Research Report, NKFI-6; Project ID: 101845

Fitting multiple surfaces with geometric constraints

1. Problem statement

Reconstructing mechanical engineering parts from measured data is an important chapter of reverse engineering (digital shape reconstruction). A reconstructed CAD model must reflect the original design intent and accurately satisfy various engineering constraints for subsequent downstream applications. Unfortunately, naively reconstructed models will not be sufficiently good and further efforts are needed to `repair' these. Several problems may occur due to the noise of measured data, the tolerance-driven methods of mesh repair and segmentation, and the numerical nature of surface fitting. The surfaces will not be perfectly aligned or connected smoothly; perpendicularity and concentricity will be set only by loose tolerances and so on.

The technique to overcome these difficulties is *constrained fitting*. Assume that after some preliminary surface fitting the most likely engineering constraints have already been detected. Then another round of fitting is performed to *re-approximate* simultaneously *multiple sets* of point regions, while constraints are enforced. Take, for example, a best-fit cylinder with an axis direction (0; 0.06; 0.97) and a radius 49:8. Having detected two related constraints, the given data will be re-approximated by a constrained cylinder, having a perfect axis (0; 0; 1) and radius 50.

The formal definition of the mathematical problem – with some simplification – is the following. Given a set of surfaces *s in S* and a vector *a* containing all associated parameters. Each surface is going to approximate a set of points denoted by P_s, and the importance of the surfaces is weighted individually (w_s). The fitting equation can be written as

$$f(a) = \sum_{s \in S} w_s \sum_{p \in P_s} d(p,s)^2$$

and the constraint equation is $c(a) = 0$. d denotes the distance between a data points and a surface. The elements of the unknown parameter vector *a* involve all surfaces, and once the system of equation is solved it fully determines all corresponding surface geometries. We need to find a solution *a*, that minimizes *f* while $c=0$. The standard solution would be that of the Lagrangian multipliers with $n+k$ equations leading to a multidimensional Newton-Raphson iteration, however, in the general case the constraints may contradict or may not be independent. This is why a special alternative method was proposed (see earlier [Benkő et al, 2002]) solving $c(a) = 0$ and $f(a) = min$ simultaneously by iteration applying linear approximation for *c* and quadratic for *f*.

2. Results

In the intermediate reports we have already described how the project progressed in details. Here we just picked the most important results, and also supplemented figures in the Appendices to demonstrate various aspects of the constrained fitting project.

1. The key of the whole project was to develop a technique by means of which *large constraint systems* with various curve and surface geometries can be defined and solved efficiently. Details can be found in our papers.

2. The application of *auxiliary elements* is a key concept in this project; by means of them the system of equations can be drastically simplified and the solution becomes computationally more efficient. Auxiliary elements include unknown single points, points with direction vectors, points with parameter values, curves with normal sweeps, and curves with parameterizations, and so on. A simple example of joining three arcs represented by measured data is shown in Appendix A. Their smooth connection was achieved by constrained fitting and using auxiliaries.

3. An important part of the project was to develop a technique where *hypothetical constraints* can be automatically included into the constraint system, or just disregarded. For example, setting an angular tolerance of one degree, all surfaces where the preliminary fitting produced an angle between 89 and 91 will constitute a constraint equation in the system, and the related surfaces will be forced to be accurately perpendicular.

4. An interesting chapter of our research was to detect and enforce *global constraints*, such as best-fit (aligned) coordinate systems, construction grids, symmetry planes, etc. The main difficulty here is that these global properties can be detected only in "approximate" sense due to the inaccuracies, moreover, these typically relate only to certain subparts of the model. Examples can be found Appendix B.

5. Representing multi-sided free-form surfaces by means of boundaries and cross-derivatives is a tough problem with lots of components including the domain, the blending functions, the parameterization of the ribbon interpolants and so on. We have investigated this problem area and our related publications were successfully received at international forums. Our results include various formulations that define transfinite and control point based multi-sided patches. Two examples are shown in Appendix C.

6. *Constrained mesh parameterization* was also an interesting chapter. Here we wish to parameterize, with other words *flatten*, a 3D triangular mesh in such a way, that certain geometric features are retained in 2D. Examples include mapping to a straight line, preserving planar curves to be identical with their 3D image after flattening, preserving developable subregions, and so on. Some examples are given in Appendix D.

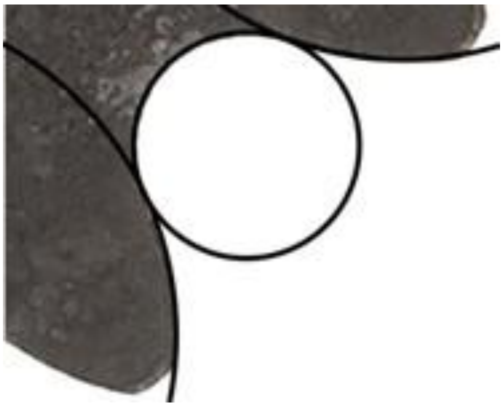
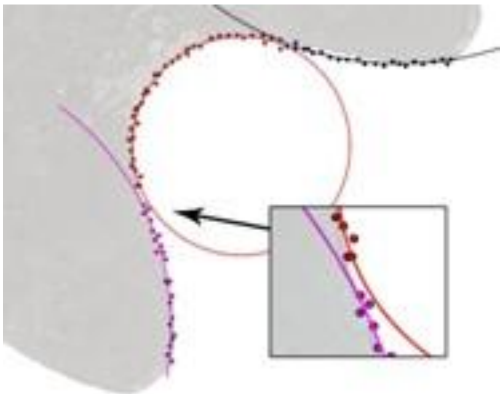
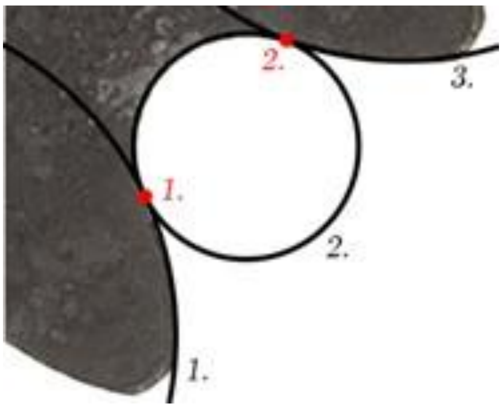
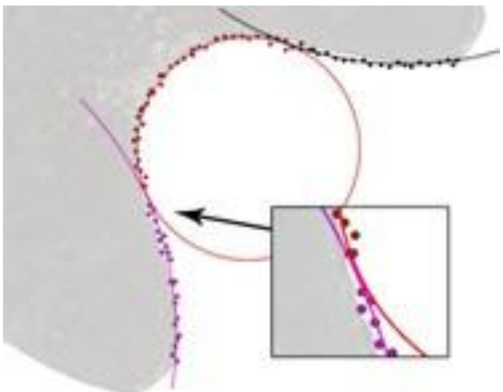
7. Applying *constraints for free-form curves and surfaces* has been a totally unexplored research area, and the project produced exciting new results. The basic difficulty is that the geometric entities where the constraints are enforced may be unknown; for example, we may require that two surfaces will intersect with 90 degree along their unknown intersection curve, then the computation of the actual intersection curve and the constraint will be satisfied simultaneously by appropriately modifying the control points of the preliminary surfaces. A nice example of enforcing a smooth connection along an unknown curve between an independently fitted cylinder and an overlapping free-form surface can be found in Appendix E.

8. This project required a very significant *3D program development*. The constrained fitting techniques must have been demonstrated in 3D with an advanced graphical user interface and analyzed by computer programs of high complexity. We have made serious efforts to implement this test environment and ran lots of tests using simulated and measured data sets. This system produced many of the figures in our papers, and also the figures in the Appendices.

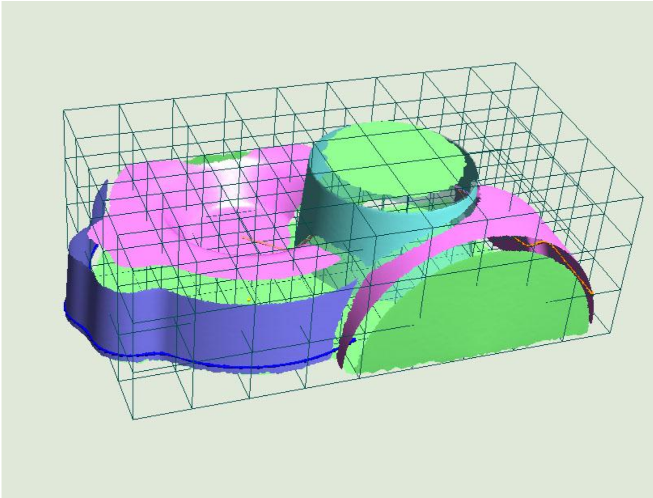
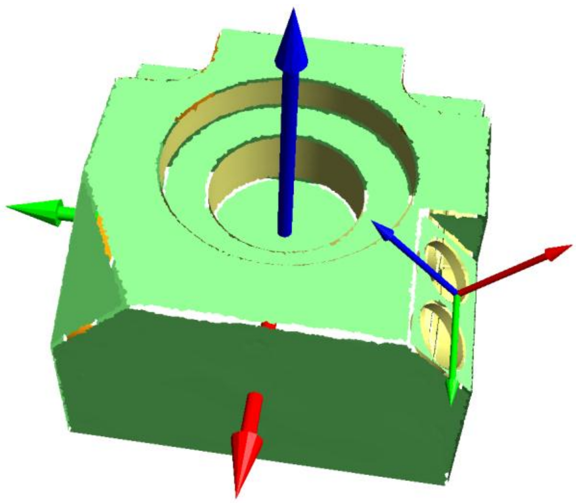
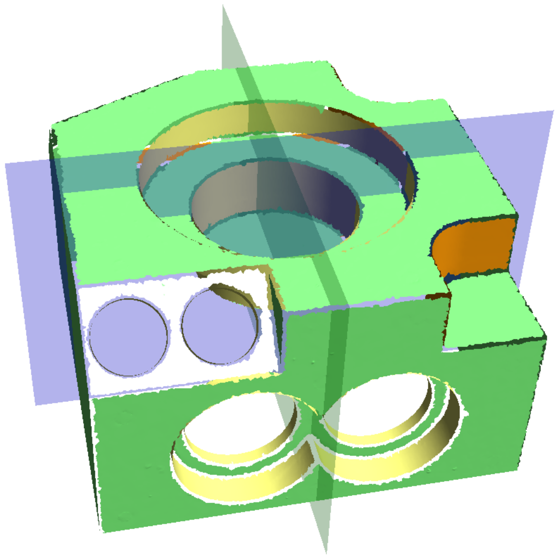
3. Publications

A detailed list of our publications can be found elsewhere in this closing document. Altogether 26 papers were published: 5 papers in leading technical journals with impact factor including Computer Aided Design, Computer Aided Geometric Design, Graphical Models and Computer Graphics Forum (Total IF: 6.335); 5 international conference papers; 14 domestic conference papers and 2 TDK papers (student research competition). The publication of our results has been completed in 2016, except one journal paper being under submission to the CAD Journal.

Appendix A: Auxiliary elements and hypothetical constraint satisfaction

<table>
<tr><td colspan="2"></td></tr>
<tr><td colspan="2">A simple three-arc example for constrained fitting with auxiliary elements</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Three ideal arcs</td><td>Three arcs fitted independently</td></tr>
<tr><td>
</td><td></td></tr>
<tr><td>Three arcs with positional and tangential constraints</td><td>Three arcs with constraints enforced; auxiliary elements for the common point and tangent were applied</td></tr>
</table>

Appendix B: Global constraints

<table>
<tr><td colspan="2"></td></tr>
<tr><td colspan="2">Computing a best-fit construction grid</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Computing best-aligned coordinate systems</td><td>Computing best-fit symmetry planes</td></tr>
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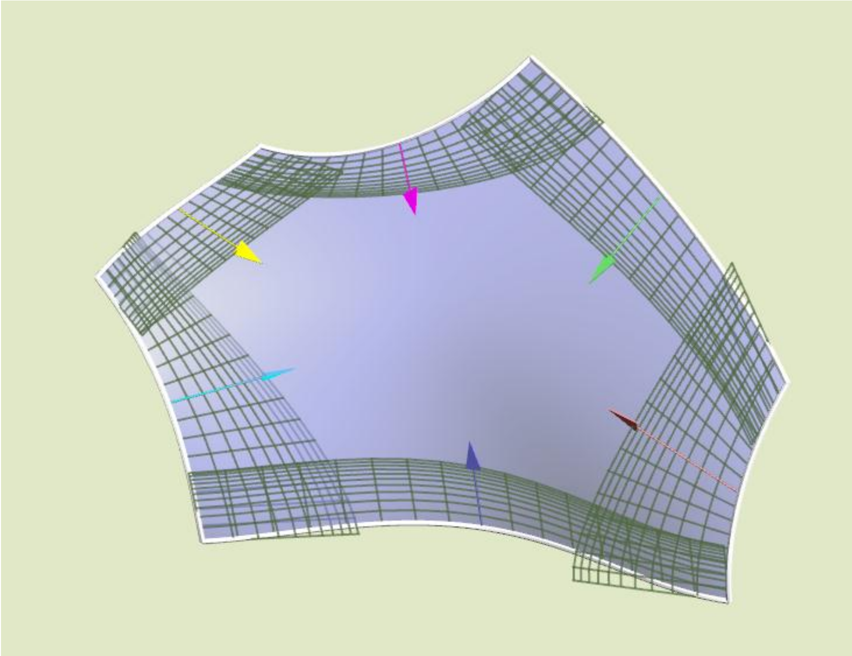
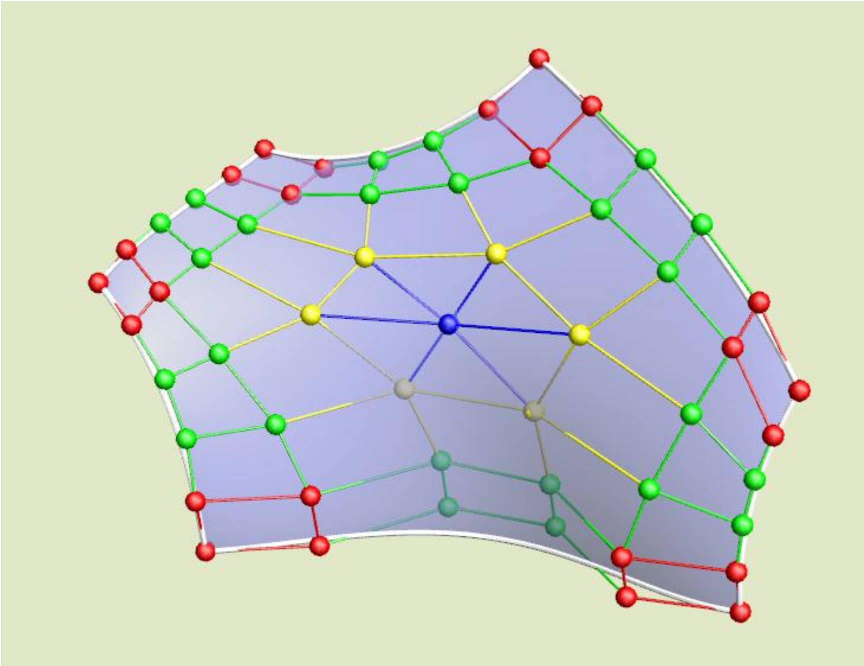
Appendix C: Multi-sided patches

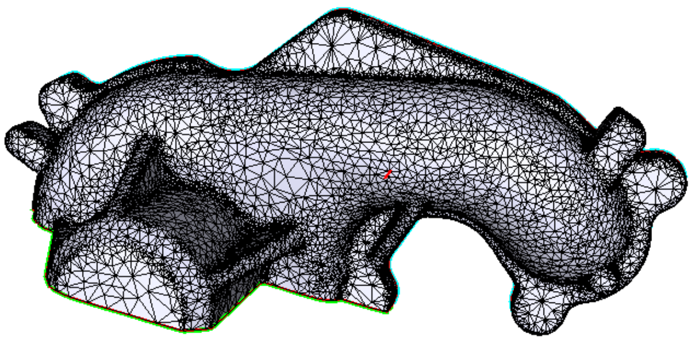
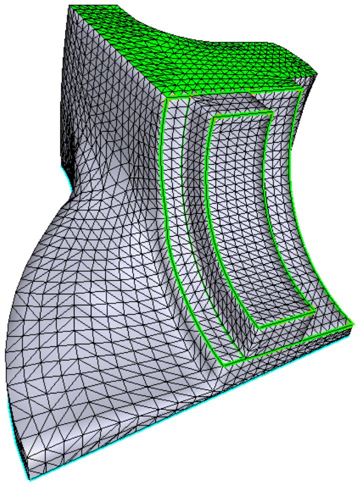
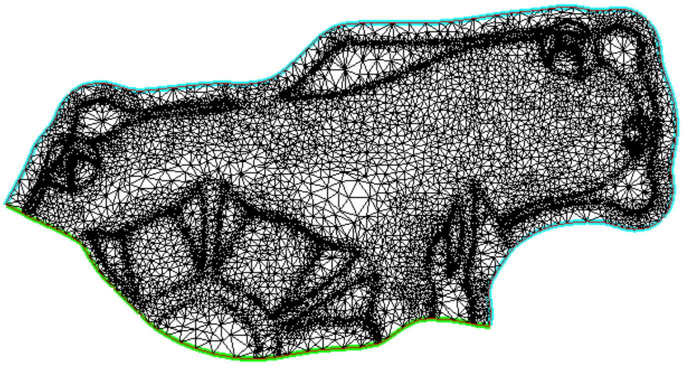
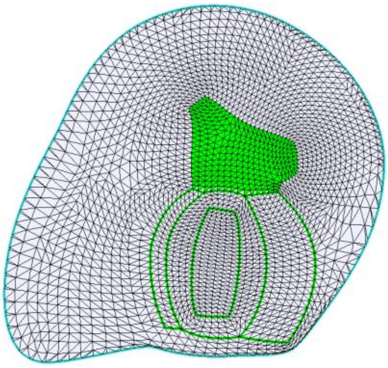
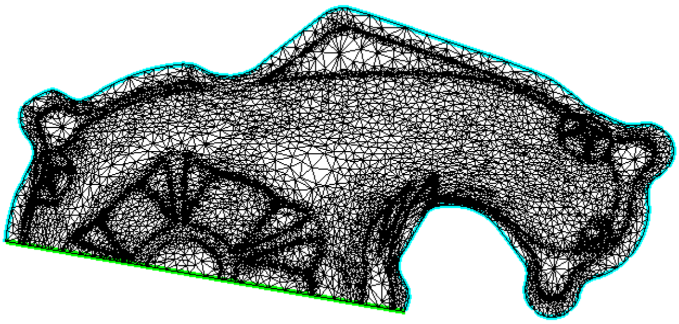
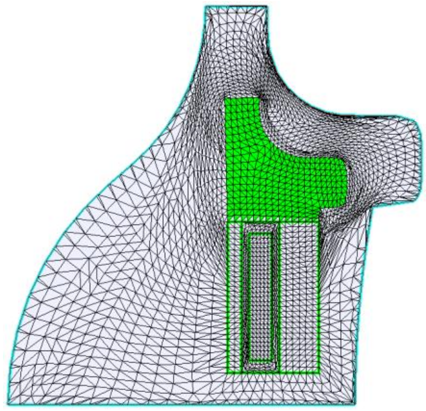


Multisided patch based on cross-derivative ribbons

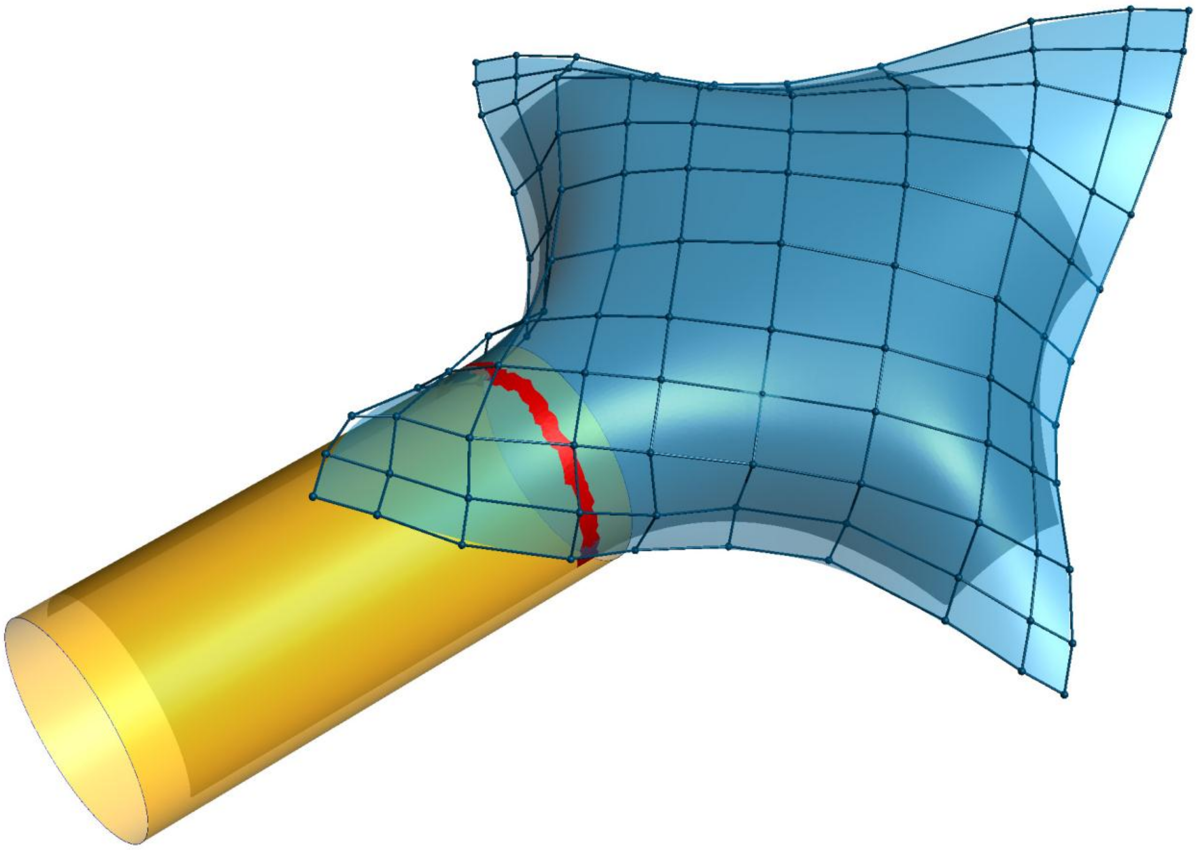


Multi-sided patch based on a Bézier-like control grid

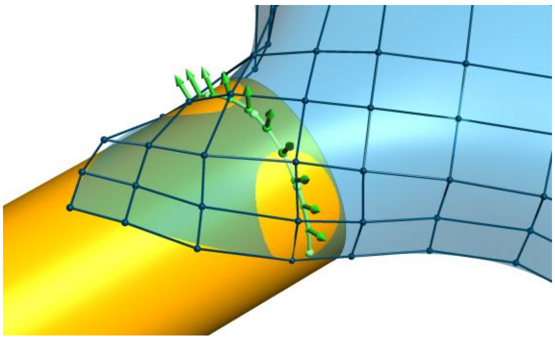
Appendix D: Mesh-parameterization with constraints

	
3D Test object A	3D Test object B
	
Flattening - no constraints	Flattening - no constraints
	
Flattening with constraints: retain feature curves, i.e. the profile (blue) in the symmetry plane and a straight boundary (green)	Flattening with constraints: retain feature curves and a selected area

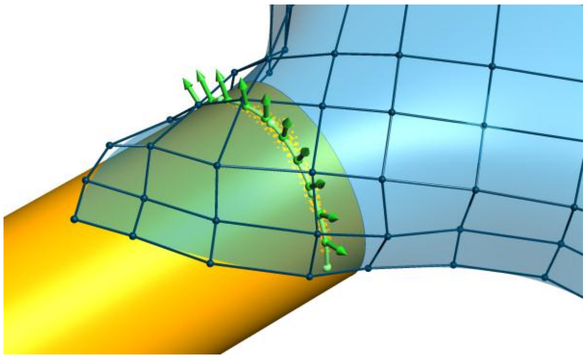
Appendix E: Free-form fitting with constraints



A cylinder and a free-form surface fitted to a segmented point-cloud independently



Two surfaces without constraints



Two surfaces with a tangential constraint enforced