Summary of research related to OTKA grant K101244

We have carried out the following research during the past 4.5 years:

A. One dimensional Dirac electrons:

1. Topological states: The topological properties of matter can be altered by applying a time-periodic perturbation, and inducing a Floquet-band structure, the temporal analogue of Bloch states. Using this method, we have investigated the fate of the edge state of a quantum spin Hall insulator in circularly polarized electromagnetic field. A topological phase transition was identified with increasing frequency from the topologically protected Thouless' charge pumping region to dissipative charge transport with increasing frequency. Several experimental proposals were worked out to test our predictions using trapped cold atomic systems.

Our work has been extended by coupling the edge state of a spin Hall insulator to various forms of environments in the presence of circularly polarized light. A Lindblad type equation is developed to determine the fermion occupation of the Floquet bands. We find by using analytical and numerical methods that 2-photon transitions lead to a mixing of the band occupations, hence the light induced photocurrent is in general not perfectly quantized in the presence of finite coupling to the environment, although deviations are small in the adiabatic limit.

Inspired by the prospect to use classical electromagnetic fields in engineering the topological properties of matter using the Floquet theory, we have investigate the effect of quantized electromagnetic fields by focusing on the quantized light-matter interaction on the edge state of a quantum spin-Hall insulator. A Dicke-type superradiant phase transition occurs at arbitrary weak coupling, the electronic spectrum acquires a finite gap and the resulting ground state manifold is topological with Chern number ± 1. When the total number of excitations is conserved, a photocurrent is generated along the edge, being pseudo-quantized in the low frequency limit, and decaying with the inverse frequency for high frequencies. The photon spectral function exhibits a clean Goldstone mode, a Higgs like collective mode at the optical gap and the polariton continuum.

In a closed ring, penetrated by a magnetic flux, persistent currents can flow. We have investigated the persistent current in strictly one-dimensional Dirac systems within two different models, defined in the continuum and on a lattice, respectively. We have mainly focused on the effect of a single magnetic or nonmagnetic impurity in the two systems. In the continuum Dirac model, an analytical expression for the persistent current flowing along a ring with a single delta-like magnetic impurity was obtained after regularization of the unbounded negative energy states. The predicted decay of the persistent current agreed with the lattice simulations. The results were generalized to finite temperatures. To realize a single Dirac massless fermion, the lattice model breaks the time-reversal symmetry, and, in contrast with the continuum model, a pointlike nonmagnetic impurity can lead to a decay in the persistent current.

Dynamical phase transitions (DPTs) occur after quenching some global parameters in quantum systems, and are signalled by the nonanalytical time evolution of the dynamical free energy, which is calculated from the Loschmidt overlap between the initial and time evolved states. Recently, it was suggested that DPTs are closely related to equilibrium phase transitions (EPTs) for the transverse field Ising model. By studying a minimal model, the XY chain in a transverse magnetic field, we show analytically that this connection does not hold generally. We present examples where DPT occurs without crossing any equilibrium critical lines by the quench, and a nontrivial example with no DPT but crossing a critical line by the quench. Although the nonanalyticities of the dynamical free energy on the real time axis do not indicate the presence or absence of an EPT, the structure of Fisher lines for complex times reveals a qualitative difference.

Topology and non-equilibrium dynamics are two vividly investigated fields of physics, with no strong bonds between them. We have studied the non-equilibrium time evolution of a variety of one and two dimensional systems (including the famous SSH model, Kitaev-chain, Haldane model, p+ip superconductor, etc.) following a sudden quench of some parameters. We prove analitically that topology-changing quenches are always followed by non-analytical temporal behaviour of return rates (logarithm of the Loschmidt echo), referred to as DPTs in the literature. Similarly to edge states in topological insulators, DPTs can be classified as being topologically protected or not. In 1D systems the number of topologically protected non-equilibrium time scales are determined by the difference between the initial and final winding numbers, while in 2D no such relation exists for the Chern numbers. The singularities of dynamical free energy in the 2D case are qualitatively different from those of the 1D case, the cusps appear only in the first time derivative.

2. Luttinger liquids: We have investigated a finite duration interaction quench in a one dimensional interacting electron gas, i.e. in a Luttinger liquid (LL). The statistics of work done on the system contains features related to both an adiabatic process and a sudden quench, and reveals several universal regimes. Our analytical results were benchmarked against numerically exact simulations, thus extending the LL universality class to the non-equilibrium situation as well.

We have determined the Loschmidt echo, the overlap of the initial and final wavefunctions of a Luttinger liquid after an interaction quench. It was found to decay exponentially with system size and exhibits universal behaviour: the steady state exponent after quenching back and forth n-times between 2 LLs (bang-bang protocol) is 2n-times bigger than that of the adiabatic overlap, and depends only on the initial and final LL parameters. These analytical results on the Luttinger model were benchmarked by numerically exact results on the XXZ Heisenberg model, using matrix product state based methods. We have also proposed an experimental setup consisting of a hybrid system containing cold atoms and a flux qubit coupled to a Feshbach resonance to measure the Loschmidt echo using rf spectroscopy or Ramsey interferometry.

In addition to global quenches, local ones are also able to reveal subtle structures in materials. We have investigated the Loschmidt echo of Luttinger liquids after a spatially inhomogeneous interaction quench. In studying the Luttinger model, we obtained an analytic solution of the bosonic Bogoliubov-de Gennes equations after quenching the interactions within a finite spatial region. As opposed to the power law temporal decay following a potential quench, the interaction quench in the Luttinger model leads to a finite, hardly time dependent overlap, therefore no orthogonality catastrophe occurs. The steady state value of the Loschmidt echo after a sudden inhomogeneous quench is the square of the respective adiabatic overlaps. Our results were checked and validated numerically on the XXZ Heisenberg chain.

Recently, work statistics performed by a non-adiabatic process has received considerable attention due to its relevance in non-equilibrium statistical mechanics, cold atomic systems etc. Following our previous works, we have studied the escort probability distribution function of work done during an interaction quantum quench of Luttinger liquids, which parallels remotely to ideas proposed by Tsallis in the field of non-extensive statistical mechanics. The resulting escort probability distribution function of work done crosses over from the thermodynamic to the small system limit with increasing escort parameter. From its characteristic function, the diagonal Rényi entropies and the many body inverse participation ratio (IPR) are determined to evaluate the information content of the time evolved wavefunction in terms of the eigenstates of the final Hamiltonian. The hierarchy of overlaps is dominated by that of the ground states. The IPR exhibits a crossover from Gaussian to power law decay with increasing interaction quench parameter.

Our previous results were extended to finite temperatures. To this end, we have studied the non-equilibrium dynamics of the Luttinger model after a quantum quench, when the initial state is a finite temperature thermal equilibrium state. The diagonal elements of the density matrix in the steady state show thermal features for high temperature initial states only, otherwise retain highly non-thermal character. The time evolution of Uhlmann fidelity, which measures the distance between the time evolved and initial states, is evaluated for arbitrary initial temperatures and quench protocols. In the long time limit, the overlap between the time evolved and initial system decreases exponentially with the temperature with a universal prefactor. Within perturbation theory, the statistics of final total energy and work are numerically evaluated in the case of a sudden quench, which yield identical distributions at zero temperature. In both statistics, temperature effects are more significant in small systems. At non-zero initial temperatures, the Dirac-delta peak at the adiabatic ground state energy difference remains present in the probability distribution of the total energy, but disappears from the work distribution.

In general, LLs arise by coupling left- and right-moving particles through interactions in one dimension. This most natural partitioning of LLs was investigated by the momentum-space entanglement after a quantum quench using analytical and numerical methods. We showed that the momentum-space entanglement spectrum of a LL possesses many universal features both in equilibrium and after a quantum quench. The largest entanglement eigenvalue is identical to the Loschmidt echo, i.e. the overlap of the disentangled and final wavefunctions of the system. The second largest eigenvalue is the overlap of the first excited state of the disentangled system with zero total momentum and the final wavefunction. The entanglement gap is universal both in equilibrium and after a quantum quench. The momentum-space entanglement entropy is always extensive and saturates fast to a time independent value after the quench, in sharp contrast to a spatial bipartitioning.

B. Two dimensional Dirac systems:

1. Graphene: We have investigated the effect of non-magnetic impurities on the local density of states and Friedel oscillations in graphene using analytical and numerical methods. Using the scale invariance of the Dirac equation, we have calculated the optical conductivity in a general two-band model. Interestingly, any two dimensional, scale invariant theory gives a featureless, constant optical conductivity, similarly to graphene.

We have studied the manipulation and movement of Dirac points in the Brillouin zone by the electron-electron interaction within leading order perturbation theory. When two linearly dispersing Dirac points meet at the merging point, an infinitesimal interaction was shown to cause opening of the gap or splitting of the Dirac points, depending on the inter- or intrasublattice nature of the merging and the sign of the interaction. The topology of the spectrum can therefore be efficiently changed by simply tuning the interaction between particles, as opposed to the usual careful band structure engineering. We have illustrated this general phenomena around the merging transition of one, two, and three dimensional Dirac-Weyl fermions. We have also studied a simple Weyl-like Hamiltonian that describes the quadratic band-crossing in three dimensions, and its stability under interactions was addressed.

We have continued our investigation of the generalized spin-S Dirac equation. Several lattices, such as the dice or the Lieb lattice, possess Dirac cones and a flat band crossing the Dirac point, whose effective model is the pseudospin-1 Dirac-Weyl equation. We investigated the fate of the flat band in the presence of disorder by focusing on the density of states (DOS) and dc conductivity. While the central hub-site does not reveal the presence of the flat band, the sublattice resolved DOS on the non-central sites exhibits a narrow peak, whose width was very sensitive to the strength of disorder. Although the group velocity is zero on the flat band, the dc conductivity is strongly enhanced and eventually diverges with decreasing disorder due to interband transitions around the band touching point between the propagating and the flat band. Generalizations to higher pseudospin are given.

2. Topological insulators: We have investigated the fate of a generalized quadratic band crossing in the presence of electron-electron interactions. A single valley quadratic band crossing in two dimensions is known to have a generic instability towards a quantum anomalous Hall (QAH) ground state for infinitesimal repulsive interactions, which is protected by time reversal and rotational symmetries, as was demonstrated for bilayer graphene and other systems theoretically. We introduce a generalization of a quadratic band crossing which is protected only by rotational symmetry. By focusing on the representative case of a parabolic and flat band touching, which also allows for a straightforward lattice realization using the so-called dice or T_3 lattice, the interaction induced nematic phase was found to become the dominant instability in certain parts of the phase diagram already at weak coupling, by competing successfully with the QAH insulator. The full phase diagram of the model, together with its topological properties, was mapped out using a perturbative renormalization group, strong coupling analysis, the mean-field theory. Interestingly, the Berry flux varies continuously in the single flavour limit with various control parameters.

A single valley quadratic band crossing in two dimensions was investigated in the presence of long range interactions. Unlike when only short range interactions are present, both nematic and quantum anomalous Hall states appear at weak interactions, separated by a narrow coexistence region, whose boundaries mark second and third order quantum phase transitions. After an interaction quench, the QAH order exhibits three distinct regions: persistent or damped oscillations and exponential decay to zero. In contrast, the nematic order always reaches a non-zero stationary value through power law damped oscillations, due to the interplay of the symmetry of the interaction and the specific topology of the quadratic band touching.

C. Three dimensional Weyl and other systems:

1. The non-equilibrium dynamics beyond linear response of Weyl semimetals is studied after a sudden switching on of a DC electric field. The resulting current is a nonmonotonic function of time, with an initial quick increase of polarization current followed by a power-law decay. Particle-hole creation à la Schwinger dominates for long times when the conduction current takes over the leading role, with the total current increasing again. The conductivity estimated from a dynamical calculation within a Drude picture agrees with the one obtained from Kubo's formula. The full distribution function of electron-hole pairs changes from Poissonian for short perturbations to a Gaussian in the long perturbation (Landau-Zener) regime. The vacuum persistence probability of high energy physics manifests itself in a finite probability of no pair creation and no induced current at all times.

The hyperfine interaction between nuclear spins and Weyl fermions was investigated. Since the density of states in Weyl semimetals varies with the square of the energy around the Weyl point, a naive power counting predicts a $1/T_1T \sim E^4$ scaling with E the maximum of temperature (T) and chemical potential. By carefully investigating the hyperfine interaction between nuclear spins and Weyl fermions, we find that while its spin part behaves conventionally, its orbital part diverges unusually with the inverse of energy around the Weyl point. Consequently, the nuclear spin relaxation rate scales in a graphene like manner as $1/T_1T \sim E^2 \ln(E/\omega_0)$ with ω_0 the nuclear Larmor frequency. This allows us to identify an effective hyperfine coupling constant, which is tunable by gating or doping, which is relevant for decoherence effect in spintronics devices and double quantum dots where hyperfine coupling is the dominant source of spin-blockade lifting.

The dynamic spin susceptibility (DSS) has a ubiquitous Lorentzian form in conventional materials with weak spin orbit coupling, whose spectral width characterizes the spin relaxation rate. We show that DSS has an unusual non-Lorentzian form in topological insulators, which are characterized by strong SOC. At zero temperature, the high frequency part of DSS is universal and increases in certain directions as ω^{d-1} with $d = 2$ and 3 for surface states and Weyl semimetals, respectively, while for helical edge states, the interactions renormalize the exponent as $d = 2K - 1$ with K the Luttinger-liquid parameter. As a result, spin relaxation rate cannot be deduced from the DSS in contrast to the case of usual metals, which follows from the strongly entangled spin and charge degrees of freedom in these systems. These parallel with the optical conductivity of neutral graphene.

2 .Topological excitations keep fascinating physicists since many decades in various fields of physics. While individual vortices and solitons emerge and have been observed in many areas of physics, their most intriguing higher dimensional topological relatives, skyrmions (smooth, topologically stable textures) and magnetic monopoles, which emerge almost necessarily in any grand unified theory and responsible for charge quantization, remained mostly elusive. We have proposed that loading a three-component nematic superfluid such as ^{23}Na into a deep optical lattice, a Mott insulating core is created at the center of the trap, which allows for creating and manipulating topologically stable skyrmion textures in its surface and investigate their properties in detail. We show furthermore that the spectrum of the excitations of the superfluid and their quantum numbers change dramatically in the presence of the skyrmion, and they reflect the presence of a trapped monopole, as imposed by the skyrmion's topology.

Other: Last but not least, B. Dóra has prepared and successfully defended his DSc dissertation to the Hungarian Academy of Sciences with the title "Non-equilibrium dynamics of low dimensional quantum systems". The topics of the dissertation overlap significantly with the work related to the present OTKA grant.