FINAL REPORT

on the project OTKA PD 105442

Project title:	**Tyre induced self-excited vibrations of vehicles**
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The aim of the project was the investigation of simple, low degree-of-freedom mechanical models of tyres and vehicle systems. The application of the formally developed delayed contact model was in focus, since it was suspected that unexplored parameter domains of self-excited vibrations could be detected by this contact model. These vibrations can be relevant from the point of view of the lateral stability and the rolling resistance of the vehicles.

Two main topics were planned to be investigated: the stability analysis of different vehicle manoeuvres using the time delayed contact model, and the analysis of the regenerative effect of the tyre carcass. Finally, a third topic became also involved in the project: the stability analysis of the skateboarding with special respect to the reflex delay in the human control, which makes the skateboard to a delayed non-holonomic system. The results related to these topics are summarized in the following subsections.

1 Extension of vehicle models to realistic manoeuvres

In the first year of the project the so-called bicycle model of a passenger car was investigated with the help of the delayed tyre model. The linear stability of the rectilinear motion of the vehicle was analysed and unstable parameter domains were identified, where the self-excited lateral vibrations of the vehicle emerge. Basic formulas were also derived that help to define the most critical speed domains. The results were published in a journal paper [1].

In the second year, the more complex car-trailer combination was in the focus of the research. This model is well-known in vehicle dynamics, and it was analysed in several studies, but only the linearized form of the equations of motion could be found in the literature. Thus, the equations of motion were derived even for the case when the geometrical nonlinearities are not negligible. The model was also extended by means of the implementation of the delayed tyre model. The resulted system of integro-differential and partial differential equations was transformed to delay differential equations (DDEs) and the linear stability of the rectilinear motion was investigated. Stability charts were constructed, where the effect of the vehicle speed and the effect of the trailer’s payload location were demonstrated. On the one hand, the most relevant stability boundaries, which were also detected by former studies, were exactly reproduced. A closed-form formula was determined to locate the critical payload position that can be used as a basic rule in practice. To verify the results, a simple experiment was built, namely, the lateral motion of a small toy vehicle with elastic tyres was analysed on a conveyor belt. Detailed experimental results of other researchers also confirmed the theoretical results. On the other hand, it was verified as well that small unstable parameter domains exist for small vehicle speeds, and in these domains the rectilinear motion is linearly unstable even for the optimal payload position. Tyre deformations were also calculated numerically and the effect of the tyre parameters was analysed, namely, different parameter set-ups were used for the tyres of the vehicle meantime the variation of the linear stability chart was observed. The contact patch length and the stiffness of the trailer's tyre were modified in order to simulate the case when the inflation pressure of the trailer's tyre changes. It was established that there exist certain (maybe 'extreme') parameter set-ups, which can lead to kind of worst case scenarios. This can be originated in the interactions of the tyres, namely, the stability depends on whether the tyres can excite each other or not, which corresponds to the wave lengths of the tyre deformations. The results were published in a conference abstract [2] and in conference proceedings [3], [4]. A manuscript was also submitted to a journal (see [5]).

In the third year, a mechanical model of a towed wheel was extended in order to examine the stability of the cornering manoeuvre. The elasticity of the king pin was considered both for the longitudinal and lateral directions. Equations of motion were determined with taking into account the geometrical nonlinearities. It was proved by means of linear stability analysis and numerical bifurcation methods that self-excited vibrations can emerge at lower speed in case of cornering than in case of rectilinear motion. The results were published in a conference proceeding [6], and in a conference abstract [7].

2 Tyre models with other regenerative effects

First, a very simple tyre model was constructed to investigate the regenerative effect of the tyre carcass on the dynamics of shimmying wheels. The mass of the tyre was neglected, and the tyre threads were modelled as massless continuum element supported by distributed spring and damper. So, the non-zero deformation at the trailing edge of the contact patch can reach the leading edge due to the exponential relaxation of the tyre thread deformation meantime the thread element travels along the circumference of the tyre. It was proved that the damping can have even disadvantageous effect on the stability at very high speeds. Thus, a critical damping was determined for the investigated parameter set-up, which can guarantee the highest critical speed, over it shimmy occurs. The results were published in [8].

The mechanical model of the tyre carcass was improved by taking into account the distributed mass of the tyre. In this model the thread elements can also vibrate after they leave the contact patch at the trailing edge. Since the tyre carcass realizes a moving continuum in the caster-fixed coordinate system, the description of the carcass deformation requires special attention. The equations of motion for the carcass deformation were derived by different methods: by means of Newton's second law and Hamilton's principle.

For zero damping, the travelling wave solution of the tyre carcass deformation was constructed by means of the Duhamel-Neumann formula even for the case when the inertial effects (e.g. the Coriolis force in the moving caster-fixed system) are not neglected. By means of the travelling wave solutions for the lateral deformations of the contact patch and the tyre carcass, a system of DDEs can be composed, which make the analytical calculations to be more manageable. The characteristic exponents of the linearized system were calculated and the parameter domains of self-excited vibrations were identified. Two different suspensions of the tyre were investigated: a caster-wheel system and a laterally elastic wheel suspension. In both models, unknown critical parameter domains were discovered, but the results on the caster-wheel system showed more fascinating dynamical properties. It was proved that the vibration of the tyre carcass can appear in a wide speed range. Of course, if the damping effect of the tyre was also implemented in the model, these domains became much irrelevant. Nevertheless, in point of view of the noise generation of the tyre, the detected critical domain can be very important since the road surface roughness can excite the detected vibration modes even in the cases when the corresponding characteristic exponents are situated on the left half of the complex plane. In the work plan of the project, experiments were planned to capture the suspected self-excited vibrations but the theoretical results proved that the vibration amplitude are very tiny, and noise measurements in silence room can only validate the results. This would have required such expensive laboratory experiments, which was not planned in the project proposal, and consequently, it could not be financialized by this project. So, the experimental results of other researcher were used to check the weaknesses of the developed tyre carcass model.

The results were published in conference proceedings [9] and [10]. Based on these publications a journal paper is going to be written.

3 Rolling wheels and time delay

Another interesting mechanical model was also investigated in the project, namely, the skateboard-skater system was in question. The analysis of the dynamics of the skateboard is an exciting topic on the field of non-holonomic systems due to the special steering mechanism of the skateboard. The steering angle of rolling wheels is influenced by the tilting angle of the board, which gives interesting behaviour for the skateboard. Moreover, the tilting angle of the board is controlled by the skater, and the reflex delay of the human rider makes the system to be a delayed non-holonomic system. Previously, the effect of the reflex delay was not considered in any model of the skateboard and there is no validated explanation for the lateral instability of the skateboard at higher speeds however this phenomenon is well-known.

A mechanical model of the skateboard-skater system was constructed. A delayed PD controller was implemented and linear stability analysis was performed. The effect of the reflex delay was investigated, and an ultimate time delay was calculated over it the rectilinear motion cannot be stabilized for any speeds. It was established that skateboarding can be performed with larger time delays in certain speed ranges than the simple human balancing. It was proved that the skater has to tune the control parameters (i.e. the control gains P and D) as the speed of the skateboard changes. It was also established that in some speed ranges the skateboard is stable without control. Namely, even at high speeds, the skater could switch off the control loop; and by this, the negative effect of the reflex delay in the control loop can be eliminated.

An enhanced mechanical model of the skateboard-skater system was constructed as well, in which different control strategies were implemented. Various stability charts were determined and such differences were identified with respect to the control strategies, which can help to obtain some information about the human control by means of experimental tests.

Different parts of the results on this topic were published on conferences ([11], [12], [13], [14], [15]). Two journal papers are going to be submitted.

Publications

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