## Final report on the NKFIH project no. 104178

The results of the project were published in 74 papers in prestigious international journals such as Ann. Math., Adv. Math., Trans. Amer. Math. Soc., Ergodic Theory and Dynamical Systems, C. R. Math. Acad. Sci. Paris, Israel J. Math., Proc. Cambridge Math. Philos. Soc., etc., from which the very first one is the most prestigious mathematics journal. The researchers reported on the results at numerous international conferences, many times as plenary or invited speakers, and András Máthé was invited to the International Congress of Mathematicians 2018, which is one of the highest honours a mathematician can achieve.

Numerous papers solved long-standing open problems, the most famous of which is clearly the circle-squaring using measurable pieces, but members of our group also answered a question of the Bolyai Prize winner S. Shelah, and settled a 80-year-old problem of Kolmogorov.

### First we describe the results of the last year of the project

Z. Buczolich answered a question raised by C. Cuny and M. Weber by showing that omega(n), the number of distinct prime factors of n, and Omega(n), the number of distinct prime factors of n counted according to multiplicity are good weighting functions for the pointwise ergodic theorems in L^1.

Z. Buczolich and G. Keszthelyi studied properties of the skew tent map. They investigated the connection between the Ljapunov exponent and the entropy. Ljapunov exponent is usually hard to calculate, but using the implicit derivative of isentropes makes it much easier. It has turned out that Ljapunov exponent is not constant on the isentropes (unlike entropy). Also the relation between the number of absolutely continuous invariant measures of the tent map and the existence of a Markov partition was studied.

T. Keleti (with P. Smerkin) proved new lower bounds on the dimensions of distance sets of planar sets. Falconer's long standing Distance Set Conjecture states in the plane that any Borel set of Hausdorff dimension at least 1 has distance set of Hausdorff dimension 1. The best estimate for such a distance set is due to J. Bourgain, who proved that its Hausdorff dimension is at least 1/2+epsilon for some very small epsilon, and no better estimate was known even if we assume that the given Borel set has Hausdorff dimension strictly larger than 1. One of the main results obtained by Keleti and Shmerkin states that any planer Borel set of Hausdorff dimension larger than 1 has distance set of Hausdorff dimension larger than 37/54.

M. Elekes and D. Nagy wrote a long survey paper about Christensen's notion of Haar null subsets of (not necessarily locally compact) Polish groups. The paper also presents several recently introduced ideas, including the dual notion of Haar meager sets. They proved some results in a more generality than the original versions, and proved some results for Haar meager sets which had previously only been known for Haar null sets.

M. Elekes, V. Kiss and Z. Vidnyánszky (with U. B. Darji and K. Kalina) used Christensen's notion of Haar null sets to investigate the structure of the random element in various homeomorphism groups. They gave a characterization of the non-Haar null conjugacy classes for the groups of order-preserving homeomorphisms of the interval and the circle. They also showed that apart from the classes of the multishifts, every conjugacy class is Haar null in the group of the unitary transformations of the separable, infinite-dimensional Hilbert space.

They also investigated the analogous result for automorphism groups of countable first-order structures, hence developing a dual theory to that of Kechris and Rosendal. They generalized theorems of Dougherty and Mycielski about $S_\infty$ to arbitrary automorphism groups of countable structures, isolating a new model theoretic property, the Cofinal Strong Amalgamation Property. A complete description of the non-Haar null conjugacy classes of the automorphism groups of $(\mathbb{Q}, <)$ and of the random graph was given, in fact, they proved that every non-Haar null class contains a translated copy of a non-empty portion of every compact set. As an application they affirmatively answered the question whether these groups can be written as the union of a meagre and a Haar null set.

Duparc introduced a two-player game for a self-map f of the Baire space in which Player II has a winning strategy iff f is Baire class 1. V. Kiss showed that an analogous game can be used to characterize Baire class 1 functions between arbitrary Polish spaces.

V. Kiss (with L. Levine and L. Tóthmérész) generalized a result of L. Levine to show that the scaled activity diagrams for a sequence $G_n = G(n, p)$ of Erdős-Rényi random graphs equipped with a suitable sequence of chip-distributions converge to a devil's staircase, almost surely. For the proof to go through, they introduced parallel chip-firing on graphons and used that the sequence $G_n = G(n, p)$ converges to the constant p graphon almost surely.

M. Elekes and M Poór determined several well-known cardinal invariants of the Haar null sets.

An n-variable associative function is called reducible if it can be written as a composition of a binary associative function. G. Kiss (with G. Somlai) summarized the known results when the function is defined on a totally ordered set and is nondecreasing. In their main result they showed that every associative idempotent and nondecreasing function is uniquely reducible.

G. Kiss provided some visual characterizations of associative quasitrivial nondecreasing operations on finite chains. He also gave a graphical interpretation of bisymmetric quasitrivial nondecreasing binary operations on finite chains. Finally, he estimated the number of functions belonging to the above classes.

G. Kiss (with E. Gselmann and Cs. Vincze) investigated polynomially linked functional equation introduced by B. Ebanks. By finding a one-to-one correspondence between some univariate and multivariate functions, they characterized the solutions of this functional equation. Applying spectral analysis and spectral synthesis they also provided an alternative characterization and they showed that all additive solutions of such equations are derivations of higher order.

Z. Vidnyánszky (with P. Komjáth, I. Leader, P. A. Russell, S. Shelah, and D. T. Soukup) continued the research on partition properties of the real numbers. Answering a question of N. Hindman, I. Leader and D. Strauss they proved that it is consistent that, under certain large cardinal assumptions, for any coloring of the reals with finitely many colors there exists an infinite X so that the coloring is constant on X+X.

Z. Vidnyánszky (with S. Todorcevic) investigated the notion of Borel chromatic numbers of Borel graphs, that is, the definable analogs of the classical notions. They proved that, unlike the case of uncountable Borel chromatic numbers, it is impossible to find a simply definable collection of Borel graphs so that a Borel graph has infinite Borel chromatic number if and only if it contains a homomorphic copy of a member of the collection. Their result, besides answering several open questions and having importance in the theory of Borel graphs, could potentially be used to exclude the existence of certain graph coloring algorithms.

**In the remaining part of the report we repeat the description of the results of the previous years**

A complex valued function defined on an Abelian group G is said to be a local polynomial, if its restriction to every finitely generated subgroup of G is a polynomial. M. Laczkovich proved that local spectral synthesis (that is, spectral synthesis using local polynomials instead of polynomials) holds on every Abelian group having countable torsion free rank. More precisely, there is a cardinal \omega _1 \le \kappa

$\le 2^\omega$ such that local spectral synthesis holds on an Abelian group G if and only if the torsion free rank of G is less than $\kappa$.

Let $\Omega$ be an uncountable and algebraically closed field. M. Laczkovich proved that every ideal of the polynomial ring $R=\Omega[x_1, x_2,... ]$ is the intersection of ideals of the form $\{f \in R : D(fg)(c)=0$ for every $g \in R\}$, where D is a differential operator of locally finite order, and c is a vector with values in $\Omega$. This result generalizes a classical theorem of Krull to polynomial rings of countably infinitely many variables.

G. Kiss and M. Laczkovich investigated the functional equation $\sum_{i=1}^n a_i f(b_i x+c_i y )=0$, where $a_i, b_i, c_i \in \mathbb{C}$, and the unknown function f is defined on the field $K=\mathbb{Q}(b_1,..., b_n, c_1,..., c_n)$. (It is easy to see that every solution on K can be extended to $\mathbb{C}$ as a solution.) Let $S_1$ denote the set of additive solutions defined on K. They proved that $S_1$ is spanned by $S_1 \cap \mathcal{D}$, where $\mathcal{D}$ is the set of the functions $\phi \circ D$, where $\phi$ is a field automorphism of
$\mathbb{C}$ and D is a differential operator on K. A linear functional equation is normal, if its solutions are generalized polynomials. Let S denote the set of solutions of a given normal equation defined on K. They also showed that S is spanned by $S\cap \mathcal{A}$, where $\mathcal{A}$ is the algebra generated by $\mathcal{D}$. This implies that if S is translation invariant, then spectral synthesis holds in S. The main ingredient of the proof is the observation that if V is a variety on the Abelian group $(K^* )^k$ under multiplication, and every function $F \in V$ is k-additive on $K^k$, then spectral synthesis holds in V. They gave several applications, and described the set of solutions of equations having some special properties (e.g. having algebraic coefficients etc.).

G. Kiss also considered a certain class of linear functional equations. He investigated the case when the coefficients of the parameters are algebraic numbers.

G. Keszthelyi and Z. Buczolich worked on their joint paper "Monotonicity of equi-topological entropy curves for skew tent maps in the square". Unfortunately for certain parameter values unexpected difficulties showed up, and they still work on simplifying the necessary computations. Hopefully, the paper will be submitted in 2014.

Z. Buczolich (with S. Seuret) constructed measures supported in [0,1] with prescribed multifractal spectrum. Moreover, these measures are homogeneously multifractal, in the sense that their restriction on any subinterval has the same multifractal spectrum as the whole measure. They also found a surprising constraint on the multifractal spectrum of a HM measure: the support of its spectrum within

[0,1] must be an interval. This result is a sort of Darboux theorem for multifractal spectra of measures. This result is optimal, since they constructed a HM measure with spectrum supported on [0,1] union with the point 2. Using wavelet theory, they also built HM functions with prescribed multifractal spectrum.

More than 80 years ago Kolmogorov asked the following question. Let E be a measurable subset of a plane with finite Lebesgue measure. Is it possible to contract E to a polygon so that the loss of the measure is arbitrarily small? Richárd Balka, András Máthé and Márton Elekes answered this question in the negative by constructing a bounded, simply connected open counterexample. Their construction can easily be modified to yield the analogous result in higher dimensions.

G. Kiss (with G. Somlai) settled an old problem by proving that in every dimension except perhaps 1, 2 and 5 a ball can be decomposed into finitely many (but at least two) congruent pieces. They also found a linear bound for the minimal number of required pieces.

András Máthé has recently answered an old question of Alberti, Csörnyei and Preiss by showing that every purely unrectifiable Borel set in the plane can be covered by an open set which intersects the graph of every 1-Lipschitz function in arbitrarily small measure.

It is a classical deep result of Bourgain and Marstrand that if a subset of the plain contains circles around all points of the plane then the set must have positive Lebesgue measure. Tamás Keleti with Dániel Nagy and Pablo Shmerkin studied the case when instead of circles we have squares (more precisely the boundaries of axis-parallel squares) around each point of the plain. They noticed that in this case not only the Lebesgue measure can be zero but even the Hausdorff dimension can be one. Their main result is that the minimal possible Minkowski dimension of such a set is 7/4.

It is a well known fact that in non-locally compact Polish topological groups there is no Haar measure. However, Christensen showed that the notion of a Haar null set can be generalised. M. Elekes and Z. Vidnyánszky have answered an old question of Mycielski by proving that there exists a Haar null set without a $G_\delta$ hull. With the same method they also solved a problem from D. Fremlin’s problem list.

Richárd Balka and Márton Elekes (with U. B. Darji) studied topological properties of functions in C[0,1] with respect to the above notion of Haar nullness. The classical Bruckner-Garg Theorem characterizes the level sets of the generic function in C[0,1] from the topological point of view. They proved that the set of functions for which the Bruckner-Garg-type characterization holds is not Haar null, and its complement is not Haar null either.

By iterating the concept of topological Hausdorff dimension, U. B. Darji and M. Elekes defined the so called n-th inductive topological Hausdorff dimension. Richárd Balka proved that this is precisely the right notion to describe the Hausdorff dimension of the fibers of the generic continuous map from K to R^n for every compact metric space K and positive integer n. This generalizes a result of B. Kirchheim (case of K=[0,1]^m) and a result of R. Balka, Z. Buczolich and M. Elekes (case of n=1).

Following the work of Kuratowski, Laczkovich, Komjáth, Elekes-Steprans and Elekes-Kunen, M. Elekes and Z. Vidnyánszky investigated the possible order types of linear orderings consisting of Baire class 1 functions (i.e. pointwise limits of continuous functions) ordered by the pointwise ordering. They eventually managed to find the long-sought complete characterisation of these order types.

In their seminal paper A. Kechris and A. Louveau introduced three rank functions measuring the complexity of a Baire class 1 function. M. Elekes, V. Kiss and Z. Vidnyánszky managed to find very well-behaved generalisations in the case of the Baire class \alpha functions. As an application they answered a problem of Elekes and Laczkovich concerning the so called solvability cardinals, arising from paradoxical geometric decompositions. They also found that certain other very natural generalisations surprisingly turn out to be degenerate.

An old result of Zamfirescu says that for most convex curves C in the plane, most points lie on infinitely many normals to C, where most is meant in Baire category sense. M. Laczkovich (with I. Bárány) strengthened this result by showing that `infinitely many' can be replaced by `continuum many' in the statement. They also proved further theorems in the same spirit.

The classical Denjoy-Young-Saks theorem gives a relation, here termed as the Denjoy property between the Dini derivatives of an arbitrary one variable function that holds almost everywhere. Concerning the possible generalizations to higher dimensions, Besicovitch proved that there exists a continuous function of two variables such that at each point of a set of positive measure in some directions the bilateral Denjoy property is violated. M. Laczkovich (with Á. K. Matszangosz) showed that for two variable continuous functions it is possible that on a set of positive measure there exist directions in which even the one-sided Denjoy behaviour is violated.

Let B(t) be a linear Brownian motion and let a>1/2. R. Balka (with Y. Peres) proved that if A is a random set such that B: A->R is either a-Hölder continuous or of bounded variation (more specially, monotone), then the Hausdorff dimension of A is at most 1/2 almost surely.

R. Balka and M. Elekes (with Udayan B. Darji) investigated the Hausdorff (and packing) dimension of fibers and graphs of the prevalent continuous function f in $C(K,R^d)$, where K is an uncountable compact metric space. They proved that the prevalent f has many fibers with almost maximal Hausdorff dimension and its graph is of maximal Hausdorff dimension. This generalizes theorems of Dougherty, and Bayart and Heurteaux. They showed that for the prevalent f in C[0,1] the level set $f^{-1}(y)$ has Hausdorff dimension 1 for almost all y according to the occupation measure. They proved that it is not possible to replace occupation measure with Lebesgue measure in the above statement by generalizing a theorem of Antunovic, Burdzy, Peres and Ruscher.

R. Balka, Z. Buczolich and M. Elekes introduced a new concept of dimension for metric spaces, the so called topological Hausdorff dimension. They examined the basic properties of this new notion of dimension, compared it to other well-known notions, determined its value for some classical fractals, and also considered fractal percolation. They also show that the topological Hausdorff dimension is precisely the right notion to describe the Hausdorff dimension of the level sets of the generic continuous function defined on a compact metric space.

Z. Buczolich and G. Keszthelyi investigated the following problem: Suppose that we have a compact Abelian group G and a measurable function f defined on G. Suppose that the Birkhoff averages of f converge for a set of positive measure of group rotations. Does it imply that f is integrable? We showed that the answer is positive for locally connected groups, but negative if the dual group contains ``infinitely many multiple torsion". However, for the group of p-adic integers there is still a positive result.

Z. Buczolich (with S. Seuret) constructed a homogeneously multifractal measure with spectrum supported by the union of [0,1] and {2}. They also provided details of the construction of a strictly monotone increasing monohölder function which has exact Hölder exponent one at each point.

M. Elekes and T. Keleti considered decompositions of the real line into pairwise disjoint Borel pieces so that each piece is closed under addition, and they investigated the possible number of pieces. They found a model of mathematics in which this number can be strictly between countable and continuum.

T. Keleti (with D. T. Nagy and P. Shmerkin) continued their research and wrote up a paper on the following problem. Let A and B be two subsets of the plane such that B contains the boundary (or in another version the vertices) of an axis parallel square around every point of A. How small A can be if the size of B is given? Here size refers to one of cardinality, Hausdorff dimension, packing dimension, or box dimension, and it turned out that the answers are different for different dimensions.

T. Keleti wrote up his results on his following conjecture: If A is the union of line segments in R^n, and B is the union of the corresponding full lines then the Hausdorff dimensions of A and B agree. He showed that his conjecture would imply the Kakeya conjecture for Minkowski dimension, and he also proved that his conjecture holds in the plane.

G. Kiss investigated de Rahm-like curves from the multifractal analysis point of view in a joint work with B. Bárány and I. Kolossváry. They studied the local Hölder exponent of certain class of fractal curves, which can be considered as the natural generalization of de Rham-curves. They also gave a condition under which the curve is non-differentiable but the Lebesgue-typical Hölder-exponent is strictly bigger than 1.

G. Kiss continued his research concerning linear functional equations. In a joint paper with A. Varga and Cs. Vincze they determined the exact solutions in the cases when the number of coefficients is small.

V. Kiss (with L. Tóthmérész) answered a question of H. Lenstra by showing that computing the rank of a divisor on a graph is NP-hard. They translated this problem to a problem about chip-firing games using the duality between these frameworks discovered by Baker and Norine. It follows from their result that computing the rank of a divisor on a metric graph or on a tropical curve is also NP-hard.

V. Kiss and Z. Vidnyánszky disproved a conjecture of T. Keleti by constructing a set consisting of finitely many unit squares in the plane with perimeter-to-area ratio exceeding 4. They showed that the analogous conjecture is false for regular triangles as well.

A. Máthé (with L. Grabowski and O. Pikhurko) studied measurable equidecompositions; i.e. measurable variants of the Banach-Tarski paradox, Hilbert's third problem and Tarski's circle-squaring problem. Let us say that two sets are equidecomposable if one set can be cut it into finitely many pieces such that after moving/rotating the pieces they can be reassembled to get the other set. They showed that in the 3-dimensional space (and in higher dimensions) every two bounded measurable sets with non-empty interior are equidecomposable. They also showed that the disc in the plane is equidecomposable to a square with measurable pieces using translations only (extending M. Laczkovich's solution to Tarki's circle-squaring problem).

Just as for dense graphs, a limit object for finite partially ordered sets (posets) can be defined. A. Máthé (with J. Hladký, V. Patel and O. Pikhurko) answered a question of S. Janson about the limit object of posets. They gave an analytic and a combinatorial proof that every such limit object can be extended to be a total order (that is, every limit object can be embedded into the object of standard order on [0,1] equipped with the Lebesgue measure).

Z. Vidnyánszky (with B. Farkas and Y. Khomskii) investigated the complexity and forcing properties of certain ideals (i.e., families of sets closed downwards and under finite unions) on the natural numbers. They introduced the notion of a "mixing real" that seems to be a new type of generic real.

A result of Kechris and Louveau states that each real-valued bounded Baire class 1 function defined on a compact metric space can be written as an alternating sum of a decreasing countable transfinite sequence of upper semi-continuous functions. Moreover, the length of the shortest such sequence is essentially the same as the value of certain natural ranks they defined on the Baire class 1 functions. V. Kiss generalized this result to arbitrary Polish spaces. Also, as in a previous paper with M. Elekes and Z. Vidnyánszky, he used topology refinement methods to prove analogous statements about Baire class xi functions.

V. Kiss (with B. Hujter and L. Tóthmérész) analyzed the following problem: given two chip-distributions on a digraph, decide whether the first one can be reached from the second one by playing a legal chip-firing game. They showed that this problem can be decided in polynomial time for Eulerian digraphs, even if the digraph has multiple edges. They also showed that if the target distribution is recurrent, that is, it is reachable from itself, then the problem can be decided in polynomial time on a general digraph.

Z. Buczolich and G. Keszthelyi submitted their first paper concerning equi-topological entropy curves for skew tent maps in the square. In this paper they study analytic properties of the auxiliary function Theta. They show that close to (1,1) the equi-topologiacl entropy curves hit the diagonal almost perpendicularly. Answering a question of M. Misiurewicz they show that for the kneading sequence RLLRC these curves are not exactly orthogonal to the diagonal, though there examples like RLC when it is.

A 1-avoiding set is a set that does not contain pairs of points at distance 1. It is not known what the maximal density of such a subset of $R^n$ is. Erdős conjectured that in the plane it is strictly less than 1/4. With the additional but natural assumption that the 1-avoiding set displays block structure (i.e., is made up of blocks such that the distance between any two points from the same block is less than 1 and points from distinct blocks lie farther than 1 unit of distance apart from each other) T. Keleti, M. Matolcsi, F. M. d. O. Filho and I. Z. Ruzsa proved that the density is indeed strictly less than 1/4 (and less than $1/2^n$ in $R^n$). They also improved the current best estimate for the density of an arbitrary 1-avoiding subset of the plane by showing that it is always less than 0.258795.

M. Elekes and Z. Vidnyánszky continued their work on the regularity properties of Christensen's notion of Haar null sets. Using a method developed earlier they proved that certain naive modifications of this notion behave badly, namely these collections of sets are not closed under unions. This generalizes a result of Elekes and Steprans.

R. Balka determined various dimensions of graphs of a prevalent continuous map from a compact metric space K to $R^n$. In case of box dimension this generalizes a result of Gruslys, Jonusas, Mijovic, Ng, Olsen, and Petrykiewicz, while in case of packing dimension this extends a theorem of Balka, Darji, and Elekes. For Hausdorff dimension an easier proof is given for a theorem of Balka, Darji, and Elekes.

Given a sequence of n real numbers $\{S_i\}_{i<n}$, the longest weakly increasing subsequence is $i_1<i_2<...<i_L$ with $S_{i_k} \le S_{i_{k+1}}$ and L maximal. R. Balka (with O. Angel and Y. Peres) considered the case when $\{S_i\}_{i< n}$ is a random walk on R with increments of mean zero and finite (positive) variance. It is well known that the expected value of L is at least $cn^{1/2}$. Their main result is that the expected value of L is at most $n^{1/2+ o(1)}$, establishing the leading asymptotic behavior. In the case of a simple random walk they obtained the better lower bound $cn^{1/2} \log(n)$.

R. Balka and A. Máthé (with O. Angel and Y. Peres) proved restriction theorems for fractional Brownian motion. Let $B: [0,1]\to R$ be a fractional Brownian motion with Hurst index a. Almost surely, if B is of bounded variation on a time set A then the upper Minkowski dimension of A is at most $\max\{1-a,a\}$. Furthermore, almost surely, if B is b-Hölder continuous on a time set A for some $b>a$ then the upper Minkowski dimension of A is at most $1-a$. The above theorems are sharp and answer questions of Kahane and Katznelson.

Kaufman's famous dimension doubling theorem states that a two-dimensional Brownian motion almost surely doubles the Hausdorff dimension of all time sets A simultaneously, that is, the exceptional null set does not depend on A. R. Balka (with Y. Peres) considered whether the one-dimensional (fractional) Brownian motion defined on a deterministic, closed time set D almost surely doubles the Hausdorff/packing dimension of all subsets A of D simultaneously. The sets D for which the above statement holds are characterized in the case of packing dimension by introducing a new concept of fractal dimension. In case of Hausdorff dimension a sufficient condition is given, and the characterization is complete if D is sufficiently homogeneous (e.g. self-similar).

Following the work of Keleti and Elekes on sets which can be "measured" by translation invariant measures, A. Máthé showed that the union of two sets measured by general Hausdorff measures need not be measured by any translation invariant measure. Inspired by certain subgroups of Banach spaces, he also constructed a compact subset of the real line which is not a union of countably many measured sets.

A. Máthé (with M. Doležal and J. Hladký) studied random sampling of graphons and determined the (asymptotic almost sure) size of the largest clique in dense inhomogeneous random graphs.

A. Máthé (with S. Baker and J. Fraser) studied inhomogeneous self-similar sets (which satisfy a fixed point equation as self-similar sets but with an extra condensation set present). They gave sharp estimates of the upper box dimension of these sets in terms of the dimensions of the homogeneous counterpart and the condensation set.

A. Máthé constructed a measure in the three dimensional space which has two independent 1-dimensional Alberti representations but has no 2-dimensional Alberti representation. Roughly speaking, it is a measure which can be represented by Lipschitz curves in two different ways, but it cannot be represented by Lipschitz surfaces. This result is related to the theory of currents in geometric measure theory.

G. Kiss answered a question of Szostok related to functional equations which were motivated by approximate integration.

M. Elekes (with D. Soukup, L. Soukup and Z. Szentmiklóssy) extended results of Rado and answered questions of Rado, Gyárfás and Sárközy. Namely, they proved that if the edges of a complete k-uniform hypergraph on a countably infinite set are colored with r colors then the vertices can be partitioned into r monochromatic tight paths with distinct colors (a tight path is a sequence of distinct vertices such that every set of consecutive vertices forms an edge). They also showed that if the edges of a complete graph on omega_1 are colored with two colors then omega_1 can be partitioned into two monochromatic transfinite paths with distinct colors.

M. Elekes (with J. Steprans) investigated the so called cardinal invariants of the Hausdorff measures. They fit these invariants into the famous Cichon Diagram. As corollaries, they solved problems of Fremlin, Humke-Laczkovich, and Zapletal.

M. Elekes, V. Kiss and Z. Vidnyánszky (with U. B. Darji and K. Kalina) investigated the size of conjugacy classes of certain homeomorphism and automorphism groups with respect to Christensen's notion of Haar null sets. They gave a full description in several important groups, such as the homeomorphism group of the unit interval, the automorphism group of the random graph, or the rational numbers (with the usual ordering) etc. They also generalized the results of Dougherty and Mycielski.

K. Héra and M. Laczkovich investigated the Kakeya problem for circular arcs. They proved that if a circular arc has angle short enough, then it can be continuously moved to any prescribed position within a set of arbitrarily small area.

K. Héra and M. Laczkovich (with M. Csörnyei) studied another variation of the Kakeya problem. We say that a planar set A has the Kakeya property if there exist two different positions of A such that A can be continuously moved from the first position to the second within a set of arbitrarily small area. We prove that if A is closed and has the Kakeya property, then the union of the nontrivial connected components of A can be covered by a null set which is either the union of parallel lines or the union of concentric circles. In particular, if A is closed, connected and has the Kakeya property, then A can be covered by a line or a circle.

G. Kiss and M. Laczkovich (with Cs. Vincze) considered the discrete Pompeiu problem on the plane. A finite subset E of the Euclidean plane has the discrete Pompeiu property with respect to isometries (similarities), if, whenever a function f is such that the sum of the values of f on any congruent (similar) copy of E is zero, then f is identically zero. It is shown that every parallelogram and every quadrangle with rational coordinates has the discrete Pompeiu property w.r.t. isometries. The weighted version of the discrete Pompeiu property is investigated as well. It is shown that every finite linear set with commensurable distances has the weighted discrete Pompeiu property w.r.t. isometries, and every finite set has the weighted discrete Pompeiu property w.r.t. similarities.

Roslanowski and Shelah proved that there is a null, but non-meager subgroup of the Cantor group and of the reals, on the other hand it is consistent with ZFC that in these two groups every meager subgroup is null. They asked if this also holds in all locally compact groups. Using techniques related to Hilbert's fifth problem M. Poór answered this in the affirmative.

Z. Buczolich studied the singularity (multifractal) spectrum of the convex hull of the typical/generic continuous functions defined on the d-dimensional cube. This paper generalizes a result of A. M. Bruckner and J. Haussermann. It is shown that the generic functions coincide with their convex hull only on a set of zero Hausdorff dimension. On the boundary of the cube the Hölder exponent is zero. There is a d-1-dimensional set where the Hölder exponent is one. At most points the exponent is infinite.

Suppose that f belongs to a suitably defined complete metric space of Hölder alpha-functions defined on [0,1]. Z. Buczolich was interested in whether one can find large (in the sense of Hausdorff or lower/upper Minkowski dimension) sets A in [0, 1] such that the restriction of f on A is monotone, or convex/concave. Some of the results are about generic functions. This direction of research continues work of M. Elekes, A. Máthé and R. Balka also supported by this grant.

Z. Buczolich presents the construction of Hölder alpha functions f, 1<alpha<2 such that the restriction of f is not convex, nor concave on any set of upper Minkowski dimension larger than alpha-1.

K. Héra and T. Keleti (with M. Csörnyei and A. Chang) studied the minimal Hausdorff dimension of the union of scaled and/or rotated copies of the k-skeleton of a fixed polytope centered at the points of a given set. For many of these problems, they showed that a typical arrangement in the sense of Baire category gives minimal Hausdorff dimension. In particular, this proves a conjecture of R. Thornton. Their results also show that Nikodym sets are typical among all sets which contain, for every point x of R^n, a punctured hyperplane H\{x} through x. With similar methods they also constructed a Borel subset of R^n of Lebesgue measure zero containing a hyperplane at every positive distance from every point. T. Keleti wrote a survey paper that contains among others these results and also the results of a previous paper of the current project by T. Keleti, D. Nagy and P. Shmerkin.

K. Héra, T. Keleti and A. Máthé proved that for any 0<k<n and s at most 1 the union of any nonempty s-Hausdorff dimensional family of k-dimensional affine subspaces of R^n has Hausdorff dimension k+s. This generalizes a result of Falconer and Mattila who proved this for k=n-1.

M. Elekes and V. Kiss (with H. Nobrega) defined a new very natural rank for the Baire class 1 functions using infinite games, and investigated how this rank is related to the classical ones.

A. Máthé (with E. Csóka, L. Grabowski, O. Pikhurko and K. Tyros) proved a Borel version of the Lovász local lemma. That is, they showed that, under suitable assumptions, if the set of variables in the local lemma has a structure of a Borel space, then there exists a satisfying assignment which is a Borel function. The main tool developed for the proof is a parallel version of the Moser-Tardos algorithm which uses the same random bits to resample clauses that are far enough in the dependency graph.

In an earlier paper R. Balka, Z. Buczolich, and M. Elekes described the Hausdorff dimension of the level sets of a generic real-valued continuous function (in the sense of Baire category) defined on a compact metric space K by introducing the notion of topological Hausdorff dimension. Later on, R. Balka extended the theory for maps from K to $R^n$. As the main goal of the paper R. Balka generalized the relevant results for topological and packing dimensions and obtained new results for sufficiently homogeneous spaces K even in the case case of Hausdorff dimension.

Let $X_n$ be either a sequence of arbitrary random variables, or a martingale difference sequence, or a centered sequence with a suitable level of negative dependence. R. Balka (with T. Tómács) proved Baum-Katz type theorems by only assuming that the variables $X_n$ satisfy a uniform moment bound condition. They proved that this condition is best possible even for sequences of centered, independent random variables. This leads to Marcinkiewicz-Zygmund type strong laws of large numbers with estimate for the rate of convergence.

Let $X=(X_1,...,X_d)$ be a Gaussian random field from $R^n$ to $R^d$ such that $X_1,...,X_d$ are independent, centered Gaussian random fields with continuous sample paths. Let f be a Borel map from $R^n$ to $R^d$ and let A be an analytic set in $R^n$. As the main goal of the paper R. Balka determined the almost sure value of the packing dimension of the image and graph of X+f restricted to A under a very mild assumption. This generalizes a result of Du, Miao, Wu and Xiao, who calculated the packing dimension of X(A) if $X_1,...,X_d$ are independent copies of the same Gaussian random field $X_0$. Provided that X is a fractional Brownian motion, the result is new even if n=d=1 and f is continuous, and even if f=0 in the case of graphs. R. Balka also obtained the sharp lower bound for the packing dimension of the graph of a fractional Brownian motion X over A in terms of the Hurst index of X and the packing dimension of A. The analogous result for images was obtained by Talagrand and Xiao.

M. Elekes and D. Nagy collected the most important results about Haar null (in the sense of Christensen) and Haar meager sets into a survey paper. New results were also proved, and several known results were generalized.

D. Nagy continued to investigate a problem that was examined earlier in a paper of M. Elekes and Z. Vidnyánszky. This earlier paper had proved a result which states that (in any abelian Polish group) there exists a Haar null set (in the sense of Christensen) which cannot be embedded into a G_\delta Haar null set. Their construction used several methods which do not allow estimating the Borel class. In the case of the (abelian Polish) group Z^\omega Nagy was able to replace these methods and construct a G_\delta\sigma\delta Haar null set that is not contained in any G_delta Haar null set.

G. Kiss (with G. Somlai) generalized the characterization of associative, monotone, idempotent functions having neutral elements on the closed interval [0,1] proved by Martin, Mayor, Torrens for n-variable functions and for any nonempty interval. They also investigated generally the case when the underlying set is a totally ordered set.

G. Kiss (with B. Bárány and I. Kolossváry) investigated the pointwise regularity of zipper fractal curves generated by affine mappings. Under the assumption of dominated splitting they calculated the Hausdorff dimension of the level sets of the pointwise Hölder exponent for a subinterval of the spectrum. As an application they studied the local Hölder exponent of certain class of fractal curves, which can be considered as the natural generalization of de Rham curves. They also gave a condition under which the curve is non-differentiable but the Lebesgue-typical Hölder-exponent is strictly bigger than 1.

G. Kiss (with Cs. Vincze) considered a sufficient and necessary condition for the existence of inhomogeneous linear functional equations on the subfields of the complex numbers by applying spectral analysis on discrete Abelian groups. This work also contains the solution of Szostok's problem about the equations of approximative integration.

G. Kiss (with Cs. Vincze) studied whether it is possible to consider all of the solutions of inhomogeneous linear functional equations. They showed that if the functional equation is defined on a subfield of the complex numbers that have transcendence degree 1, then the all of the solutions can be given by an algebraic method from the parameters. If the transcendence degree is greater than 2, then there are equations which cannot be described only from the parameters, not even with knowledge of the automorphism solution of the equation. The method based on the application of spectral synthesis.