Final report on

the research project OTKA K 105433 titled

Stabilization of unstable dynamical systems by delayed feedback controllers

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Results in publications and projects

In the 4-year period of the project, **25 journal articles (sum impact factor: 36,07), 20 conference proceeding papers and 2 book chapters** were published with the contribution of the members of the OTKA project. Furthermore 12 other conference presentations were given with an abstract of 2 page or shorter. In the list of publications we gave only the final publication of the results (for instance, if a conference proceedings paper was published later as a journal article, then we gave only the journal publication). The list of publication contains 23 journal articles, 12 conference papers and 2 book chapters.

The members of the research group submitted **3 PhD theses**. Daniel Bachrathy received his PhD in 2013, Ambrus Zelei received his PhD in 2015 (although these PhD theses are not entirely associated to the current OTKA project, the project definitely helped in the final stage of their PhD work and in the preparation of the theses). David Lehotzky submitted his PhD dissertaion in 2016 and the PhD defense will be in 2017. Furthermore, the project leader Tamas Insperger received the **title of Doctor of Hungarian Academy of Sciences** in 2015.

The project leader, Tamas Insperger, gave a **plenary talk** at the XII. Hungarian Conference on Mechanics with the title Dynamic models of human balancing and gave a 60-minute talk as **invited speaker** at Workshop on systems with delays and noise in Nagoya (costs for this latter workshop was fully covered by the Japanese organizers) in 2015.

The project leader, Tamas Insperger, organized three minisymposia during the project:
(1) Minisymposium on "Intermittent Balance Control with Reflex Delay" at the SIAM 2013 SIAM Conference on Applications of Dynamical Systems, Snowbird, Utah, USA, 2013.
(2) Minisymposium on "Systems with Time Delay" at the 8th European Nonlinear Dynamics Conference (ENOC 2014), Vienna, Austria, 2014.
(3) Minisymposium on "Machine Tool Chatter" at the 12th IFAC Workshop on Time Delay Systems, Ann Arbor, Michigan, USA, 2015.

As a partial outcome of the OTKA project, the project leader received the grant by the **MTA Lendület Programme** and established the MTA-BME Lendület Human Balancing Research Group.

Detailed results

During the 4-year project, the research group worked on the following tasks.

1, Stability analysis of time-delay systems

There exist several techniques for the stability analysis of autonomous (time-invariant) delay-differential equations. The interest in developing numerical methods for the stability analysis of delay-differential equations is reflected in the high number of journal papers published in this area in the last 15 years. In the project, we gave an overview of the existing methods, developed some new techniques and performed a comparison to other methods in the literature.

First, a special version of the method of the weighted residuals, called **pseudospectral tau approximation**, was developed and described in detail with special attention to the choice of the test functions and trial functions. It was shown that, for certain engineering problems, the convergence of the proposed method is better than those of the techniques available in the literature. The results were published in the prestigious International Journal of Numerical Methods in Engineering.

Another method, the least-square spectral element method was analysed in detail and presented as a conference paper at 12th IFAC Workshop on Time Delay Systems (Lehotzky D.). This method was found to have no advantages over other method (such as the above-mentioned pseudospectral tau approximation), therefore it was not elaborated further.

As a byproduct of the study of other existing numerical methods, the well-known spectral element method was extended to time-periodic delay-differential equations with multiple and distributed delays. The corresponding results were published in the Communications in Nonlinear Science and Numerical Simulation. The method was then also extended to systems with discontinuous time-periodicity in an efficient way in order to provide an efficient tool to the stability prediction of machine tool chatter. These results were published in International Journal of Advanced Manufacturing Technology.

Stability analysis of periodic delay differential equations by Magnus expansion was also investigated in the early stage of the project. The application of the Magnus expansion on periodic time-delayed differential equations is proposed, where an approximation technique of Chebyshev Spectral Continuous Time Approximation (CSCTA) is used to convert a system of delay-differential equations (DDEs) into a system of ordinary differential equations (ODEs). The solution of the resulting ODE was then obtained via the Magnus expansion. The stability and time response of this approach are investigated on two examples and compared with known results in the literature. This task was performed in cooperation with prof. Eric A. Butcher (New Mexico State University).

2, Stabilizability of unstable systems with feedback delay

There are many different concepts to stabilize unstable systems with feedback delays. One goal of the project was to determine the control algorithm, which is able to stabilize unstable open-loop system in the presence of maximum feedback delay.

One of the most well-known predictive controller is the **Smith predictor**. We performed a detailed time-domain analysis of the Smith predictor for a second-order plant with feedback delay. We demonstrated that the Smith predictor is sensitive to infinitesimal parameter mismatches between the internal model and the actual system. Furthermore, we showed that the original Smith predictor can stabilize unstable plants for some extremely detuned internal model parameters. Thus the general concept that the original Smith predictor is not capable to stabilize unstable systems is technically not true. The final results were published in the International Journal of Dynamics and Control.

Another predictive controller is the so called **finite spectrum assignment** (FSA). The FSA controller predicts the actual state of the system over the delay period using an internal model of the

real system. If the internal model is perfectly accurate then the feedback delay can be compensated. However, parameter mismatches of the internal model or implementation inaccuracies of the control law may result in an unstable control process. We analyzed the stabilizability of an undamped second-order system for different system and delay parameter mismatches with special attention to theoretical stability and robustness against implementation inaccuracies of the control law. We showed that, for small parameter uncertainties, the FSA controller allows stabilization for significantly larger feedback delays than a conventional delayed proportional-derivative-acceleration controller do. The final results were published the Journal of Vibration and Control.
Experimental analysis of the act-and-wait control concept was performed in case of digital effects and feedback delay. A 2-page abstract was written and a conference talk was given at the 8th European Nonlinear Dynamics Conference (Vienna, Austria, July 6-11, 2014). Currently, we are working on the journal publication of these results.

3, Machine tool chatter - Stability prediction

We have developed a new efficient frequency-domain technique to the stability analysis of time-periodic delayed dynamical systems. The method is based on higher-order approximation of Hill's infinite determinant. The parameter points, where the number of unstable Floquet multipliers changes, are computed by the Multi Dimensional Bisection Method. The stability boundary lines are selected by a numerical algorithm. The method can be used to determine the stability diagram for machining operations, and provides an alternative to the time-domain semi-discretization method. The results were presented at the ASME IDETC/CIE 2013 conference by Daniel Bachrathy.
Stability behavior of milling processes performed by conventional, variable helix and serrated milling tools was analyzed. A **general milling model** linked to a non-proportionally damped dynamic system was considered. Extended multi frequency solution and semi-discretization are implemented and used to calculate the stability of stationary milling. Measurements were performed in industrial environment and we validated that the general numerical algorithm is able to predict the stability conditions of milling processes carried out by cylindrical cutters of optional geometry. Both the calculations and the measurements confirm that, for roughing operations, the highest stability gain can be achieved by serrated cutters. It is also demonstrated that variable helix milling tools can achieve better stability behavior only if their geometry is optimized for the given cutting operation. The results were published in the CIRP Annals-Manufacturing.
Machine tool chatter arising in an interrupted turning process was investigated in a strong industrial context with a complex flexible part within the frame of an informal cooperation with French partners. A detailed analysis of the real cutting process is performed with special respect to the geometrical defects of the part in order to highlight the source of machine tool vibrations. The analysis is completed by simple models to estimate the forced vibrations in interrupted turning, the gyroscopic effect, and the mode coupling using a new simplified formulation. Stability analysis of this model is performed by the semi-discretization method. A sensitivity analysis shows the effect of the value and the orientation of the geometrical defects for low speed conditions. Then this result are extrapolated to high-speed conditions to look for possible new stable cutting conditions and shows a period doubling flip instability. The final results have been published in the International Journal of Advanced Manufacturing Technology.

4, Machine tool chatter - Control via digital controller

Machining operations subjected to digital control can be described by hybrid equations involving point-delay terms due to the surface regeneration and digital delay terms caused by the piecewise constant control force. Because of the digital controller, the system is time-periodic at the sampling period of the controller. We have developed an improved semi-discretization method (one and two-point methods) to the numerical stability analysis of these hybrid systems. We determined the stability charts of the delayed oscillator and the turning process for a digital PD controller. The

results were presented as a book chapter in the new series of Advances in Delays and Dynamics by Springer in 2014.
We have introduced the terminology of **stabilizability diagrams**, which presents the critical depth of cut, which limits the stabilizability of the machining process for a given spindle speed in the sense that machining operation at larger than the critical depth of cut cannot be stabilized by the applied digital controller for a fixed sampling period. We showed that the resulted stabilizability diagram shows some similarities to the traditional stability lobe diagram of machining processes. The final results were published the International Journal of Dynamics and Control.
A theoretical framework was developed to the chatter control for turning operations using the drives in the feed direction. The results were published as a Hungarian conference paper at the XII. Hungarian Conference on Mechanics (Lehotzky D.).

5, Machine tool chatter - Distributed delay models
In machine tool chatter models, the cutting force is often modelled as a concentrated force acting at the tool tip. Models, where the **cutting force is distributed** along the rake face of the tool are only available in the literature for turning operations. As generalization of these models, we developed models for milling operations, where the force distribution becomes also time-dependent due to the continuously entering and exiting cutting teeth. It was shown, that this, so-called short regenerative effect has a stabilizing effect on the process especially for low spindle speeds. This implies that the short regenerative effect may be an alternative explanation to the phenomenon called process damping. The results of the linear stability analysis were published in the journal Periodica Polytechnica and at XII. Hungarian Conference on Mechanics as a conference paper (Molnár T.).
Nonlinear analysis of the dynamics of orthogonal cutting operations with distributed cutting force was performed based on the available results related to concentrated cutting force. The results showed that the subcritical sense of the Hopf bifurcation arising along the linear stability boundaries is preserved for distributed-force models. The final results were published in the Journal of Computational and Nonlinear Dynamics.
A detailed model was developed, which combines the distributed cutting force with state-dependent delays. Since the contact length on the tool's rake face depends on the uncut chip thickness, the governing equations involve a distributed delay term with a state-dependent limit in the integral. It was shown that this state-dependency does not affect linear stability as opposed to existing models in the literature. Numerical analysis showed however that the global dynamics may strongly be affected: the subcritical bifurcations may turn to supercritical ones if this type of state-dependent distributed delay is involved into the model. The results have been published the journal Nonlinear Dynamics.
An improved estimation of the bifurcation curve and the **bistable zone** in turning processes was established by assuming some typical features of the system behavior with respect to the bifurcation parameter. The results were published in the journal Electronic Journal of Qualitative Theory of Differential Equations in 2016.

6, Machine tool chatter - Robust stability analysis
Although there are several efficient numerical techniques to the stability analysis of the governing equation of machine tool chatter, the predicted stability lobe diagrams often do not match experimental cutting tests. One reason for this is the uncertainties of the measured FRF. The effect of the modelling imperfections of turning operations on the stability of the machining process was analyzed using the semi-discretization method and the single-frequency solution and the results was published at the 12th IFAC Workshop on Time Delay Systems (Hajdu D.). Similar analysis for milling operations (where the multi-frequency solution was used) was published at the ASME 2015 International Design Engineering Technical Conferences (Hajdu D.).
Based on the combination of **pseudospectra and stability radii for time-delay** systems and the single frequency method for the stability analysis of machining operations, a new model was

developed for the robust stability analysis of machining operations with respect to the uncertainties of the measured FRF. The results were published in the International Journal of Advanced Manufacturing Technology.

7, Machine tool chatter - process damping effects

The stabilizing effect of process damping at low cutting speeds for regenerative machine tool vibrations of milling processes was investigated using detailed model of the relation between the cutting force and the chip thickness. Process damping is induced by a **velocity-dependent cutting force model**, which takes into account that the actual cutting velocity is different from the nominal one during machine tool vibrations. The chip thickness and the cutting force are calculated according to the direction of the actual cutting velocity. This results in an additional damping term in the governing delay-differential equation. In the literature, this term is often assumed to be constant and is considered to improve stability properties at low spindle speeds. Here, we showed that the velocity dependent cutting force model captures the improvement in the low-speed stability only for turning operations and milling with large radial immersion, while it results in a negative process damping term for low-immersion milling. Consequently, an extended process damping model is needed to explain the low-speed stability improvement for low radial immersion milling. The results were published in the International Journal of Advanced Manufacturing Technology.

8, Modelling the mechanism of human balancing

This task was performed in cooperation with prof. John Milton (Claremont Colleges, USA), who spent his sabbatical at the department of Applied Mechanics, BME between December 2013 and June 2014. The main goal of this research is to explore the control mechanism of the human nervous system for simple tasks such as stick balancing. Experimental observations show that human subjects cannot balance sticks of length shorter than 25-30 cm due to the relatively large **reaction delay**. We investigated the effects of **sensory input uncertainty** on the stability of time-delayed human motor control by calculating the minimum stick length that can be stabilized in the inverted position for a given reflex delay. We considered five control strategies often discussed in the context of human motor control: three time-invariant controllers (proportional-derivative, proportional-derivative-acceleration, model predictive) and two time-varying controllers (act-and-wait and intermittent predictive controllers). The uncertainties of the sensory input are modeled as a multiplicative term in the system output. We found that for a typical range of uncertainty the **model predictive controller** is the most robust controller. The results were published in the journal Biological Cybernetics.

We have also analyzed the condition for stabilization by some basic controllers, such as PD, PID, PDA and **PIDA controllers** for a simplified balancing model. We pointed out that while the acceleration feedback improves stabilizability (the critical time delay limiting stabilizability is increased by 40%), the integral term does not improve stabilizability at all. The results were published in the journal Biomechanica Hungarica.

Several articles in the literature related to human balancing applies an approximation of the delayed term by Taylor series expansion, therefore we made a short analysis about the justification of this approximation. We demonstrated through some simple second-order scalar systems that low-order Taylor series expansion of the delayed term approximates the asymptotic behavior of the original delayed system only for certain parameter regions, while for high-order expansions, the approximate system is unstable independently of the system parameters. The results were published in the Journal of Computational and Nonlinear Dynamics.

A simplified model of human balancing with special attention to the feedback delay and the **sensory dead zones** were analyzed in order to understand the basics of the balancing mechanism. Although Newtonian dynamics is described by second-order systems, here we reduced the analysis to a first-order system, which can be considered as a projection to the unstable manifold of the open-loop system. We showed that in addition to limit cycle and chaotic (“micro-chaos”) oscillations, transiently stabilized balance states are possible even though both the open-loop and the closed-loop systems are globally unstable. This result may open new section in human balance control. The possibility that human falls can be an intrinsic component of neural control of balance may provide new insights into how the risk of falling in the elderly can be minimized. The results were published in the SIAM Journal Applied Dynamical Systems.

We have also written a book chapter about the development of strategies to minimize the risk of falling. The corrective movements made by humans to maintain balance are small amplitude, intermittent and ballistic. Small amplitude, complex oscillations (micro-chaos) frequently arise in industrial settings when a time-delayed digital processor attempts to stabilize an unstable equilibrium. Taken together these observations motivate considerations of the effects of a sensory threshold on the stabilization of an inverted pendulum by time-delayed feedback. In the resulting switching-type delay differential equations, the sensory threshold is a strong small-scale nonlinearity which has no effect on large-scale stabilization, but may produce complex, small amplitude dynamics including limit cycle oscillations and micro-chaos. We explored the mathematical relationship between a scalar model for balance control and the micro-chaotic map that arises in considerations of digitally controlled machines. We have published these results as a chapter in the book *Mathematical approaches to biological systems: Networks, oscillations and collective phenomena* edited by prof. T. Ohira and prof. T. Ozawa.

New models were developed for stick balancing, which take into account the inertia of the human arm, the sensory dead zones and the physiological limitations of the human arm in the form of constraints to the **maximum acceleration and the maximum jerk**. The results were published in the prestigious Journal of the Royal Society Interface.

A new approach, namely, **robust stability of human mediolateral balance** control was also investigated by the research group. The main challenge was the application of the concept of pseudospectra and stability radii for real-valued structured perturbation of certain parameters in the model. Previous results in the literature were available only for complex-valued unstructured perturbations. The analysis ended up in a surprising result: narrow stance widths are more robust to parameter variations than wide ones. The results were published in the IEEE Transactions on Neural Systems and Rehabiliation Engineering.

9. Connected cruise control of vehicular strings

The similarity between the governing equations of delayed feedback systems and the traffic control in case of connected vehicles opened a new area for the members of the research group: control of vehicle platoons. If the vehicles are connected by wireless information transmission, then the process delay is of key importance. If the human drivers directly control their own vehicle then the human reflex delay should be involved into the model. We analyzed a model where a predictor is used to estimate the actual signals in case of loss of information (e.g., due to packet loss). In this model no significant improvement was found, which can be attributed to the fact that so-called **string stability** (magnifying effect of the accelerations along vehicle string) plays a more important role than **plant stability** (self-excited vibrations between vehicles). The results were published at 12th IFAC Workshop on Time Delay Systems (Molnar T) and a journal paper is under preparation.