propagation due to transformation was analysed.

According to the investigations, it was found that the rotational sequence incorporates the errors of the third axis into the measurements of the two more accurate axes, resulting in a notable drop of accuracy in total. Therefore it is suggested to perform the analysis in the gradiometer-fixed reference frame. It results in an unusual solution, as all the processing equations should beforehand be rotated to the gradiometer-fixed reference frame and filtered to the MBW. So the system of processing equations become time-variable and each epochs of observation must be treated separately. With this manner, however, the loss of accuracy can be avoided.

5th topic: On the computation of truncation coefficients for the extended Stokes function [14], [15]

Due to the most recent satellite missions, such as GOCE, the knowledge of the gravity field is improved both in accuracy and in resolution (i.e. the maximal degree and order of the spherical harmonics determined in the model). Their use for several applications requires the evaluation of truncation coefficients that are products of high degree Legendre polynomials and a kernel function over a given domain. The oscillating characteristics of the integrands make to evaluate them difficult. An accurate quadrature is derived for fast computation of these integrals based on the Glaser-Liu-Rokhlin root finding algorithm and the Gauss-Lobatto quadrature between the roots. The algorithm has successfully been applied to eliminate the known instability of earlier recursive computation techniques, which is demonstrated for high degrees and altitudes.

6th topic: Temporal variations of gravity [11], [6], [10], [1], [4], [2], [3]

Some years ago there were attempts to use GOCE observations for determining temporal variations of the gravity field. It was found, however, that even though the GOCE observations are obviously affected by the temporal varying gravity, the temporal variation content of the GOCE signal alone is not sufficient for determining temporal variation of geophysical processes, only by combining them with GRACE observations.

According to that, investigations with temporal gravity field variation have been performed. With the use of GRACE gravity field models, hydrological investigations were performed for the basin of the La Plata river [11],[10]. According to our results, some days delay could be detected between the precipitation and the runoff, i.e. the mass surplus (detected by GRACE) was followed by a water level maximum (detected by gauges) some days later. It means that the water storage capacity of underground reservoirs in this basin has a small effect on the runoff. GRACE gravity field model data has also been used to determine mass variations over the Antarctic continent for investigating ice mass balance variations [6]. It was found that the Antarctic continent is either quite stable in its ice mass, or the presently available observations and models are not accurate enough to deliver reliable estimates of ice mass change. The only conspicuous ice mass changing process in this continent is the accelerating melting in the West Antarctica region.

In practice, temporal gravity variations are estimated based on sampled data. In the case of satellite-borne techniques, the sampling occurs along the orbit of the satellite, while in the case of terrestrial measurements, usually measurements in a monitoring network in repeated measurement campaigns

at certain epochs are performed. In both cases, the discrete signal is an approximation of the continuous, real signal, which in any cases aliases the gravity variation estimate. For periodic temporal variations this error source is discussed in general, and applied for the GOCE satellite and for other geodetic measurements as well, c.f. [1] and [3]. A very similar sampling error can be defined in the space domain as well, i.e. the description of surfaces by point-wise data may be improper, as indicated by [4]. A supplementary tool for dealing with observed and averaged gravity signal using the Legendre polynomials were developed by [2].

Postscript

As a closing of the OTKA project, a summarizing study on the GOCE satellite and the related researches in Hungary were presented in [9]. In order to make the results easily accessible by the public, a website has been created: <http://www.geod.bme.hu/~fl/otka106118/otka106118.html>

The website is purposely in Hungarian language to reach the Hungarian audience. The site provides direct links to the related papers as well.

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