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The importance of research of large finite structures can be hardly overestimated in the age of computers. Finite mathematics has plenty of applications in coding theory, discrete optimization, communication, scheduling, complexity of computations, theory of algorithms and in general in computer sciences.

In this proposal the PI and his colleagues considered the problem of finding some regular substructures in graphs, hypergraphs and in geometrical structures. These questions are called Ramsey and Turan type problems. Traditionally it is one of the strongest area of research in Hungary, it is sufficient to mention the name of Denes Konig, Paul Erdos, or Endre Szemeredi and Laszlo Lovasz.

A summary of the activities supported by the grant:

This was a successful project.

During this time the PI was invited 7 times to Oberwolfach Workshops (one of the most prestigious invitation only workshops). Furedi maintains a very extensive research activity, published several results.

Many manuscripts completed in this project have already been appeared, or accepted mainly in the most prestigious journals of the field like Journal of Combinatorial Theory or the Combinatorics, Computing and Probability, Discrete Mathematics.

A few of the results achieved:

1. A modern proof of the Erdos-Simonovits stability result. The original theorem is a typical stability result, it states that if the size of an F-free graph is close to the optimal then its structure is close to the extremal graph, i.e., to the Turan graph. There were many rather technical proofs by Bollobas, Bondy, Chvatal, Szemeredi, Ishigami and others so the short manuscript [33] is a result of 50 years of research. It builds on the insight of Erdos and then applies Szemeredi's regularity. It is a 'book proof', recently Sergey Norin (an acknowledge researcher on the field) called the PI's result 'the nicest proof he read in the last two years'.

2. The classical Erdos-Gallai theorems from 1959 states that the most edges in n-vertex graphs not containing a k-vertex path has at most (k-2)n/2 edges. This result was one of the sources of the famous Erdos-Sos conjecture for trees. These were sharpened later by Faudree and Schelp, Woodall, and Kopylov to two-connected graphs.

The PI (with coauthors A. Kostochka, and J. Verstraete) [47] proved a stability version.

This was such an unexpected result (with a highly nontrivial proof) that the 32 page manuscript had been invited into the 50th year anniversary volume of the Journal of Combinatorial Theory.

3. The PI (with coauthor Z Maleki from Isfahan University) gave an asymptotic formula for the minimum number of edges contained in triangles among graphs with n vertices and e edges. The main tool was a generalization of Zykov's symmetrization method that can be applied to several graphs simultaneously.

These result already generated much further research. Gruslys and Letzter, using a refined version of the symmetrization method, proved our conjecture for all for all n > n_0. Grzesik, P. Hu, and Volec, using Razborov's flag algebra method extended the results for all odd cycles. There certainly will be more follow up papers.

4. The PI (with Ida Kantor from Charles University, Prague) showed that almost every graph on n vertices is an induced subgraph of some Kneser graph of rank k, such that with high probability $c_1 n/(\log n) < k < c_2 n/(\log n)$.

Graph representations are important, (e.g., in data compression) and this result is related to a result of Frieze and Reed concerning the clique cover number of random graphs.

5. The PI (with coauthor Tao Jiang) in a series of papers [22, 23, 34] verified the Erdos-Sos-Kalai conjecture concerning the Turan number of trees for a wide class of configurations. These involve the most interesting cases like (loose) paths and linear cycles. Let us note that finding a cycle is a considerable more difficult task than the case of paths.

6. Professor A. Gyarfas continued to achieve significant Ramsey type results concerning hypergraphs, especially triple systems. Questions about the structure of hypergraphs without linear cycles (paper [40]) were investigated further by Ervin Gyori and his students in Central European University. There is significant follow-up activity on problems discussed in papers [5], [11], [42] on χ-bounded families.

The Ramsey type problems studied in papers [3],[17],[13] and surveyed in [38] attracted many researchers.

7. Dr G. Sarkozy applies the most advanced methods, versions of the Szemeredi regularity lemma he proves for a wide variety of graph Ramsey problems. One of the most interesting ones are achieved by A. Gyarfas concerning the Ramsey number of large monochromatic matchings, and large connected subgraphs.

For example, he determined asymptotically the two color Ramsey numbers for bipartite graphs with small bandwidth and constant maximum degree and the three color Ramsey numbers for balanced bipartite graphs with small bandwidth and constant maximum degree.

8. Dr Ruszinko (with coauthors in [31]) introduced a random graph model on the planar lattice points, which is a combination of a fixed torus grid and some additional random edges. The random edges are called long, and the probability of having a long edge between vertices u and v with graph distance d on the torus grid is c/Nd, where c is some constant. We show that, (with high probability), that the order of the diameter is $\log N$.

Moreover, we consider non-monotonous bootstrap percolation on the same model.

We prove the presence of phase transitions in mean-field approximation and provide fairly sharp bounds on the error of the critical parameters. Our model addresses interesting mathematical questions on non-monotonous bootstrap percolation, and it is motivated by recent results in brain research, too.

All of the above mentioned works are open accessed, mainly had been put on arXiv.org.